Physics of Nanoscale Devices
Prof. Vishvendra Singh Poonia
Department of Electronics and Communication Engineering
Indian Institute of Technology, Roorkee

Lecture - 22
Modes - II

Hello everyone as all of you are aware that we are discussing the idea of Modes in a device and last couple of lectures were on introducing the modes and how to derive the expression for the modes in a device. So, today we will go a bit more into the details of this new notion the notion of modes.

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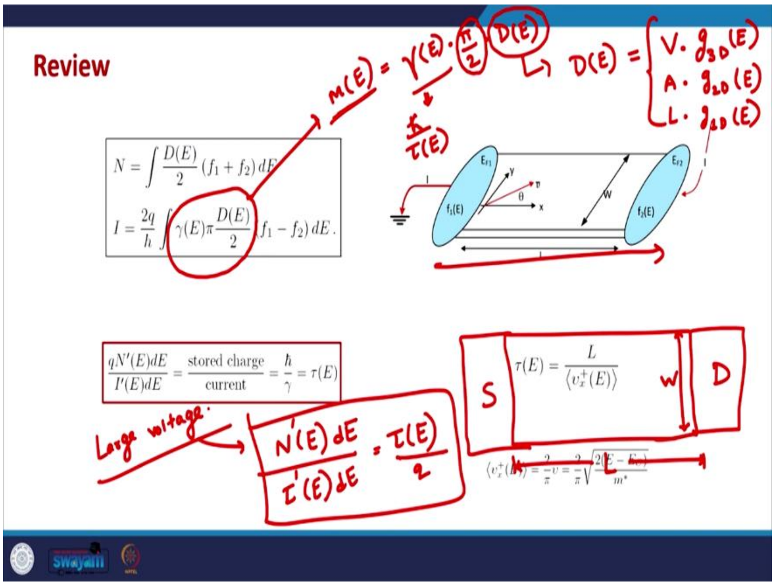

And as we are aware that this quantity in the current expression this quantity which we first saw in the current expression is defined as the modes and it depends on primarily two parameters.

One is the gamma E and second is the quantity D (E) and then we also have a constant pi by 2 in the expression for the modes. So, this D(E) is essentially the number of electronic states per unit energy in the device and it is defined as. So, if we collectively define it as for 3D, 2D and 1D this is defined as volume times $g_{3D}(E)$ where $g_{3D}(E)$ is the density of states for the 3D material area times $g_{2D}(E)$, $g_{2D}(E)$ is the density of states for the 2D material and length times $g_{1D}(E)$.

Now, we also saw that in order to derive or in order to actually see how this expression, how this quantity looks like we need to know what is the γ(E) in the device because we already know what is this D(E) parameter we just need to know the γ(E) parameter. And in order to calculate this γ(E) parameter we started with description of a 2D channel this is how it looks like this is just a slanted view of the channel.

We can also sort of take it to be like this we have a source we have a drain and we have a 2D channel there is a length and we also have width, which is denoted as W. In this kind of setup we saw that in order to calculate γ(E) or in other words in order to calculate τ(E) which is essentially $\frac{\hbar}{\gamma(\mathrm{E})}$, we need to find the ratio between the stored charge in steady state divided by the current.

So, the ratio of the stored charge and current this ratio is essentially the parameter tau E in our description in the previous class we had q in the denominator of the right hand side ok this is where this is what we had. And please also keep in mind that this was true when this was true in the case of a thought experiment that we did and in our thought experiment we applied a large voltage across the device.

So, across the device near equilibrium conditions are not true, but in the contacts the near equilibrium conditions are still true because the contacts are bulk contacts, large contacts and lot of electrons are available there the number of electrons is huge.

So, any application of voltage quickly brings or even after any application of voltage the electronic population is quickly in the equilibrium state because of the inelastic scattering in the contacts. So, this is what we had and after this what we saw was that in a ballistic case. Ballistic means when the electron is starting from the source terminal going directly to the drain terminal.

