Physics of Nanoscale Devices
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Lecture - 20
General Mode of Transport, Modes

Hello everyone. As you are aware we are discussing the General Model of Transport of electronic devices and as we have already sort of mentioned this, as I have already mentioned is this before as well that the general model of transport is applicable to specially mesoscopic devices. It is not directly applicable to devices that are of atomic dimensions, but at the same time this is also not applicable to or generally not used in macroscopic devices.

So, the devices that we use nowadays in our systems are of mesoscopic dimension and that is where this model is applicable and we encountered a new concept called the concept of modes in devices and that is what we will be discussing today. So, let me quickly review what we have discussed so far.

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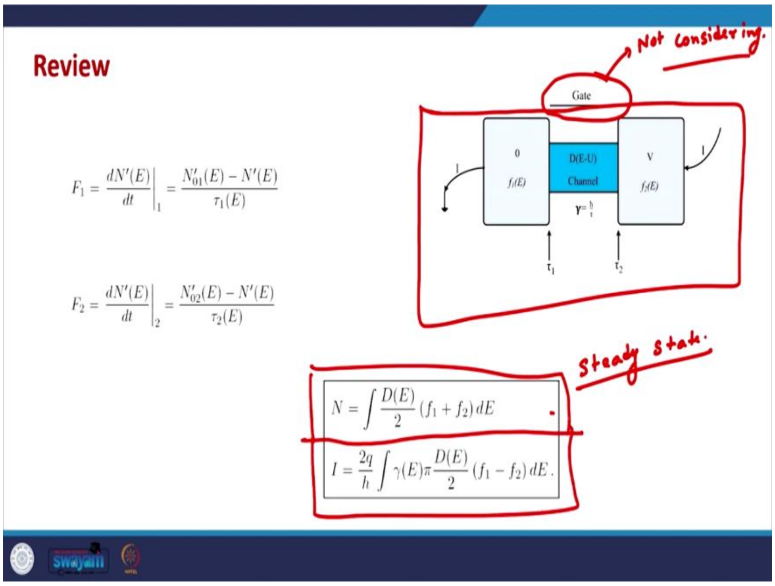

We saw that in a two-terminal device. So, we are taking a two-terminal device as the starting point to understand the transport in electronic system. So, which means that we are not considering this third terminal, this is not under consideration. So, we are not considering gate in the devices, we are just our device in discussion has a source, a drain and the channel region in between and from our analysis of the transporting devices we saw that in steady state. So, steady state is the state when the flow is constant.

So, when we have applied a battery across the device and the flow of electron and electrons and current is now constant. In steady state the number of electrons in the device is given by the this expression and the steady state current is given by this expression and these 2 are extremely important expressions.

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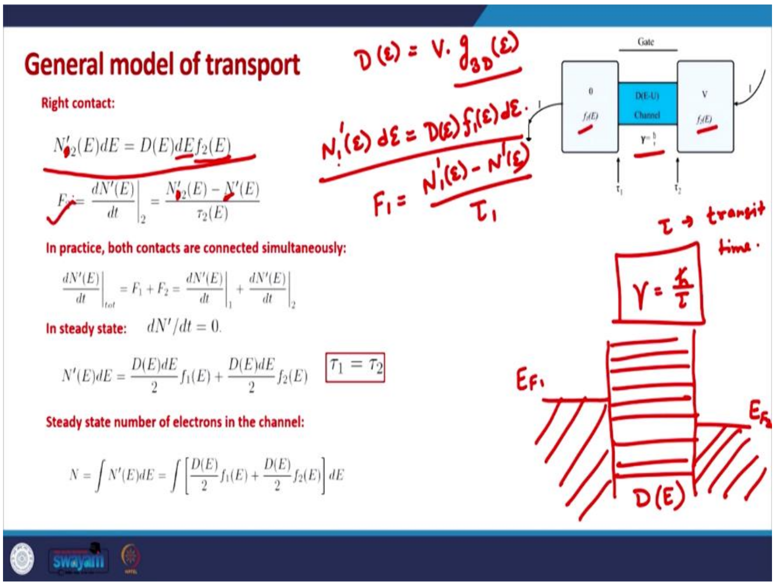

Let me quickly also go through how these were derived. So, in order to calculate the steady state number of electrons in the channel or in the device we use or what we need is, we need the density of states in the channel we need Fermi functions of the contacts and we need an additional parameter this parameter known as the transit time or the characteristics time of the channel and contact.

