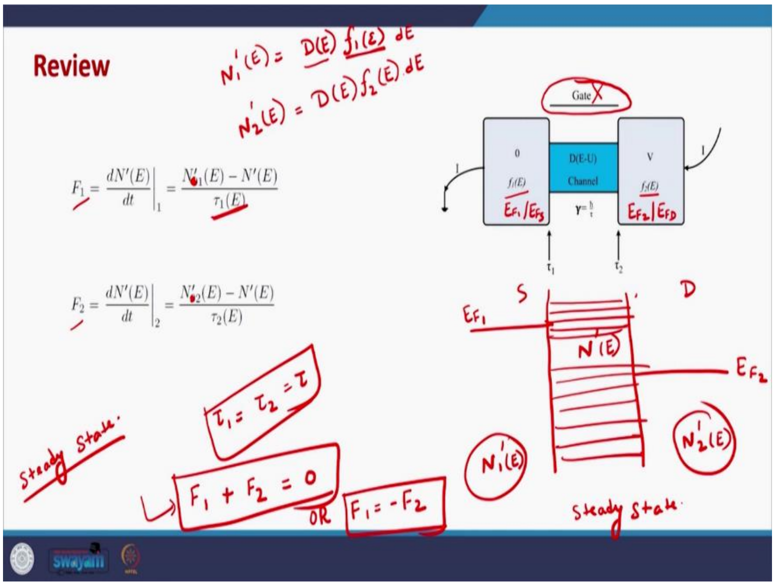
Physics of Nanoscale Devices
Prof. Vishvendra Singh Poonia
Department of Electronics and Communication Engineering
Indian Institute of Technology, Roorkee

Lecture - 19
General Model of Transport - III

Hello everyone, today we will continue our discussion of the general model of Transport and this model was essentially given by Landauer in 1950s and then it was later developed by professor Datta and later on Mark Lundstrom.

(Refer Slide Time: 00:48)



So, in this formalism of mesoscopic devices, small devices few tens of nanometers of channel the devices that have that kind of a channel we started our discussion from a two terminal device and this third terminal at the moment we are not considering.

So, please ignore the third terminal the gate terminal at the moment. And with so, generally the model is or which is actually quite close to the reality that the contacts are large and the channel is small and in that case the electrons in the contacts are characterized by their Fermi functions and Fermi level E_{F1} or E_{FS} and Fermi level here is E_{F2} or E_{FD}.

So, what we discussed in the previous class is that the source contact tries to bring the channel in equilibrium with itself which means that it tries to fill all the electronic states up to its Fermi function its Fermi level. So, it will try to sort of make sure that N_1' electrons are there in the channel, when the channel is in equilibrium with the source contact N_1' electrons will be there in the channel.

Similarly, on the drain side when the channel is in equilibrium with the drain contact N_2' electrons will be there in the channel. In steady state when both the contacts are connected the channel is not in equilibrium with any of the contacts and it is the steady state the flow is, there is a flow of electrons and the flow becomes of very quickly the flow becomes constant and that is known as the steady state.

In steady state, the number of electrons in the channel is N' and if we consider the energy range from E to E + dE, $N_1'(E)$ is the number of electrons in the source in the energy range E to E + dE; $N_2'(E)$ is the number of electrons in the drain from energy range E to E + dE and $N'(E)$ is the number of electrons in the channel in the energy range E to E + dE.

With these quantities we could write the rate equations essentially. So, the rate at which electrons are hopping from source into the channel is essentially will be given by $\frac{N_1'(E)-N'(E)}{\tau_1}$. In some cases even the characteristics time is a function of energy which means that in different energy levels in the channel, the time that the electron takes to go from the contact to the other end of the channel is a function of energy ok.