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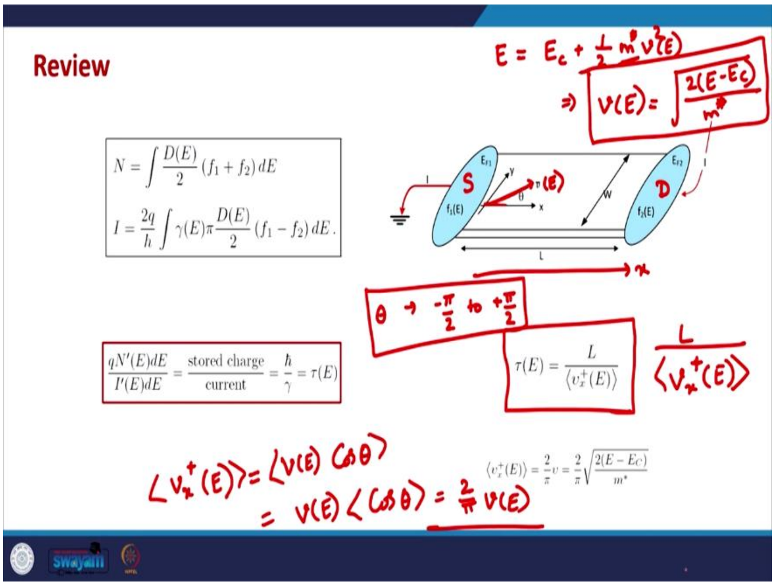

In that case this expression τ(E) actually turns out to be $\frac{L}{\langle v_x^+(E) \rangle}$ means velocity of electrons in x direction and positive means positive x direction, positive x direction is the direction from the source to the drain ok.

And average of this, average velocity of electrons in the positive x direction in the device that is what we saw and generally in order to calculate this average $v_x^+(E)$ or I would say in order to calculate $v_x^+(E)$. $v_x^+(E)$ is the velocity of an arbitrary electron that is starting from the source towards the drain.

And in a 2D channel we can assume that the electron is starting at an arbitrary angle θ with the x axis and this theta can range from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ this is the range of these θ angles v_x^+. So, the velocity if the electron is starting with velocity v_x^+ the velocity in the positive x direction will be v(E)cos θ ok. And this v(E) actually comes from the relationship of the energy of electron with the total energy of the electron with the kinetic energy of the electron.

So, which means that this v(E) is equal to please keep in mind that generally when we use when we treat electron as a classic like a classical particle or like a particle we need to have the parabolic band structure, which means the E k relationship should be a parabolic relation E should be equal to $E_c + \frac{\hbar^2 k^2}{2m^*}$ and we need to have instead of electron mass we need to have the effective mass.

So, this v(E) will be $\sqrt{\frac{2(E-E_c)}{m^*}}$ ok and the average value of this would be the average value of this ultimately this turns out to be the average value of cos theta which is as we saw in the last class is $\frac{2}{\pi}$v(E).

So, this is what we have now and where v(E) is given as by this expression ok. So, this is the ballistic case and which means that electrons are not dissipating electrons are not changing any pathway during their transport from the source to the drain ok.

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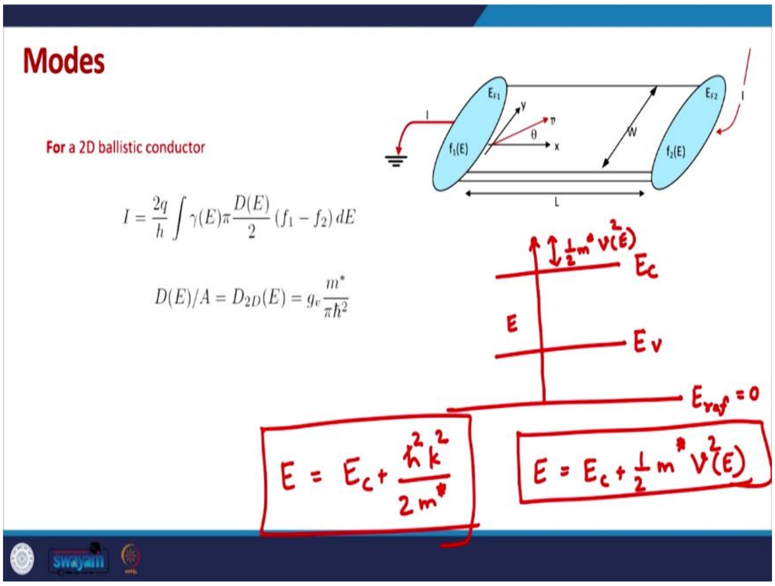

So, now with this we can see just one thing that needs to be kept in mind is that generally when we deal with electrons we have valence band we have the conduction band and if the electron is sitting somewhere here that and if this is the reference energy level let us say the E reference is somewhere below the valence band, which is the reference energy level is generally taken to be the zero energy.