So, it is the time that the electron takes to jump from the contact and pass through the channel basically inverse of this time is known as the energy broadening of the levels in the channel.

So, this is represented by gamma parameter. So, the way it was , the way we obtained these expression the expressions for steady state number of electrons and current is we first considered when the left contact is putting, when the channel is put in contact with the source contact just with the source contact, the left side this way essentially.

So, we have these are the we have the left contact, the electronic states in the channel and what this contact will try to do is it will try to fill all the electronic states up to E_{F1} level and similarly the right contact in equilibrium will try to fill all the states up to E_{F2} level. So, E_{F2} is the Fermi level of the right contact E_{F1} is the Fermi level of the left contact and the density of states in the channel is represented by D(E) essentially and D(E) is actually in a 3D channel it is volume times $g_{3D}(E)$.

So, where $g_{3D}(E)$ is the density of states that we calculated in our previous discussions ok. So, we consider what happens when the channel is put in contact, put in touch with just the left contact. Then we consider what happens when the channel is put in touch with right contact only and the number of electrons in the channel when it is only in contact with the right contact.

In that case the number of electrons in equilibrium is represented by $N_2'(E)dE$ and that is given as the density of states of the channel times the Fermi function of the right contact times the energy range. And similarly, the number of electrons if the channel is just in contact with the left contact in equilibrium will be density of states Fermi function times dE and but when the channel is in between the 2 contacts both right and left contact both source.

And drain contacts in that case the number of electrons in the channel will be somewhere between N_1' and N_2'. So, it will be slightly lesser than N_1' and slightly higher than N_2' and that is what we did in our last class as well and if that is the number is $N'(E)$ then the rate of flow of electrons from the left contact will be F1 will be $\frac{N_1'(E)-N'(E)}{\tau_1}$ ok. And similarly, the rate of flow of electron from the right contact is this and in steady state both of these rates should be equal to 0.

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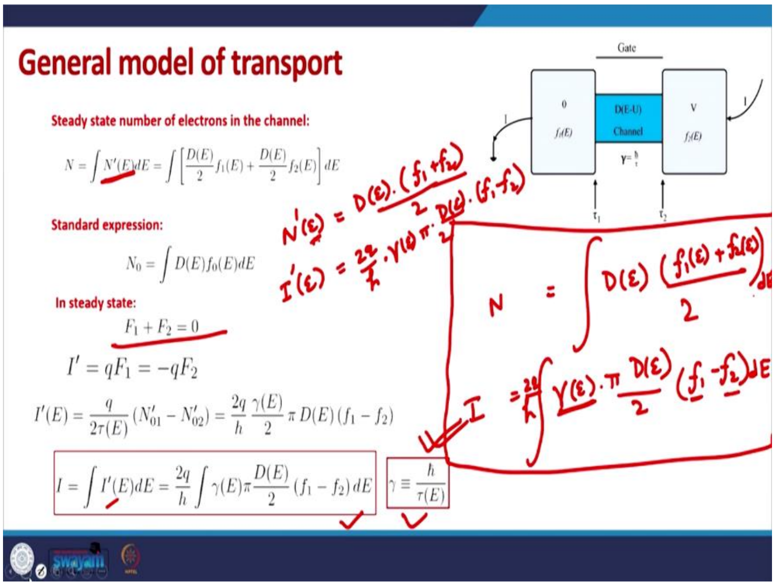

Because there is the flow of electron is constant. So, the change in the number of electrons that is flowing across the channel is 0 the change is 0 the number of electrons is not changing it is constant. So, this rates will be 0 and that way we obtain the expression for the current.

So, the expression for the steady state number of electrons is just a simple expression D(E). $[f_1(E) + f_2(E)]/2$ integral over all energy ranges and sorry that is the total number of electrons in the steady state and similarly the total current in the channel will be there is a constant factor outside $\frac{2q}{h}\int \gamma(E)\pi\frac{D(E)}{2}(f_1 - f_2)dE$ where this γ factor is essentially $\frac{\hbar}{\tau(E)}$.

So, that is what we have seen. So, far and these are please keep in mind that these are important expressions for transport. So, if we have a new material and we if we want to make a new device out of that material, we first try to find out the density of states in that material. Then we try to find out the characteristics time and if we somehow get to know about these 2 quantities, maybe from first principle simulations then we can find out the current that will be flowing across the device. If we are putting 2 contacts with Fermi functions f_1 and f_2 ok.

