Physics of Nanoscale Devices
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Lecture - 17
General Model of Transport - I

Hello everyone. As all of you are aware that today we will be starting a new topic the topic to study transport in electronic devices and. So, this transport is known as the General Model of Transport this model of transport is known as the general model of transport and we will see how it is sort of developed before going into the details of the transport let me quickly review our discussion in last couple of classes.

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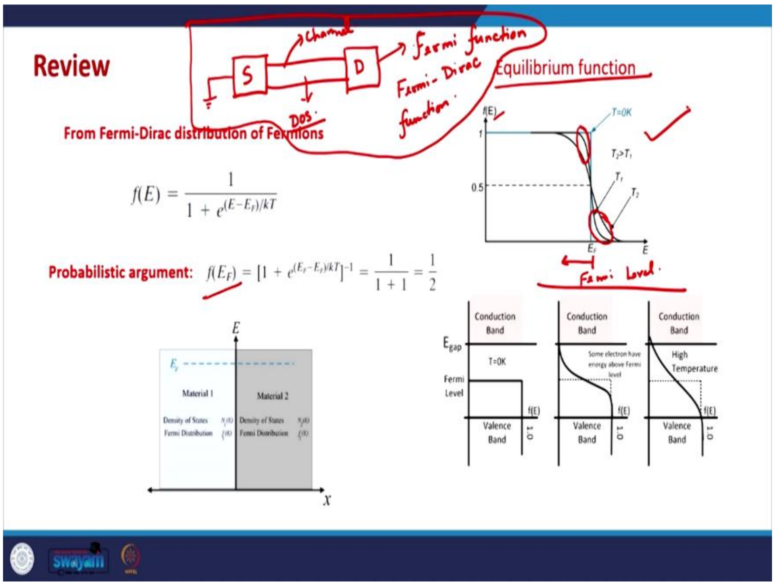

Review

From Fermi-Dirac distribution of Fermions

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

Probabilistic argument: $f(E_F) = [1 + e^{(E_F-E_F)/kT}]^{-1} = \frac{1}{1+1} = \frac{1}{2}$

Equilibrium function

We have been discussing that the number of available states in a device depend on the density of states in the channel. So, if this is a two terminal device in which we have a source, we have a drain and we have a channel in this device the number of available electronic states depend on the density of states of the channel and the electronic population in the source and the drain, the contacts basically is governed by the so called Fermi function which is also known as the Fermi-Dirac function ok.

So, these ideas we have tried to understand in a good amount of detail in last few classes and. So, now, we are in position to apply these ideas in real practical scenarios. So, that basically brings us to the transport. Let me quickly review the Fermi function the ideas of the Fermi function. Fermi function in one of the interpretation of Fermi functions, Fermi function is the probability that a state is occupied by the electron at a given temperature and this is the plot of Fermi function as versus energy as a function of temperature as well for various temperatures.

So, at T equal to 0, Fermi function is one below this level which is known as the Fermi level this E_F level is known as the Fermi level and this is one the Fermi function is 1 at T equal to 0 Kelvin below E_F energy levels which means that all energy levels up to E_F are occupied at 0 Kelvin.

As the temperature is increased some energy levels very close to the Fermi level are not fully occupied now as you can see that the Fermi function is no longer 1 here and some of the energy levels just above the Fermi level are now occupied because the probability is now non zero or the Fermi function is now non zero.

So, this idea is quite important and let me sort of rephrase this that the idea of Fermi function is an equilibrium concept that this idea is applicable only at equilibrium and as we have already discussed that equilibrium basically means that each and every process in the device or in the system is countered by the reverse process.

So, there is a balance between the forward processes and the backward processes and that state of the system is known as the equilibrium state of the system. In equilibrium, the state variables or the variables which are used to characterize the system they do not change essentially.