Then the total energy of the electrons sitting at this energy level will be E and this fraction of this energy will be the kinetic energy. So, which means that E is written as $E_c + \frac{v(E)^2}{2m^*}$ ok also as we saw in our previous discussions that in the conduction band the E k relationship can be written as E equals $E_c + \frac{\hbar^2 k^2}{2m^*}$ ok.

So, now we are in a position to explicitly calculate what is the modes number of modes in a device and. So, please keep these two expressions in mind generally near the top sorry near the bottom of the conduction band the bands are parabolic and this relationship holds and the amount of energy that is in access to the bottom of the conduction band is the kinetic energy of the electron and that is given with this relationship ok.

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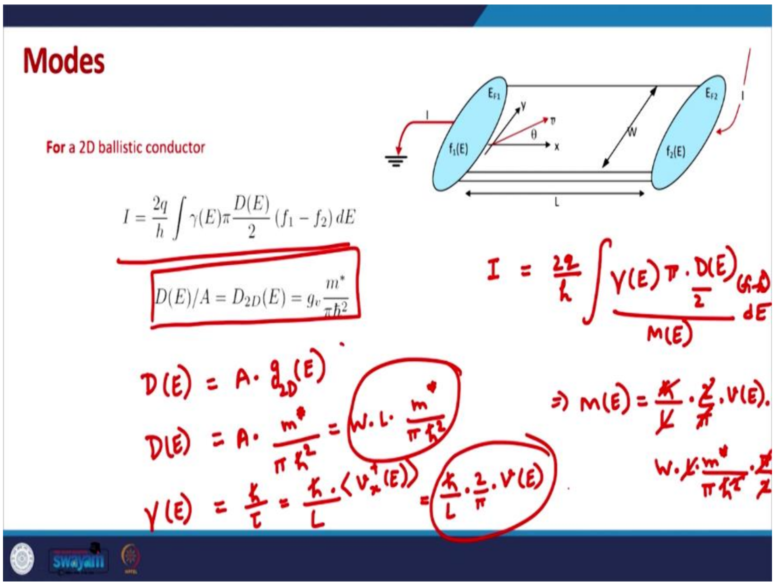

So, now let us come back to the expression for the current and that is how it looks like the current in the device any arbitrary device looks like this, this is the integration of $\frac{2q}{h}\int\frac{\gamma(E)\pi D(E)}{2}(f_1 - f_2)dE$. So, this factor M(E) this is the M(E) and a two terminal and in a 2D device this D (E) is given as in a 2D device D(E) is area times g the density of states in 2D device g_{2D}(E) which means area times $\frac{m^*}{\pi\hbar^2}$.

And γ is given as $\frac{\hbar}{\tau}$ which is essentially $\frac{\hbar}{L}v_x^+(E)$ which is equal to $\frac{\hbar}{L}\frac{2}{\pi}v_x^+(E)$. So, if we put this value of γ(E) and this value of this value of D(E) which is essentially W times L times $\frac{m^*}{\pi\hbar^2}$. So, this value of D(E) it ultimately gives us the number of modes to be $\frac{\hbar}{L}\frac{2}{\pi}v_x^+(E)$ times WL $\frac{m^*}{\pi\hbar^2}$ times $\frac{\pi}{2}$.

So, this $\frac{\pi}{2}$ and $\frac{2}{\pi}$ cancels, L L, $\hbar$ $\hbar$. So, what is left is something like this ultimately the modes can be written as if we sort of eliminate all these redundant things from here.