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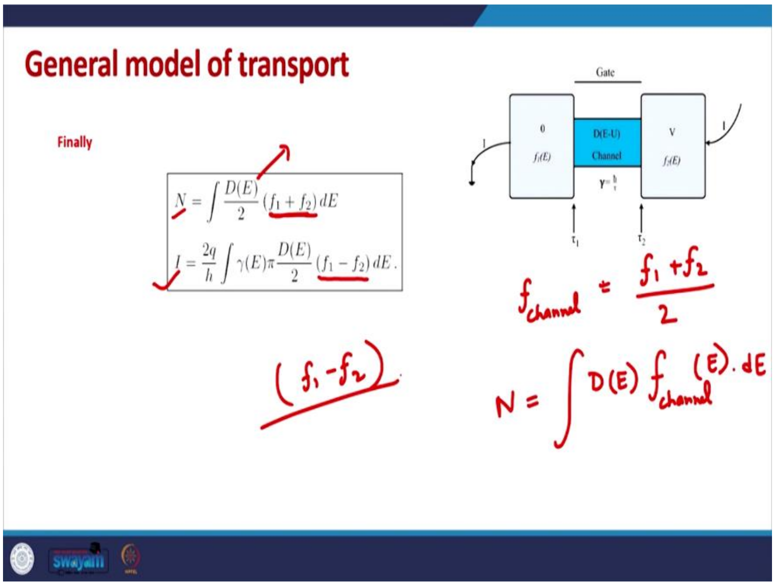

So, this is what we have seen so far in our discussion and in this there are some intuitive terms here, we can make sense intuitive sense of those terms the steady state number of electrons in the channel is essentially dependent on the density of states which is what we also expected because which anybody would expect.

Because the number of electrons will definitely be dependent on the number of allowed electronic states in the channel and that is given by the quantity dE at the same time the steady state number of electrons in the channel is directly proportional to the average of the 2 Fermi functions average of the left Fermi function and the right Fermi function $\frac{f_1 + f_2}{2}$ and that is what we also expect because f_1 is trying to fill electrons up to f_1 level up to E_{F1} level, f_2 is trying to fill electrons up to E_{F2} level.

And jointly the number of electrons will be somewhere in between which will be sort of the or the Fermi function of the channel we can say that the Fermi function of the channel is essentially the average of the Fermi function of the 2 contacts and in that case the steady state number of electrons in the channel can be written as just like this, we can write it in terms of density of states in the channel.

And the Fermi function of the channel times dE, although this is not the actual Fermi function of the channel because channel is a very small region and it is not straight forward to define a Fermi function in small devices, when we have only a small number of electrons and small number of electronic states, but this is what we can sort of phenomenologically say. And the current in the device is proportional to this quantity f_1 - f_2, this we also expected heuristically as well.

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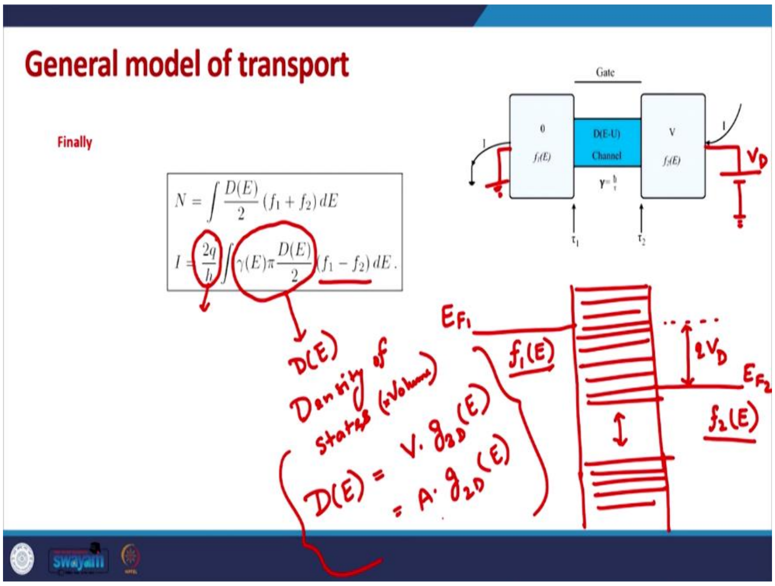

Because, if we see the energy level scheme the source contact has Fermi level E_{F1} the drain contact has Fermi level E_{F2} and let us say we have the electronic states distributed like this maybe there is a gap in between this is the gap where electrons cannot exist in the channel and the electrons are distributed according to the Fermi function $f_1(E)$ on the left contact and according to the Fermi function $f_2(E)$ on the right contact ok.