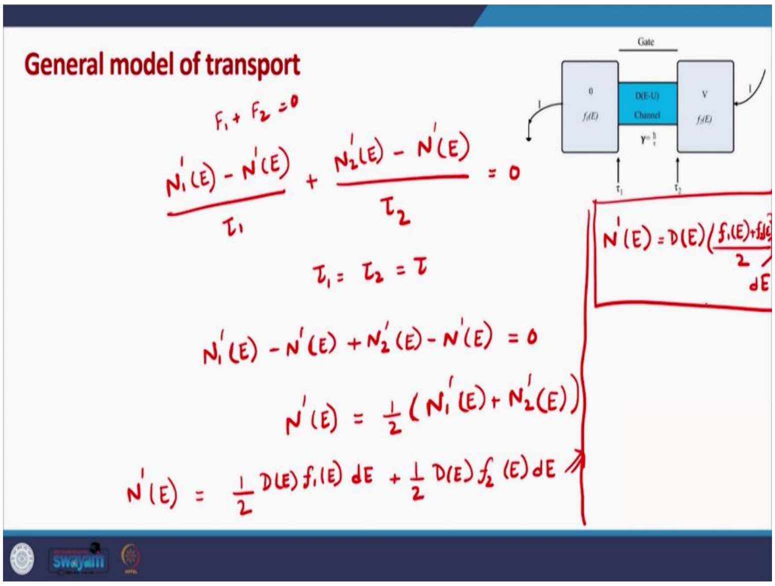
But for simplicity we assume that both the contacts are symmetric and $\tau_1 = \tau_2 = \tau$ and it is independent of energy. Similarly the rate at which electrons will be jumping from the drain contact into the channel will be given by $\frac{N_2'(E)-N'(E)}{\tau_2}$.

In steady state the net flow is constant. So, the rate at which electrons are jumping from the source into the channel will be equal to the rate at which electrons are jumping from the channel to the drain or the net rate of electrons coming to the channel is 0 . So, this can be written as $F_1 + F_2$ is equal to 0 or $F_1 = -F_2$. So, this is the steady state situation in the device ok.

In addition to this, what we know is that $N_1'(E)$ is equal to density of states in the channel times the Fermi function of the channel number of allowed electronic states in the channel times the probability that a certain state is occupied by the electron according to the Fermi function of source contact times the energy gap.

Similarly, $N_2'(E)$ is density of states of the channel times the Fermi function of drain contact times energy range dE this is what we know; in steady state we see that $F_1 + F_2 = 0$.

(Refer Slide Time: 06:20)



So, if we put these quantities so, $F_1 + F_2$ is equal to 0 and if we put F_1 is essentially $\frac{N_1'(E) - N'(E)}{\tau_1}$.

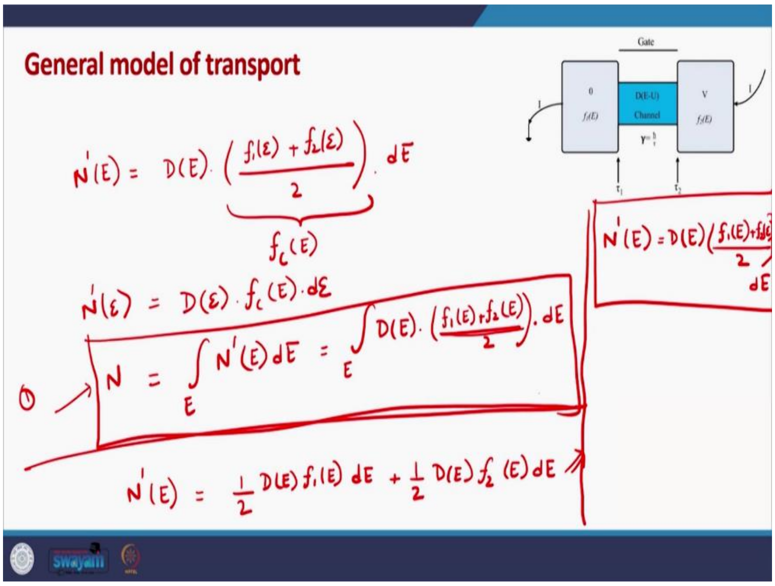
Similarly, F_2 is $\frac{N_2'(E) - N'(E)}{\tau_2}$. and that is equal to 0. So, if we assume $\tau_1 = \tau_2 = \tau$; in that case $N_1'(E) - N'(E) + N_2'(E) - N'(E) + N_2'(E) - N'E$ will be equal to 0.

So, essentially this $N'(E)$ which is the steady state concentration of electrons number of electrons in the channel in steady state will be given by $\frac{1}{2}[N_1'(E) + N_2'(E)]$. So, if we here put down the expressions for $N_1'(E)$ and $N_2'(E)$, the number of electrons in the channel in steady state in the energy range E to E + dE will be $\frac{1}{2} f_1(E) D(E) dE + \frac{1}{2} f_2(E) D(E) dE$ ok.