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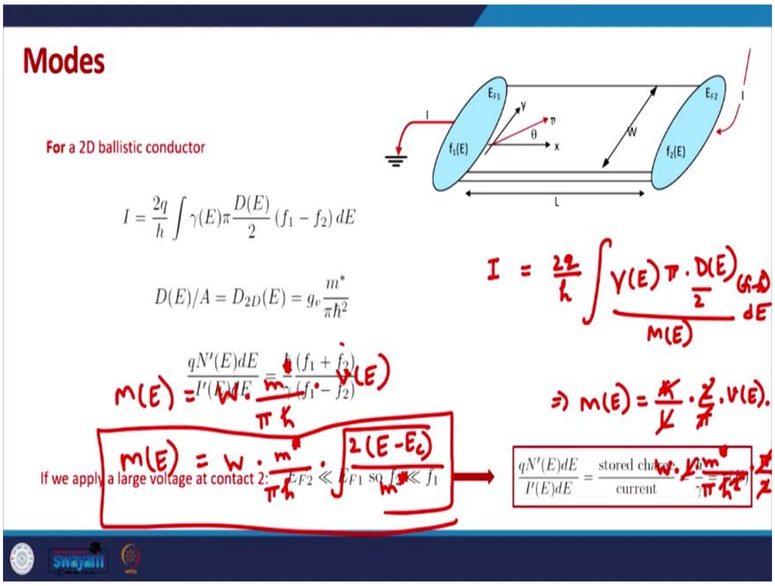

It is M(E) equals what is left here is $W\frac{m^*}{\pi\hbar}$v(E). v(E) is essentially $\sqrt{\frac{2(E-E_C)}{m^*}}$.

So, this is the number of modes in a 2D channel and as we discussed in the last class that number of modes in a channel is essentially the number of conduction pathways in the channel that are there and number of conduction pathways means that these are sort of lanes in the channel that are available for electrons to conduct essentially ok. So, this is what we have this is for the 2D channel and this is all we have seen.

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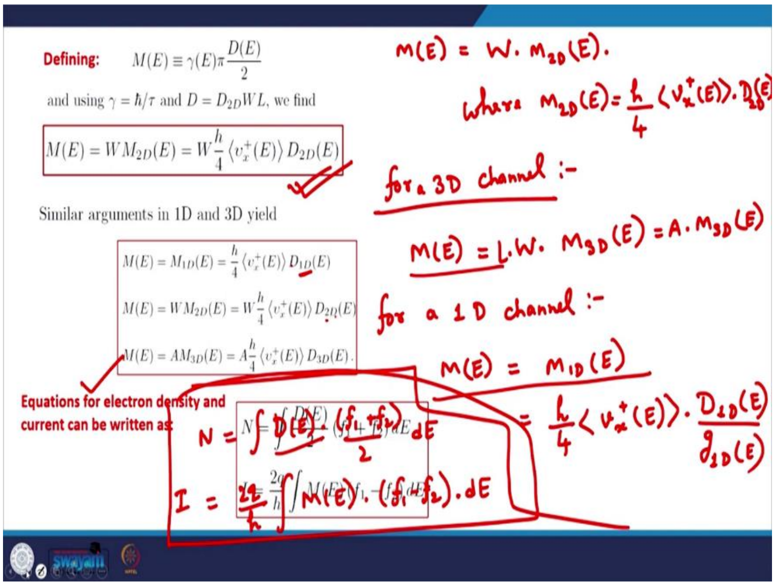

Finally, this is another way of writing the number of modes in a 2D channel like this.

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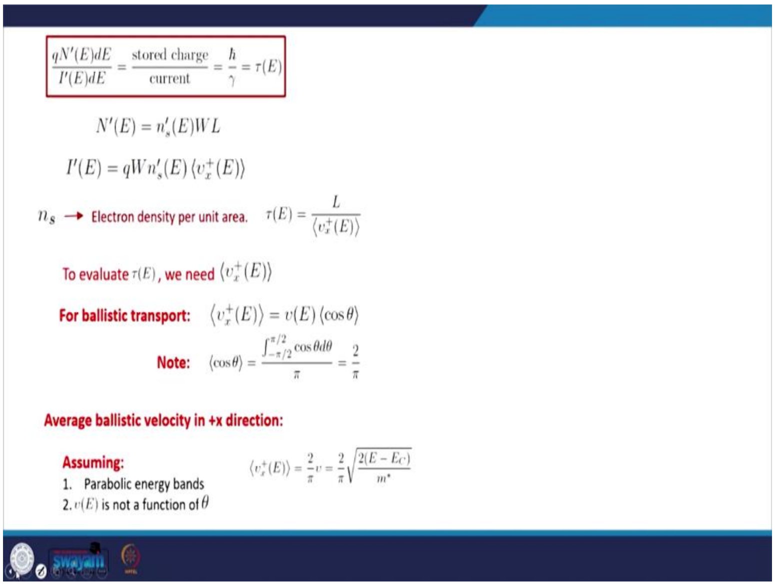

So, this expression for the modes W times $\frac{m^*}{\pi\hbar}v(E)$ this is equivalent to that expression.