So, this left contact is trying to fill the channel according to this distribution function f_1 distribution function the right contact is trying to fill the channel according to f_2 distribution function. So, the net current is essentially dependent on the difference between the 2 Fermi functions. So, if the difference is more which means that the applied voltage is more because the difference between E_{F1} and E_{F2} is dependent directly on the voltage that we apply on the device here ok.

So, if the difference is more we expect more current. So, we also expected that the current should be dependent on the difference of the 2 Fermi functions and that is what we are also obtaining from our analysis. These are 2 constants, one is the charge unit essentially the fundamental unit of charge and second is the Planck's constant q is the fundamental unit of charge and h is the Planck's constant.

But there is an additional term in the current expression here which is what we did not or what we sort of cannot directly predict just from qualitative analysis. So, this is what we obtained by doing our by this rate equation analysis and this is what we need to understand more deeply because also in this there is this term D(E). So, D(E) is as we all of us know is the density of states in the channel times volume as well. So, it is density of states time volume.

So, D(E) is for a 3D channel let us say D(E) is volume times g_{3D}(E) the and for 2D channel it is area times g_{2D}(E). So, that way it is not density of states, it is number of states per unit energy because we are now multiplying by volume in 3D case and we are multiplying by area in 2D case ok.

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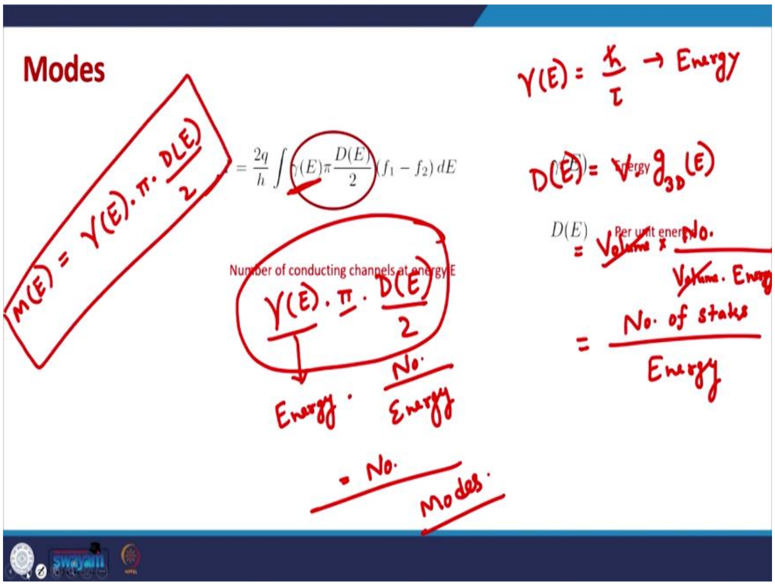

So, if we have a more sort of close look at this quantity, the first term here is γ(E) and if we at if we have a look at the units of this quantity γ(E) is essentially $\frac{\hbar}{\tau}$ and it has units of energy. So, the first term in this expression. So, this is what we are trying to understand, the first term has the units of energy here the second term is a constant again, it is just a mathematical constant the third term as we just saw D(E) is essentially for a 3D channel it is volume times g_{3D}(E) where, g_{3D}(E) is the density of states.

And if we want to see the units of this it will be the units of volume times the units of density of states is number of allowed electronic states per unit volume per unit energy. So, the units of D(E) will be number of states per unit energy. So, this has units of number per unit energy ok. So, the unit of this quantity jointly will be just number, it is just number of something. It is not number of allowed electronic states because that is the quantity dE and it is getting multiplied by γ(E).

So, it is not the number of allowed electronic states it is something else and this quantity is known as the modes essentially. So, this quantity γ(E) times π times D(E)/2 is known as the number of modes and it is generally represented as M(E).