So, this is the expanded form of the expression, that essentially takes us to this expression $N'(E)$; the number of electrons in steady state in the channel in the energy range E to E + dE will be equal to $\frac{1}{2}[f_1(E) D(E) + f_2(E) D(E)]$ ok. So, this is the expression for the number of electrons in the energy range E to E + dE in the steady state in the channel ok.

And if you look at the expression this expression is I would say intuitively justified as well; because as we also saw in the previous class that the source contact is trying to make sure that electrons are filled in the channel up to E_{F1} or it is trying to fill the channel with $f_1(E)$. The drain contact is trying to fill the channel with $f_2(E)$.

(Refer Slide Time: 10:03)



So, let me rewrite it here $N'(E)$ is $\frac{[f_1(E)D(E) + f_2(E)D(E)]}{2} dE$. So, now, when both the contacts are there in that case the channel is filled with the Fermi function which is the average of the two Fermi function. So, let us say this becomes the Fermi function of the channel in the steady state ok.

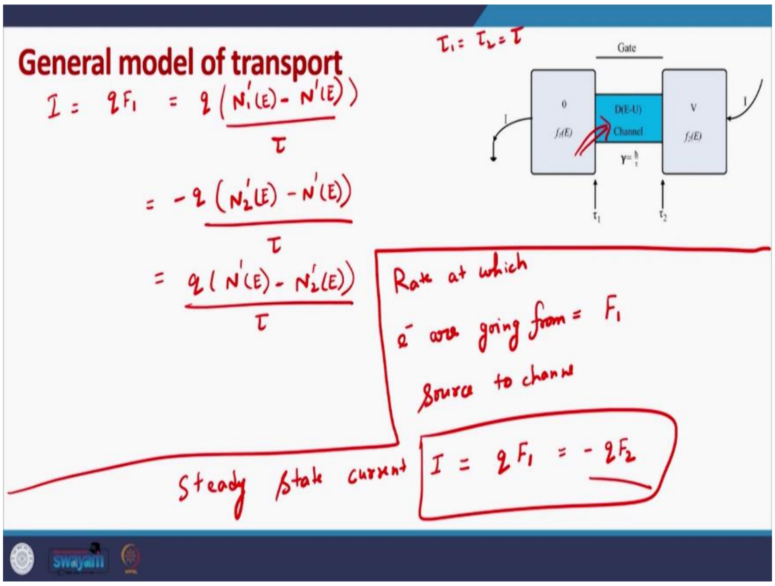
So, the total number of electrons in the channel will be now. So, the total number of electrons in the channel will be the integration of the number of electrons in the energy range over all possible energy ranges. This will essentially be integration over all allowed energy ranges $\frac{[f_1(E)D(E) + f_2(E)D(E)]}{2} dE$ ok.

So, as I said this also makes sense intuitively as well because one is trying to fill the channel with Fermi function f_1 source; the drain is trying to fill the channel with Fermi function f_2 and both of them together will fill the channel with Fermi function $f_1 + f_2 /2$ ok.

So, we have obtained the total number of electrons in the channel in steady state from this simple analysis of the rate equations in the 2-terminal device. Now the so, this was one of the first objective to find out what will be the steady state population of electrons in the channel. So, before the contacts are connected to the channel the electronic population would have been somewhat different.

When the channels are connected this will be the steady state electronic population in the channel ok. Now let us see and more importantly we also want to find out what will be the current in this device in steady state and that is what we will see now.

(Refer Slide Time: 12:56)



In steady state the rate at which electrons are going from source to channel is F_1 the rate at which electrons are going from source to channel is essentially F_1. And the rate at which electrons are going from channel to drain will be $-F_2$ as we have seen earlier.

