Physics of Nanoscale Devices
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Lecture - 21
Modes - I

Hello everyone. Welcome back and as you are aware we are discussing about the modes in a mesoscopic device and in our previous class we started our discussion from these two expressions.

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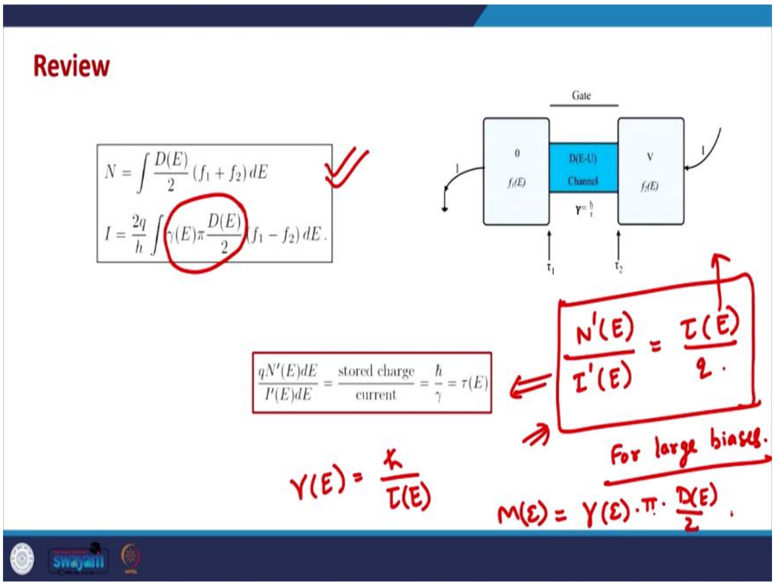

The expression for the steady state number of electrons in the channel and the steady state current and in order to sort of understand this term in better detail, we did sort of a thought experiment or we did a virtual experiment in which we took the ratio of the 2 quantities that is the number of electrons at a certain energy E with the current at that energy E ok.

So, that ratio is essentially turns out to be $\frac{\tau(E)}{q}$ which is also written here and this happens for large biases when the applied voltages are large.So for large biases this is what we obtain and this will in fact pay a way for us to calculate this characteristics time of the device this will let us calculate this parameter $\tau(E)$ and from that we can calculate the parameter $\gamma(E)$ which is essentially $\frac{\hbar}{\tau(E)}$. And from there we can calculate the number of modes which is given as $\gamma(E)$ times π times D(E)/2 ok. So, that is the sequence that we will follow.

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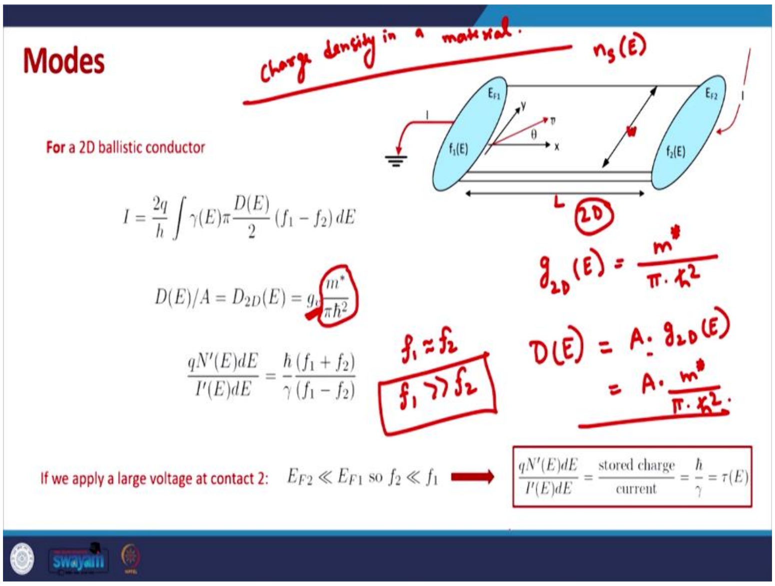

So, this we have already seen, we as a starting point we take a 2D channel and the 2D channel is taken just for the sake of better visualization because we can visualize a 2D material in a much better way. So, the channel has length L it has a width W and the density of states in the 2D channel you might recall is given as $\frac{m^*}{\pi\hbar^2}$.

So, in a 2D channel this parameter D(E) which is essentially the number of electronic states per unit energy will be area times g_{2D}(E) divided which will be area times $\frac{m^*}{\pi\hbar^2}$. So, in this expression we also in addition to this term we also have another term which is written as g_v it is known as the valley degeneracy.