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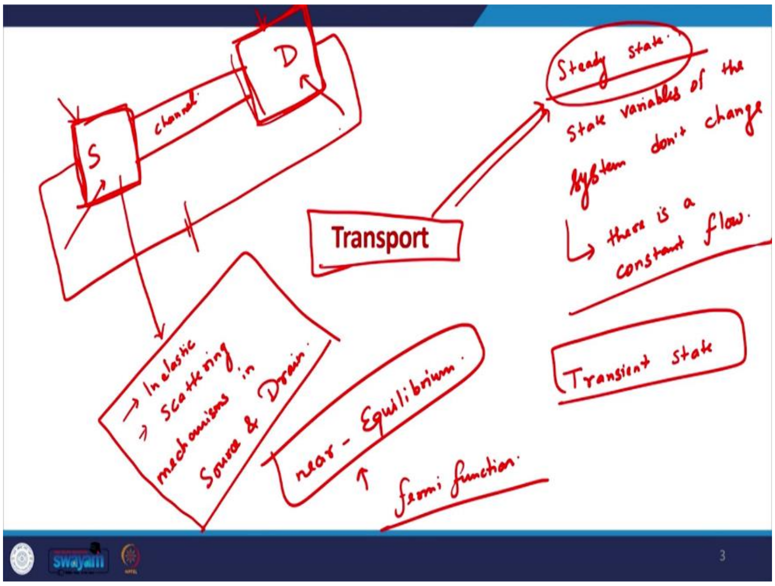

Transport

And there is a related concept of steady state; in steady state, also we can say that the state variables of the system do not change; the state variables of the system do not change, but there is a flow there is a constant flow so to say to be more precise there is a constant flow in the system.

So, what it means is that there is a forward process in the system and there might be a backward process as well one of these processes dominate which means that there is a net forward process, but. So, there is a net flow, but the flow does not change the flow is constant. So, the steady state is quite important in devices because that is the state when current is flowing through the device and that is also the state in which the idea of transport is generally studied.

There is one more state which is known as the transient state. In transient state, the state variables are changing they are going from certain values initial state to final state and this is a state of flux, this is a state of change basically and generally in stable systems the transient state does not stay for a long time; generally, the system very quickly obtain a steady state ok.

So, in transport we need to sort of keep in mind that transport, in transport there is a net flow of carriers, there is a net flow of particles in a device, net flow of electrons in our case and when that flow is constant that is known as the steady state of the system. So, generally we study transport in steady state, but the idea of Fermi level is applicable in equilibrium.

So, that is why now we need to see if we can use the idea of Fermi function in transport or not and there is an important approximation that we need to make. And this is not just an approximation, this is very close to the reality that in a practical device the source and drain regions are generally either metals or heavily doped semiconductors behaving like metals almost behaving like metals and these regions are extremely large as compared to the channel region in between ok.

So, what we can say is that there is a lot of carriers first of all in these regions and there are a lot of electronic states as well because if a material is large the electronic states will be continuous, they will be closely spaced to each other and if a material is small then the electronic states will be discrete and smaller the material more far apart they are located more far apart the electronic states are located.

So, in bulk material generally electrons can have a continuum of energy states continuum of allowed energy states and moreover there is a lot of electrons available in the source and the drain. So, that is why even in case of transport even in case when we have for example, applied a voltage across the device we have put a battery across the source and the drain in that case the battery will maintain a voltage difference between the source and the drain and that voltage difference as we have also seen in our previous classes.

That it maintains the flow of current we will also quickly review in a few minutes in a few seconds from now. So, when we have a battery and by virtue of being large there are inelastic scattering mechanisms in source and drain. So, even when there is a battery there is not a state of equilibrium even if there is a state of flow of electrons even in that case because there are lot of electrons, lot of energy levels moreover the size is big.

So, when electrons flow through these materials they can scatter in these materials and these scattering mechanisms. So, scattering can be of two types; one is elastic scattering and second is inelastic scattering. In elastic scattering the energy does not change for the electrons, in elastic scattering the energy changes.

So, what happens is because of the inelastic scattering if an electron is removed from a certain energy level it is quite likely that one of the electrons sitting in upper energy level will fill that energy level very quickly and because of the inelastic scattering the state of electrons in the source and the drain is quite near to equilibrium. So, the state in source and drain is maintained near equilibrium because of the inelastic scattering mechanisms ok.

So, even during transport when we do not have extremely high voltage applied across the device in that case the electrons in the source and the drain collectively statistically they remain near equilibrium and that is why we can apply the idea of Fermi function or we can use this notion of Fermi function to describe the distribution of electrons in the source and the drain.

So, that is one point that we need to keep in mind while studying transport. So, even though the transport is not an equilibrium phenomenon even then we can use the notion of Fermi function at least in the source and the drain ok.