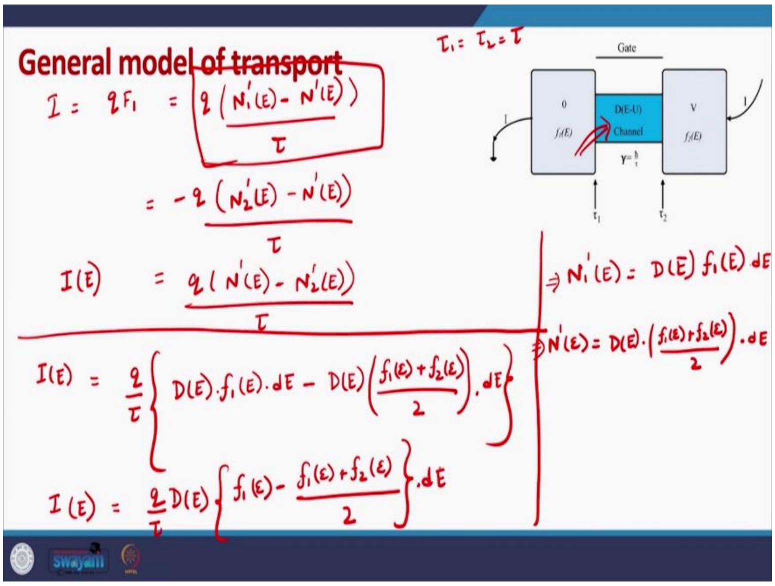
So, this is F_1 that will be F_2 and in steady state we saw that $F_1 + F_2$ is equal to 0 or $F_1 = -F_2$. So, the current in the steady state current will be current in the steady state will be steady state current will be q times F_1; because F_1 is the rate or number of electrons per unit time that is getting from source across the channel or - q F_2 that will be the current in the steady state.

Now, let us see what it turns out to be. So, I will be $q.F_1$ and F_1 if you recall is $q\,\frac{N_1'(E)-N'(E)}{\tau}$.; it can equivalently be written as from minus – $q.F_2$ it can equivalently be written as $-q\,\frac{N_2'(E)-N'(E)}{\tau}$.

As I have made it clear that $\tau_1 = \tau_2 = \tau$ is assumed. So, this will be the steady state current in the system ok. So, if we put down we have now the expression for $N_2'(E)$ and

$N'(E)$ as well from our earlier from the derivation the previous page $N_1'(E)$ is essentially D(E) times f_1 (E) into the energy range.

(Refer Slide Time: 15:56)

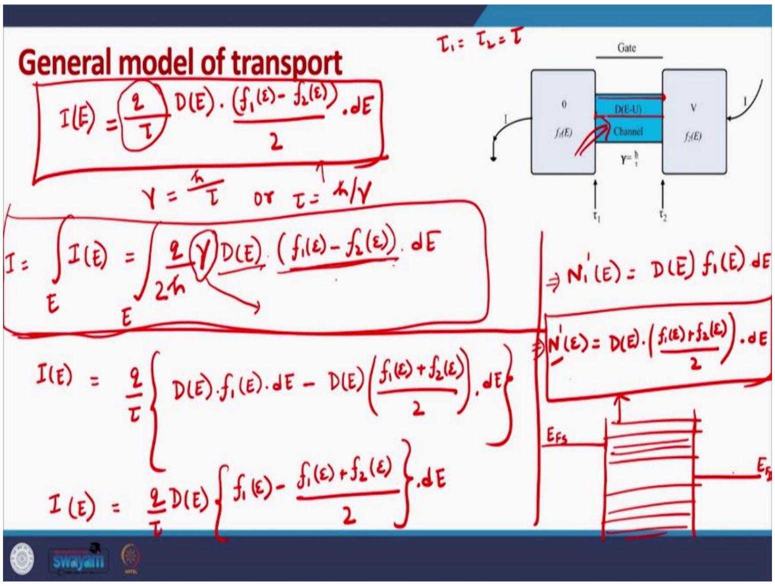


Similarly, the $N'(E)$ the steady state electronic population in the channel will be D(E) times f_1 (E) plus f_2 (E) by 2 times dE ok. Please remember that we need to write I as a function of E as well because this is the number of electrons in the energy range from I(E) to I(E + dE). So, this I(E) will be the contribution or the current contribution of the electrons in the energy range from E to E + dE ok.

So, if we take this part and put the values from here I(E) will be $\frac{q}{\tau}D(E)\left(f_1(E) - \frac{f_1(E)+f_2(E)}{2}\right)dE$ ok. So, this will be the contribution in total current from the electrons in the energy range E to E + dE on further simplifying this expression we can take dE outside

So, finally, the current due to electrons in energy range from E to E + dE which can be written as.