So, if there are multiple in the energy band diagram there are multiple valleys at the same time at the same energy level. In that case multiple electrons can stay at the same energy level and that give rise to the degeneracy and that is accounted by g_v, but for our current discussion we can ignore this term g_v and we will discuss about this in more detail later on ok.

So, as we have seen that for low voltages f_1 is almost equal to f_2 and for high voltages f_1 is extremely larger than f_2 ok. And in that case this the ratio between the steady state number of electrons to the current becomes $\frac{\tau}{q}$ and q can be taken here.

Now, you might have also seen from experiments that from certain kind of experiments we can calculate the charge density in a material. So, for example, from the Hall experiment we can calculate the, Hall effect experiment we can calculate the number of charge carriers per unit area or per unit volume.

So, generally there is this parameter known as the charge density and in 2D material case this will be known as the sheet charge density represented as n_s and this is an experimental parameter that we can calculate in characterization experiments ok. And the charge density at a certain energy will be represented as n_s(E).

Now, for large bias limit we have this ratio between the number of electronic or number of electrons at energy E to the current at that energy E ok. So, if we try to see that in terms of.

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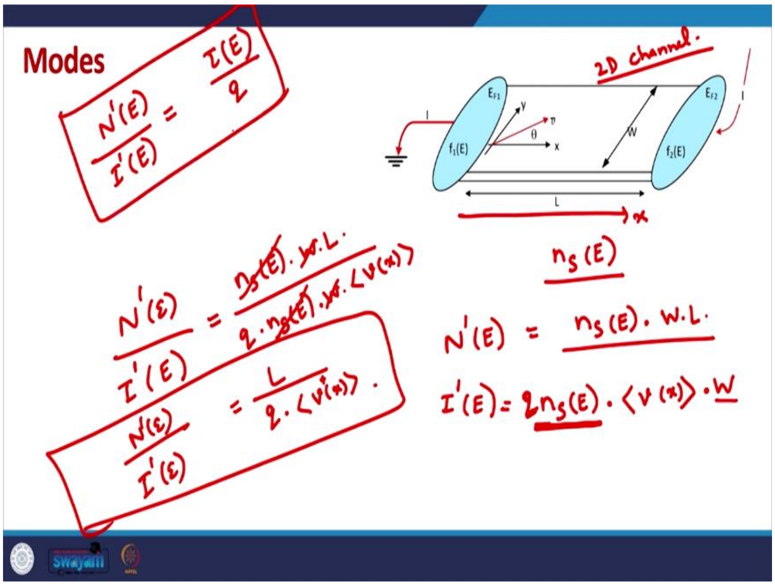

So, let us say if we have the charge density in the material n_s(E). So, in our 2D material in our 2D channel, let us say this is our 2D channel and in this channel somehow we are able to deduce the charge density to be n_s(E), then the number of electrons in steady state at the energy E will be the charge density at that energy times area and the area of a 2D material is just length times width ok.

So, this is a straight forward expression. Similarly, the steady state charge in the device can be represented as the charge density times the rate at which electrons are being eliminated from the channel or the rate at which electrons are getting across the channel and that is given by the average velocity of electron in x direction.

So, average velocity of electrons in x direction times width times q ok. So, this will be the steady state current in the device if the steady state charge density of the device is n_s(E) and this expression is quite obvious because WL times n_s(E) tells us about the number of electrons that are getting eliminated or number of electrons that exist in steady state, v_x tells us about number of electrons or the velocity of electron and that essentially is the rate at which electrons are getting swept through the device.

And if we multiply by width, it will give us the number of electrons getting across the device per unit second per unit time or per second and if we multiply that by q that will essentially give us the steady state current. So, now, these the ratio of these two terms $N'(E)/I'(E)$ is n_s(E) times W times L divided by q times n_s(E) times W times the average velocity of electrons in x direction.

So, n_s cancels n_s, W by W. So, what we are left with is L by q times v_x. So, this is what we obtain, had we not done any analysis or any quantum mechanical analysis any mathematical analysis. This is what we obtain from our experimental analysis actually.

If we do an experiment on the 2D channel and if we find out the steady state charge density or this sheet charge density in the 2D channel to be n_s, in that case the ratio of $N'(E)/I'(E)$ can be written as L by q times average of v_x.

Where v_x is the, I would better write it $< v_x^+ >$ is the velocity average velocity of electrons in positive x direction ok. And from our previous analysis we found out that the ratio of $N'(E)/I'(E)$ is essentially $\frac{\tau(E)}{q}$. So, if we equate these two expressions, if we equate these two expression one expression from our analytical calculations.