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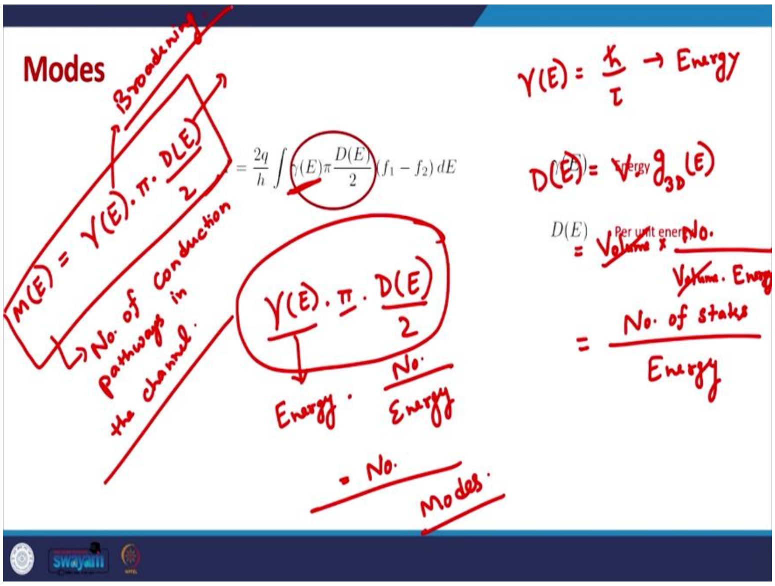

The unit of this is just number and it is the number of conduction pathways in the channel, it is just the number of conduction pathways in the channel. So, there is an interesting analogy given by Professor Supriyo Dutta in this case. So, this is as if the number of lanes in a highway. So, this can be equivalently said to be equivalent this can be said to be the number of lanes in the channel or number of pathways in the channel that the electrons can take while they are conducting current in the device ok.

And it is as you see it is expectedly dependent on the density of states, at the same time it is dependent on the broadening. So, this number of conduction pathways or number of modes are states as well as broadening taken into account together. So, because of the finite lifetime of electrons in the channel the electronic states are broadened and if we take the broadening into account the states are no longer just one single energy value, it is a broadened one and it sort of becomes a pathway and that is how we define the modes in the device ok.

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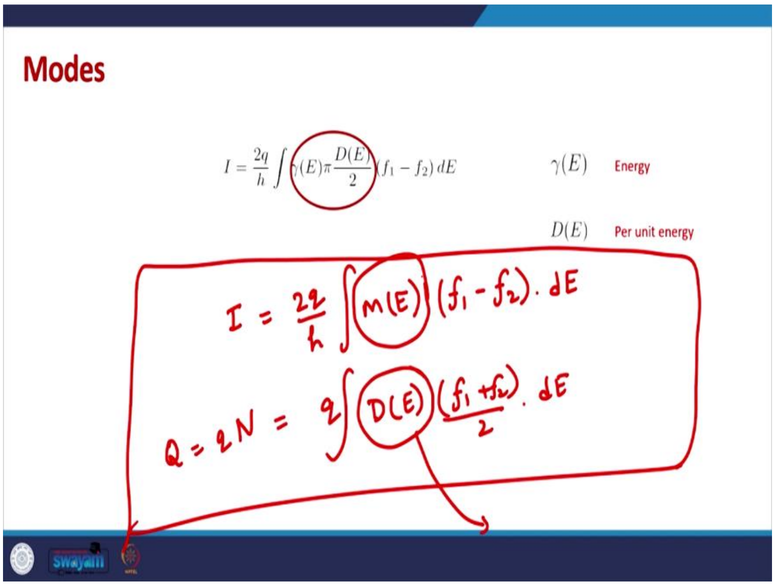

So, that is also an important concept because the current in the device actually can be represent written as M(E) times (f_1 - f_2) times dE. Whereas, the steady state charge in the device or steady state number of electrons in the device can be written as like this (f_1 + f_2)/2 times dE and number of steady state charges essentially q times N.

So, it will be Q times this. So, these are important concepts, these are important expressions and this concept we have already studied in good detail, the density of state the notion of density of states we have already understood in good details. Now is the turn to understand the modes in the devices because that is turning out to be a determining factor in current in the device ok. So, that is what we will do now. We will try to understand the notion of modes in devices more closely and for that.