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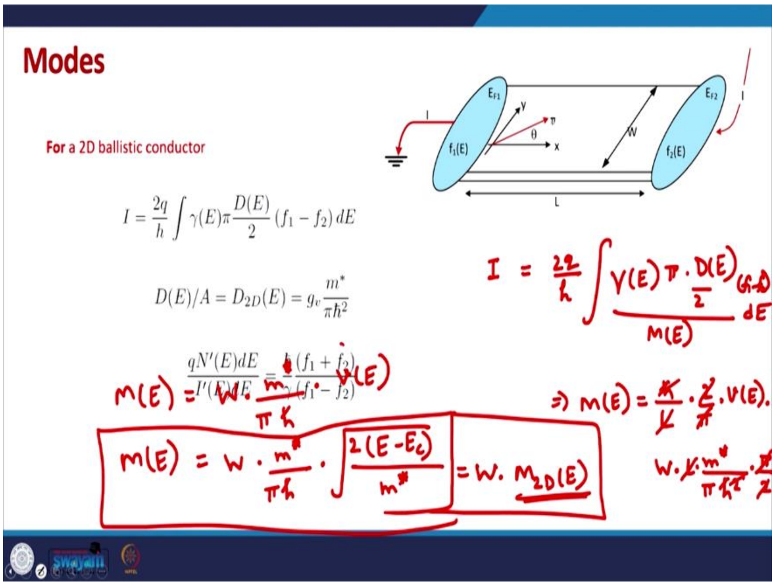

And that is also written as W times $M_{2D}(E)$ number of modes in a basically $M_{2D}(E)$ is the number of modes per unit width of the channel. So, this is this was the case with the 2D channel and this expression is exactly equivalent to this expression.

So, number of modes is in a 2D channel is in a more formal way it can be written as number of modes times W times number of modes per unit width where number of modes per unit width is h by 4 average velocity of electrons in the positive x direction times $D_{2D}(E)$ ok. Similarly the similar expression will hold true for 3D channel as well.

So, for a 3D channel total number of modes in the channel would be W times $M_{3D}(E)$ and here in addition to W we will also have length or we will have area times $M_{3D}(E)$ where this $M_{3D}(E)$ is the number of modes per unit area. And for a 1D channel total number of modes in the channel would be just $M_{1D}(E)$ and that will be $\frac{h}{4} v_x^+(E)$ times $D_{1D}(E)$ where D is essentially equal to $g_{1D}(E)$ density of states in 1 dimensional channel.

So, this is what we finally, obtain. So, here D and g which means here D with subscript 1D is essentially the density of state in the 1D channel D subscript 2D is the density of states in 2D channel and D subscript 3D is density of states in a 3D channel ok.

So we are not explicitly deriving modes for 3D channel and 1D channel I would recommend all of you to try this derivation and this is pretty much similar to the derivation that we did for the 2D channel.

In 2D channel things are I would say more can be explicitly visualized and that is why we started with a 2D channel because the length and width we can easily picturize in our minds. So, finally, the way we can sum it up is that the electron density in the device in equilibrium or the total number of electrons in the device in equilibrium can be written as D(E) times $\frac{f_1+f_2}{2}$ dE where dE is the number of electronic states per unit energy in the channel.

And the current can be written as this constant $\frac{2q}{h}\int M(E)(f_1 - f_2)dE$ and where we now know explicitly the forms of D(E) and M(E), D(E) is the is comes from the density of states and M(E) comes from the modes in the channel.

So, this is essentially the idea of the modes in a channel now there is an interesting way of looking at the modes in the channel and that is what we will do now. So, if we sort of start with this expression .