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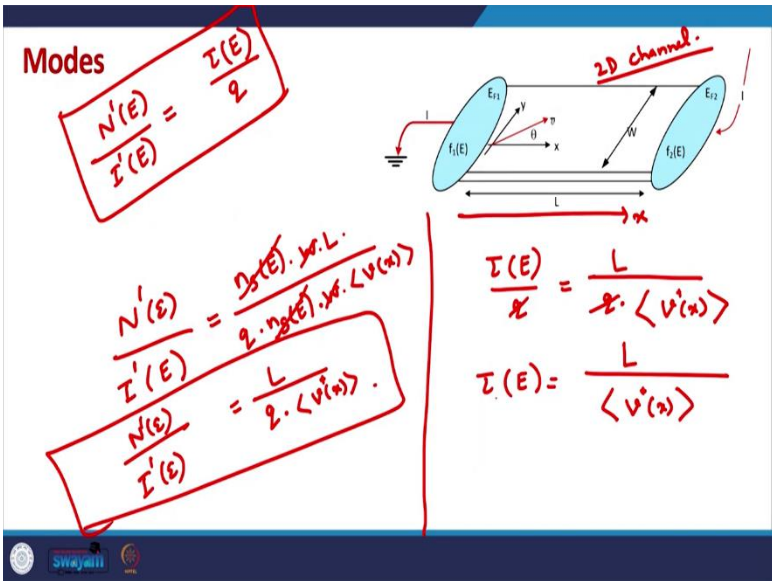

And second expression from our experimental understanding of the material. In that case we can write down $\frac{\tau(E)}{q}$ to be equal to $\frac{L}{q\langle v_x^+\rangle}$ and the transit time on an average will be or the transit time of the electron at energy level E will be $\frac{L}{\langle v_x^+\rangle}$.

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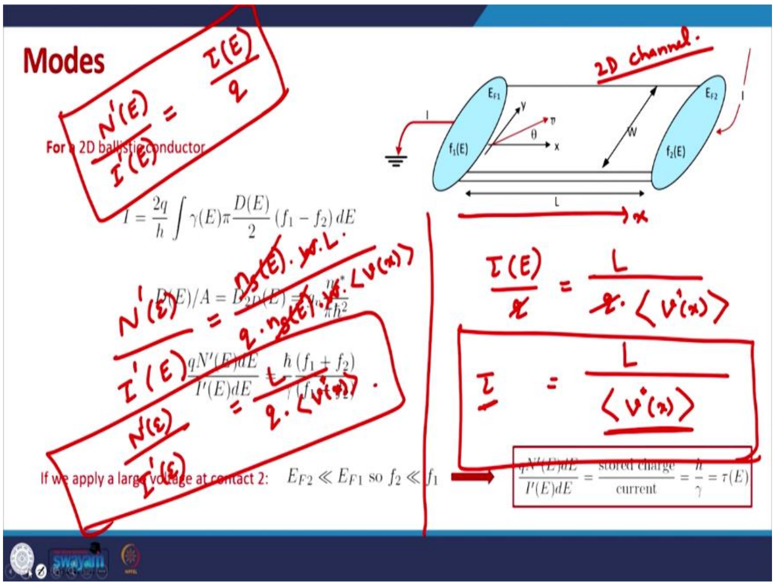

We can leave this parameter E and now this is the expression that we obtain for the transit time or for the characteristic time of the device ok. So, from this expression what we see is that, if we want to find out this characteristic time we need to calculate the average velocity of electrons in the positive x direction and that is what we will try to do in our next that is our next step so to say.

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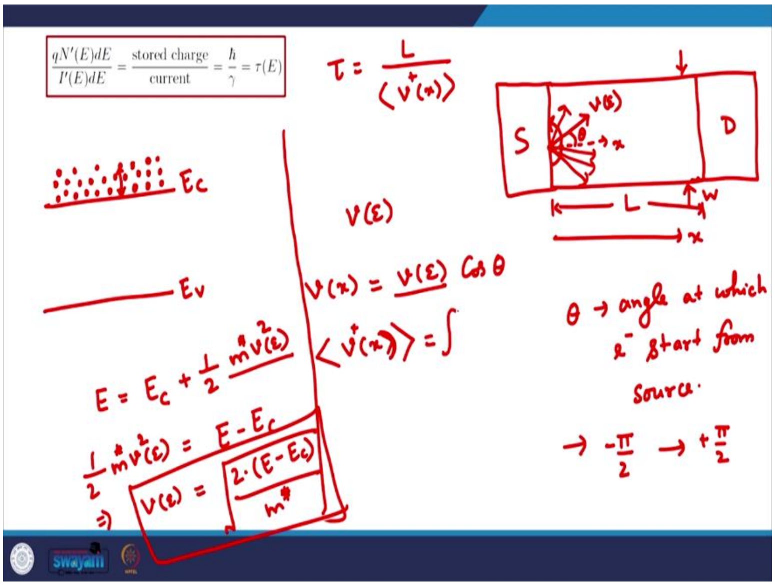