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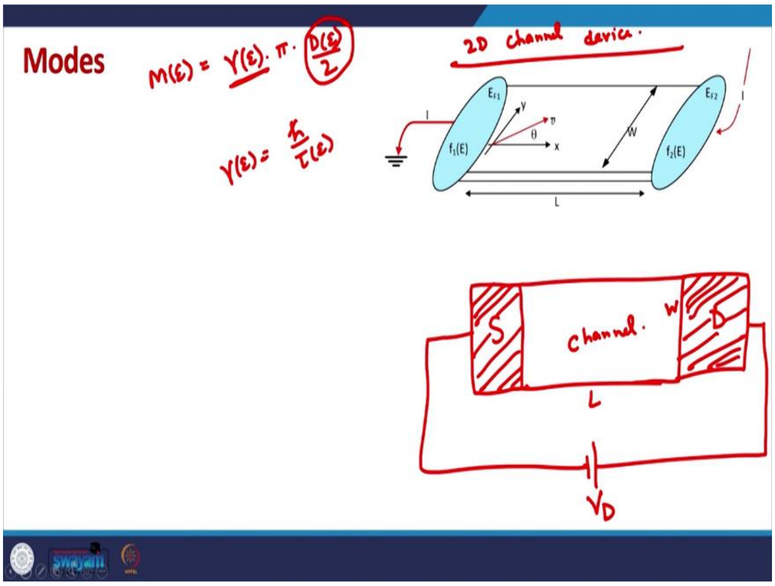

We take a 2 channel device actually, in order to understand the notion of modes more formally and you know in more details we take a 2D channel. So, in a 2D channel, this is how we can also write it we have a channel like this the source is on the left side the drain contact is on the right side and there might be a battery between the source and the drain , the voltage of the battery is represented as V_D, the source and drain regions are large as opposed to what may seem from the figure.

So, and the channel region is small, the channel is a 2D channel, only length and width are there in the channel, the third dimension is extremely small and ideally nonexistent ok. So, in this kind of device in order to understand the notion of modes. So, the modes is written as $\gamma(E)\pi\frac{D(E)}{2}$. So, we are already aware of the density of states idea of density of states. We need to more closely understand the broadening of energy levels or the transit time, because gamma E is also the transit time of the electrons.

So, that is what we need to understand more closely. And for that if we go back a little bit and we sort of do a thought experiment and in this experiment what we do is please keep in mind that this steady state number of electrons in the channel is D(E) times $(f_1 + f_2)/2$ ok. So, this is the number of electrons at this energy level, at this energy value in steady state.

Similarly, the steady state current is from this expression, this is we have already sort of seen from our previous analysis that in steady state the current can be written down as like this. So, it is essentially $\frac{2q}{h}\gamma(E)\pi\frac{D(E)}{2}(f_1 - f_2)$ ok. So, if we take a ratio of these 2 quantities.

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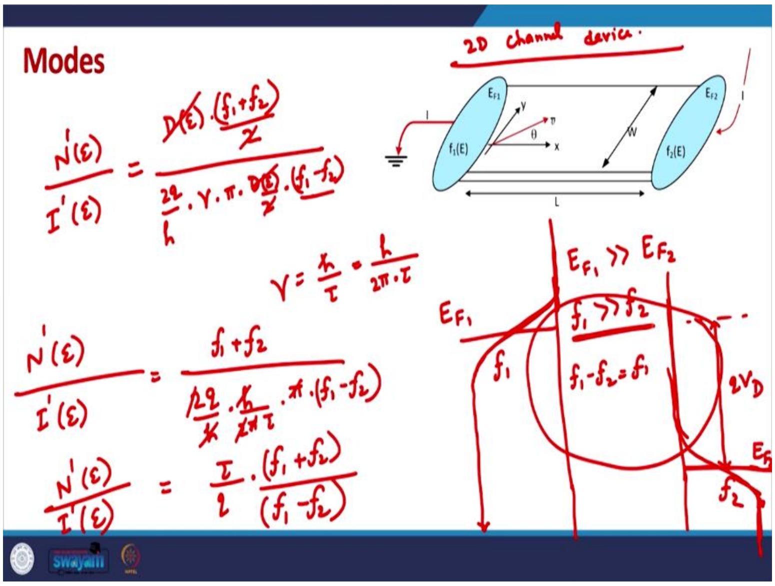

So, in our this expression if we take a ratio of steady state number of electrons and the current what would be this ratio? Steady state number of electrons at energy value E is given as D(E) times ($f_1 + f_2$)/2 and steady state current is given as $\frac{2q\gamma\pi}{h}$ times D(E)/2 times ($f_1 - f_2$) now this gamma can be written as h bar by tau and h bar is h by 2 pi times tau.