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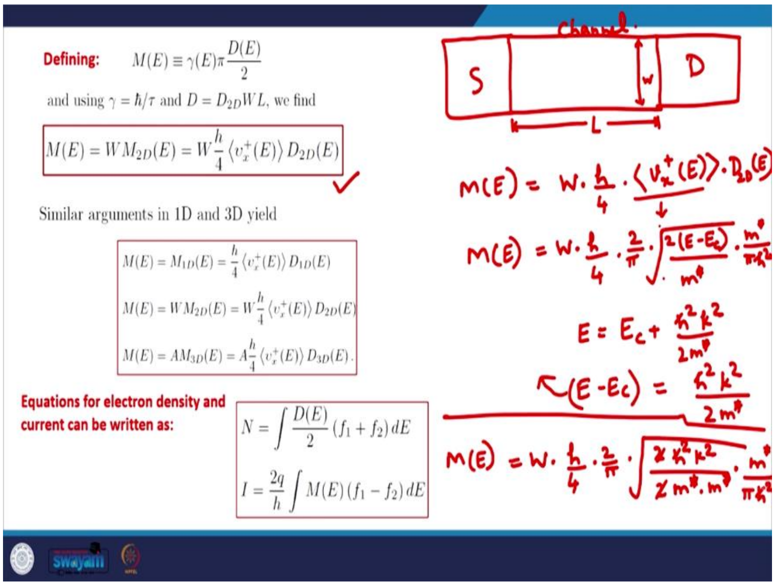

So, for example let us take the case of a 2D channel. So, the device our device looks like this, we have the length the width and the source contact and the drain contact this is the channel. The number of modes in the channel is given as from here W times $\frac{h}{4}$ average velocity in x direction $v_x^+(E)$ please work out these derivations on your own as well because until you will not understand that properly.

So, if we write this positive velocity in x direction the expression for this, this is $\frac{2}{\pi}\sqrt{\frac{2(E-E_C)}{m^*}}$ and this $D_{2D}(E)$ is $\frac{m^*}{\pi\hbar^2}$. So, putting everything together we have M(E) number of modes in a 2D device is W times $\frac{h}{4}$ times this. Now if we assume or I will not say assume because that is generally the case in most of the most of the devices near the conduction band we have parabolic relationship between E and k.

So, the E k relationship can be written as E equals $E_C + \frac{\hbar^2 k^2}{2m^*}$, where k is the wave number of the electrons which means E - Ec is essentially $\frac{\hbar^2 k^2}{2m^*}$.

So, if we put this value of E - E_C into this expression the expression of modes this will further simplify to W times $\frac{h}{4}$ times $\frac{2}{\pi}$ into E - $E_C$is $\frac{\hbar^2 k^2}{2m^*}$. So, 2, 2 will go away and here we have $\frac{m^*}{\pi\hbar^2}$ ok

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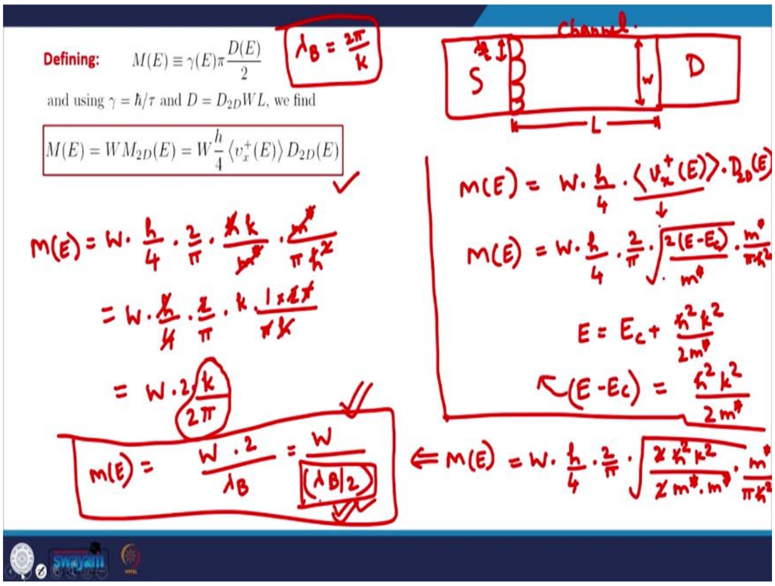

So, this is the expression for the number of modes in the 2D channel, this on further simplification becomes W times $\frac{h}{4}$ times $\frac{2}{\pi}$ now everybody comes out of the square root. So, it is $\frac{\hbar k}{m^*}\frac{m^*}{\pi\hbar^2}$. m^* and m^* go away and $\hbar$. Finally, what we have is this W times $\frac{h}{4}$ times $\frac{2}{\pi}$ into k into $\frac{1}{\pi}$ and h bar can be written as $\frac{\hbar}{2\pi}$.