So, tau E is essentially or I would say τ is L by velocity in x direction average velocity in. So, in a 2D device in a device where the channel is 2D and this is in contact with the source terminal and the drain terminal, this is the length of the device, this is the width of the device.

Width of the device is W, the length of the device is L ok. So, in this case for a typical electron for any arbitrary electron it might and if this is the positive x direction the electron can start from the source at an angle θ from the x direction with velocity v(E) ok.

So, some electron will start let us say with in this direction, some electron will start in this direction. So, this will be the various possibilities that the electron will start from the source to the drain side. Ideally this θ the angle at which electrons start from the source is this can range from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

So, this can be from this angle to right up to this angle in positive x directions ok. So, if the velocity of the electron is v(E). Let us say v(E) is the velocity of electron in a direction which is at angle θ from the x direction, then the velocity in x direction will be v(E)cosθ ok. But, what is this velocity v(E) here?

What is the velocity with which electrons start from the source side? And here the band diagram picture will be slightly helpful for us. So, in the device if this is the top of the valence band, this is the bottom of the conduction band and in this case generally as you are aware that electrons that are present in the conduction band, only those electrons are involved in the current conduction in the devices. Mostly those electrons are transporting across the device ok.

So, the electrons present in the conduction band are generally traveling through the channel ok. For the electrons in the conduction band we can write down the energy of electrons in conduction band to be E equals Ec + $\frac{mv^2}{2}$. So, the energy that is in access to the conduction band edge is generally the kinetic energy of the electrons.

So, if electron is sitting here this much will be the kinetic energy of the electron and so $\frac{mv(E)^2}{2}$ will be equal to E - Ec. And this implies that v(E) is please keep in mind that m here is m* because since we are using classical analog of electron we need to use m* instead of m. So, v(E) will be equal to $\sqrt{\frac{2(E-E_C)}{m^*}}$.

This will be the average velocity of the electrons or this will be the velocity of the electron that starts from the source with energy E and the average velocity of the electron in x direction positive x direction will be; if we take the average that will be essentially. So, please keep in mind that for positive x direction the angle theta can be from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ so, if we take the average over all possible angles.

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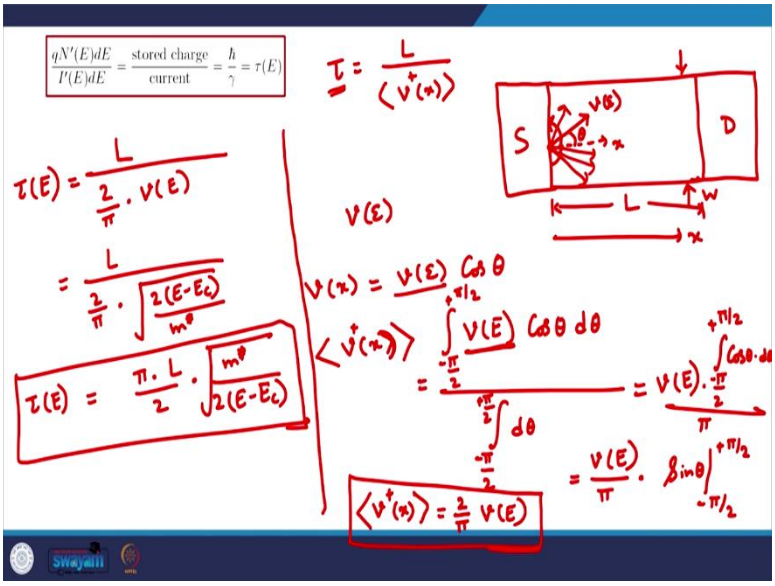

It will be essentially v(E)cosθdθ and this θ may range from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ divided by this average will be given as divided by dθ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. This will be the average velocity of electrons in positive x direction, because we are taking angles from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ and since v(E) the expression for v(E) does not involve any θ in it.