So, if we replace gamma by this expression in this 2 and 2 goes away and D(E), D(E) also cancels out. So, the ratio between the steady state number of electrons and current will be ($f_1 + f_2$) divided by $\frac{2q}{h}$ γ is $\frac{h}{2\pi\tau}$ into ($f_1 - f_2$) . So, this is what we have. So, h is cancelled by h, 2 by 2, pi by pi. So, what we are left with is $\frac{\tau}{q}$ times ($f_1 + f_2$) divided by ($f_1 - f_2$). So, this is essentially the ratio of the steady state number of electrons at a certain energy E with the current at the same energy and in our thought experiment if we apply a high voltage in the device.

So, if we apply a large voltage across this kind of channel, in that case when a large voltage is applied in that case. So, this is the device we have and if let us say this is the applied voltage q times V_D and this is high value as is also visible here, the difference between E_{F1} and E_{F2} is large in that case E_{F1} is when a large voltage is applied is very much larger than E_{F2} and that will also be the case with Fermi functions ok.

Because Fermi functions are mostly 0 above E above the Fermi level. So, after a small energy range Fermi levels are 0 above Fermi level and Fermi functions are one after a small range of energy below the Fermi level. So, this f_1 will be one for all these energies, it will be going from 1 to 0 in this energy range. Similarly, this f_2 will be one for all these energy range and it will make a transition from 1 to 0 from in this energy range ok.

So, which means that f_1 is also large as compared to f_2; $f_1 - f_2$ is essentially equal to f_1 if a very large voltage is applied because in that case this is the energy range where f_2 takes value 1 is way below then the energy range that we are interested in ok. So, in that case. So, when a large voltage is applied in that case we can approximate $f_1 + f_2$ by just f_1. So, that is what we will do and $f_1 - f_2$ you can also be approximated by just f_1.

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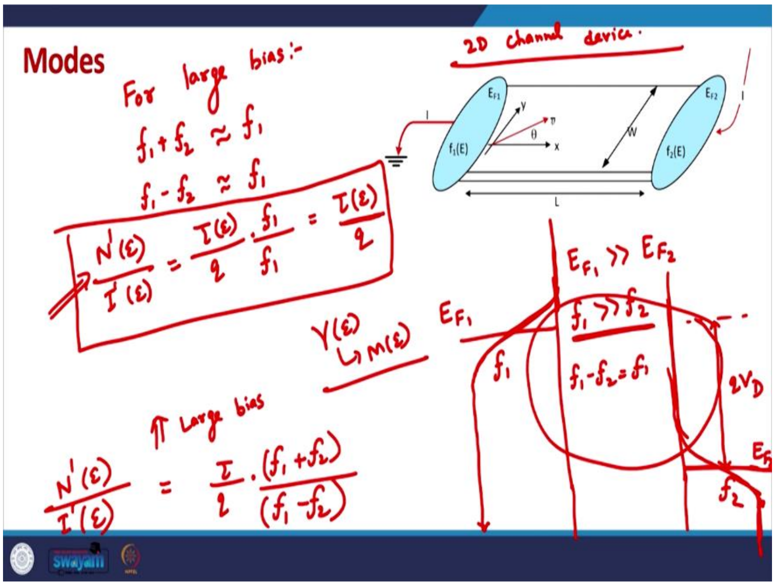

So, this is what we can write for large bias f_1 plus f_2 is almost equal to f_1 in the energy range that we are interested in and f_1 minus f_2 is also equal to almost equal to f_1 in the channel region for the energy range of our interest. So, in that case this expression essentially boils down to N'(E) divided by I'(E) for large bias case τ let us write it as a function of E as well times f_1 divided by qf_1. So, it essentially becomes τ(E) divided by q.

So, what we see from here is that the ratio of the steady state number of electrons to the current is directly proportional to the transit time or the characteristics time. So, if we can calculate this ratio properly, we can calculate the transit time and from the transit time we can calculate the energy broadening.

And from this we can calculate the number of modes. So, that is what essentially that is what we will do in our next class we will take this calculation further and do the analysis of the number of modes in more details.

Thank you for your attention, see you in the next class.