So, this further cancels out π and π 2, 2 ok. So, what is left with us is W times k/π. So, if we divide and multiply by 2 this is what is. Now please recall from the basic quantum mechanics that according to the wave particle duality if the wave number of an electron is k its de Broglie wavelength is λ_B is 2π/k ok. So, if we have this here it becomes W times two divided by λ_B or W divided by $\frac{\lambda_B}{2}$.

So, this is actually the number of modes in a 2D device, in a 2 dimensional device number of modes in a 2 dimensional device is actually width divided by the half de Broglie wavelength of the electrons and now this is an interesting result. What it says is that the number of modes in a device is essentially the number of half wavelengths that can fit into the width of that device.

So, it essentially means that it is essentially this number of half wavelengths where this is $\frac{\lambda_B}{2}$. So, what it says is that the width of a conduction pathway or. So, to say width of a highway or a lane in the channel is essentially the de Broglie wavelength divided by 2 and that is why. So, what it says is that in order for an electron to go through from to travel through the device it needs at least $\frac{\lambda_B}{2}$ space.

So, it is not a point particle it is like a wave and in more classical sense if we treat electron classically it becomes a particle that needs $\frac{\lambda_B}{2}$ space while traveling through the channel and that is why the number of conduction pathways in the channel is width divided by $\frac{\lambda_B}{2}$ in the case of a 2D channel and this is this is an interesting and important intuitive result actually.

So, please keep this in mind because this makes the idea of modes more physically clear to us, why this idea is important and how this idea is different from the density of states. As you might recall that in density of states we have the number of allowed electronic states per unit energy per unit volume and those energy states are discrete energy states there is no broadening of energy levels, there is no sort of we do not say that electron is occupying a certain space.

But while electron travels because of the finite lifetime it leads to energy broadening of levels and that broadening is roughly equal to $\frac{\lambda_B}{2}$ which is the de Broglie wavelength of the electron or in other words electron behaves like a classical particle that needs at least or that needs space of around $\frac{\lambda_B}{2}$ while travelling through the device.

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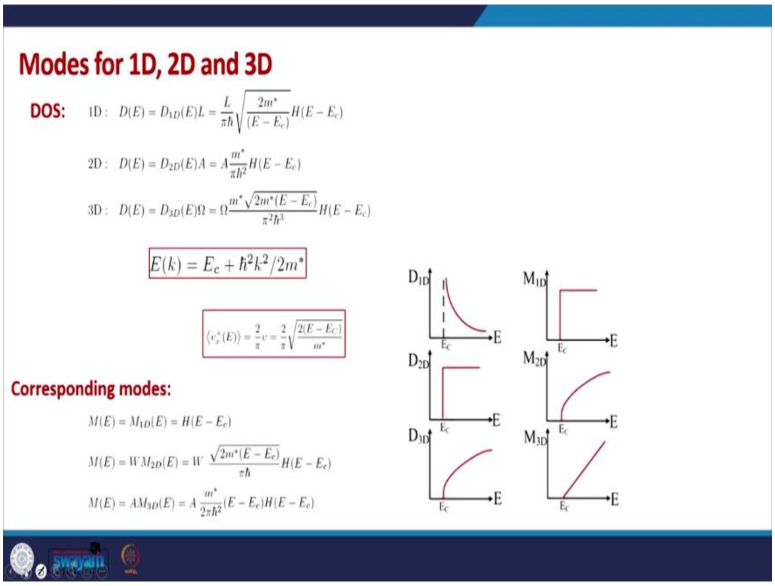

So this is an important intuitive result. And finally, this is how we can summarize our results. These expressions are the expression for the density of states in the device and these expressions are the expressions for the number of modes in the device. And where we have this parabolic band assumption which is not an unfair assumption in most of the cases and the velocity or more precisely the average velocity of electrons is given by this expression.

So, we will discuss about this in somewhat more detail in the next class and until then I will let you think about the I will let you think more about the intuitive understanding of modes in a device. So, thank you for your attention, see you in the next class.