This can be taken out and the average velocity will be just from [$-\frac{\pi}{2}$ to $\frac{\pi}{2}$] cosθ dθ, the denominator will be just this will be θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$it will just be π . So, it will be $\frac{v(E)}{\pi}$ times the; if we integrate cosθ with θ it will be sinθ and the limits of the integration are from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

So, which will essentially sin$\frac{\pi}{2}$ is 1 and sin$\frac{-\pi}{2}$ is -1. So, that will be essentially 2. So, the average velocity will be finally, $\frac{2}{\pi}$ v(E). This will be the average velocity of electrons in x direction ok. So, from this as you might have seen, we can calculate the transit time as well.

So, the average velocity in positive x direction is $\frac{2}{\pi}$ into v(E). So, the transit time will be L divided by the average velocity of electrons in the positive x direction. So, if we write it down like this, this will be L divided by average velocity which is essentially $\frac{2}{\pi}$ into v(E) and v(E) if you recall is $\frac{2}{\pi}$, v (E) is s$\sqrt{\frac{2(E-E_C)}{m^*}}$.

So, this parameter τ will be $\frac{\pi L}{2}\sqrt{\frac{m^*}{2(E-E_C)}}$. So, that is essentially what it will be for $\tau(E)$ ok.. So, now we are in a situation to calculate this parameter gamma essentially. So, since now we know the transit time in terms of energy values and the device dimension we can now easily calculate the parameter γ.

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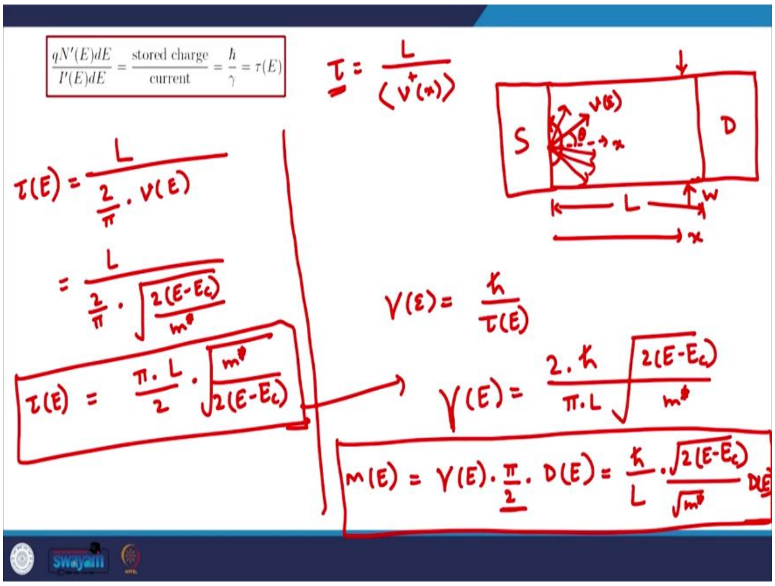

So, γ will be $\gamma(E)$ is $\frac{\hbar}{\tau(E)}$ and if we put this expression for $\tau(E)$ here, then $\gamma(E)$ will be equal to $\frac{2\hbar}{\pi L}\sqrt{\frac{2(E-E_C)}{m^*}}$ ok. So, this is the energy broadening in the channel, because of the finite lifetime of the electrons in the channel and as you might have predicted from this that now we can calculate the number of modes in the channel using this expression.

So, if you recall number of modes in the channel is $\frac{\gamma(E)\pi D(E)}{2}$ ok. So, if we put this value of $\gamma(E)$ here, it will be this $\frac{\pi}{2}$ will cancel by $\frac{2}{\pi}$, it will just be $\frac{\hbar}{L}\sqrt{\frac{2(E-E_C)}{m^*}}\ D(E)$..

Now, you might have also sort of understood why we use this factor of $\frac{\pi}{2}$ in the expression for modes, because finally, this $\frac{\pi}{2}$ gets cancelled with the $\frac{2}{\pi}$, which is there in the expression of γ(E).

So, this is how we obtain the expression for the number of modes in the channel from our analysis of the transit time in the device ok. So, although we started with a 2D channel, but this kind of treatment is fairly general and it can be generalized to 3D and 1D channels as well which we will see in the coming class.

So, I will let you think more about this expression until next class and please keep in mind all the approximations that we make while doing calculations. So, with this derivation of number of modes, we conclude this lecture and we will start from this point in the coming class.

Thank you for your attention, see you in the next class.