(Refer Slide Time: 19:01)



$$I(E) = \frac{q}{\tau} D(E)\left(f_1(E) - \frac{f_1(E) + f_2(E)}{2}\right) dE$$

This is the current due to electrons in the energy range from E to E + dE. As you can see and if you compare this to this expression here this is the electronic population in steady state in the device, this is the current contribution due to electrons in the energy range from E to E + dE in the device ok. If you see here in the expression for the steady state electronic population we have $\frac{f_1(E)+f_2(E)}{2}$.

In the current expression, we have $\frac{f_1(E)-f_2(E)}{2}$. So, that is the most striking difference in these two expressions apart from another term as well which is this; this τ can be as we know that there is a relation between Υ and τ and it is $\Upsilon = \frac{\hbar}{\tau}$. So, this I(E) can be written as.

So, just another way of writing this thing and the total current in the device will be essentially the integration of this expression over all possible energy values. So, this will be the expression for the total current; intuitively speaking, the steady state electronic concentration depends on the average of the Fermi functions $\frac{f_1(E)+f_2(E)}{2}$ and the steady state current depends on the difference of the Fermi function.

And that is also; what we discussed intuitively as well. So, what we discussed was that if we plot (Refer time: 22:00) the band structure or the energy level scheme of the device this is E_{FS}, E_{FD}; these are various states in between, the more the difference between E_{FS} and E_{FD} or E_{F1} and E_{F2} more will be the current more will be the difference in f_1 and f_2.

So, the more is the difference between the two Fermi functions the Fermi function of two contacts more will be the current in the device. However, that is not significant this difference in the Fermi function is not important for the electronic population for electronic population the average is more important ok the average of the two Fermi functions is more important. So, that is an interesting observation and that is an important point to keep in mind which is also intuitively expected ok.

So, this is what we have. So, we have ultimately obtained the expression for the current and the expression for the steady state electronic population in the device; when the two terminal device is connected to the two contacts and this is what we were trying to see right from the beginning.

I would say that these equations are among the most important equations in mesoscopic device analysis and even nanoscale devices these equations are extremely important starting points ok. So, in order to find out current in a device we need to know the density of states of the channel we need to know the Fermi functions of the contacts.

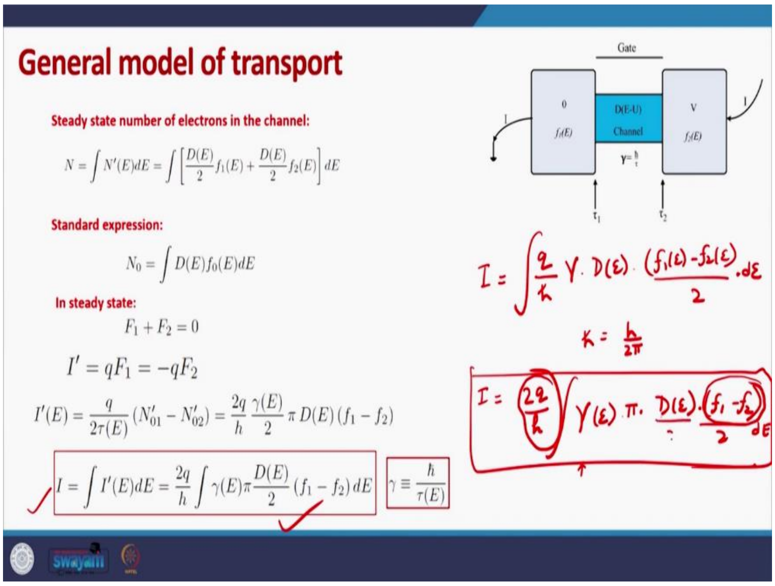
We need to know this gamma parameter which is essentially related to the transit time it also in other words captures the interaction of the contacts with the channel. So, we need to know the Fermi functions in the contacts we need to know the density of states we need to know the interaction between the contacts and the channel and there is another approximation that we need to keep in mind which I did not explicitly mentioned earlier

In these devices it is assumed that the contacts are reflection less. What it means is that the electron that travels through the channel is completely absorbed by the corresponding contact to which it strikes. Similarly if the electron travels from here to here they do not reflect the electrons back what it essentially means is that, this junction between the contacts and the channel is as perfect as it can be which is practically not the case always.

There are some imperfections, there are some dangling bonds or some states trap states or at the interface. So, those impurities or those imperfections are not taken into account in this analysis; we are considering that this junction is perfect between the contact and the channel and the electrons are travelling such that electrons are not reflected back by the contacts ok.

So, with these equations we will now see how we obtain a new idea, a new notion in the device analysis which is generally known as the modes in the channel ok.

(Refer Slide Time: 26:16)



So, this is the expression for the current ok. So, what we obtained was, we derived an expression that the total current is $\frac{q}{\hbar}$ ok into γ into D(E) into $\frac{f_1(E)-f_2(E)}{2}$ dE.

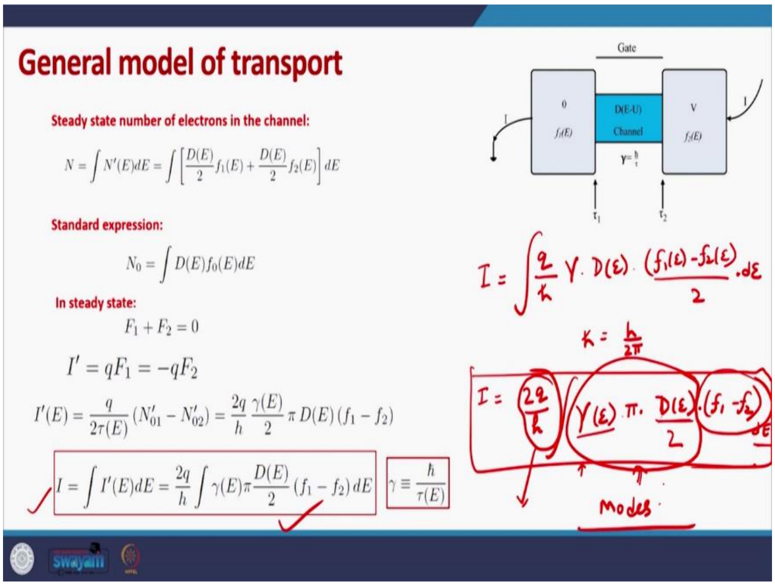
This was the total current expression that we derived it can be simplified further if we write $\hbar$ equals $\frac{h}{2\pi}$, in that case this expression becomes $\frac{2q}{h}$ and this is a constant. So, it can come out of the integral γ and it can also be a function of energy times π times D(E) times $\frac{f_1(E)-f_2(E)}{2}$ and these two f_1 and f_2 are also functions of energy we are not explicitly writing it.

But, it should be kept in mind and this is essentially the expression for the current which is written here. So, here as you can see there are. So, we need to properly understand this equation. In fact, I would suggest you to sort of remember this equation and you do not need to explicitly remember this final expression if you do the derivation yourself you will understand it clearly and you will not forget it.

So, this equation has a constant term which involves fundamental constants one is the charge of the electron second is the Planck constant in addition to that it involves the difference in Fermi functions $f_1 - f_2$ which is expected the more the difference more the current.

So, the difference is necessary for current to flow through the device. So, if there is no difference between the Fermi functions of the source and the drain the current will be 0; however, that is not the case with the steady state electronic population ok.

(Refer Slide Time: 28:55)



If we bring this 2 here and sort of remove the 2 from here, we know that this is a fundamental constant we know this is what is required, but we do not know what this quantity is this γ(E), we know separately γ(E) is the energy broadening D(E) is the density of states, π is a constant again, 2 is a number dE is the energy range, but this quantity is an interesting quantity that we need to analyze.

So, please think about this and we will discuss more about this in the coming class what this quantity is, this gives let me give you a hint this is known as the modes in the channel or the conduction pathways in the channel. Just to give an analogy it is more the number of modes more will be conduction pathways in the channel and more current will flow through the device and please keep in mind that this is different from the allowed energy states.

So, modes is not exactly equal to the allowed energy states which is given by the density of states it is different notion than that it involves density of states and the broadening of the states ok. So, we will start discussing this new concept known as the modes in the next class.

So, please do these are simple derivations with the approximations clearly in mind you can do these derivations very simply. I would recommend all of you to do these derivations on your own and then you would remember these expressions as well ok.

Thank you for your attention, see you in the next class.