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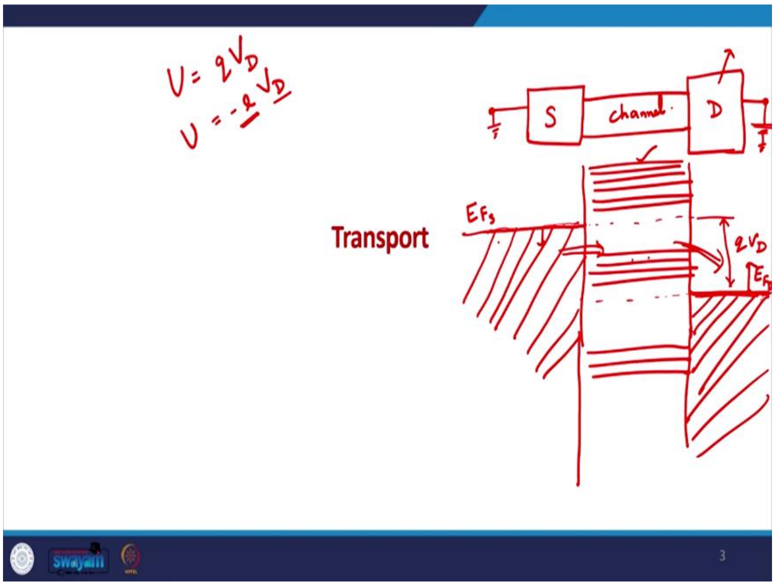

Transport

So, that is one thing. Second is if we have a device a two terminal device like this in which we have a source, we have a channel and we have a drain. So, when there is no voltage applied across the device and the source and the drain material are of the same type they are from, they are made of the same material in that case the Fermi level of the source and the Fermi level of the drain will be at the same level.

So, all the electronic states below E_{FS} the source Fermi level are filled in the source similarly all electronic states below E_{FD} the drain Fermi level are filled in the drain. In between, we have a channel region in the channel region according to the density of states of the material there might be some electronic states available at certain energy levels and at certain energy levels there might not be any available electronic state in the system. So, generally this could be typically this could typically be the scenario in devices these are the allowed electronic states in the channel.

This is the source Fermi level the drain Fermi level. Now, if a voltage is applied for example, if a positive voltage is applied on the drain terminal ok; a positive voltage difference is applied in other words across source and drain with drain being on high voltage high potential in that case this E_{FD} value this E_{FD} level would go down.

Because the potential energy of the electrons will be reduced in a positive potential because the potential energy of the electrons is q times applied voltage and q for electrons is negative q the charge of electrons is negative charge. So, a positive voltage results in reduction in the potential energy. So, when a positive voltage is applied E_{FD} which is the drain Fermi level will go down and now this is how it would look like ok.

So, this source terminal will try to fill all the energy levels up to E_{FS} in the channel, it will try to bring the channel in equilibrium with itself and the drain Fermi level will try to fill all the electronics all the electronic levels up to the drain Fermi level and all the states above the drain Fermi level it will try to make sure that those states are empty it will try to drain electrons out of those states.

So, that is why there will be an electron flow from the source to the channel and from the channel to the drain terminal. And please keep in mind that in order for this flow to be maintained, this difference between E_{FS} and E_{FD} needs to be also maintained. So, we need to maintain this constant difference q times V_D, this difference between E_{FS} and E_{FD} D and that is maintained by the battery basically ok.

If we apply a voltage for let us say for a moment and we bring the drain Fermi level down to this point and So, this is then it is left or we use for drain contact we use another semiconductor material in which the drain Fermi level is lower than the source Fermi level. In that case what will happen is electrons will go from the source terminal to the channel and from the channel to the drain and ultimately and there is no applied no external applied voltage, no battery.

If that is the case, in that case because of the electron flow from the source to the channel to the drain, this drain Fermi level will come up; this source Fermi level will go down slightly this will come up slightly and ultimately a state of equilibrium will be achieved in which E_{FS} and E_{FD} are the same. And this is what we also derived in our previous discussion that in equilibrium even if there are two different semiconductors the Fermi level is uniform across the entire device across the entire system of junctions ok.

So, this is a another thing that in order for the flow to be maintained we also need to maintain this difference between E_{FS} and E_{FD} and that is maintained by the battery basically and that is where the role of the battery basically comes in ok.

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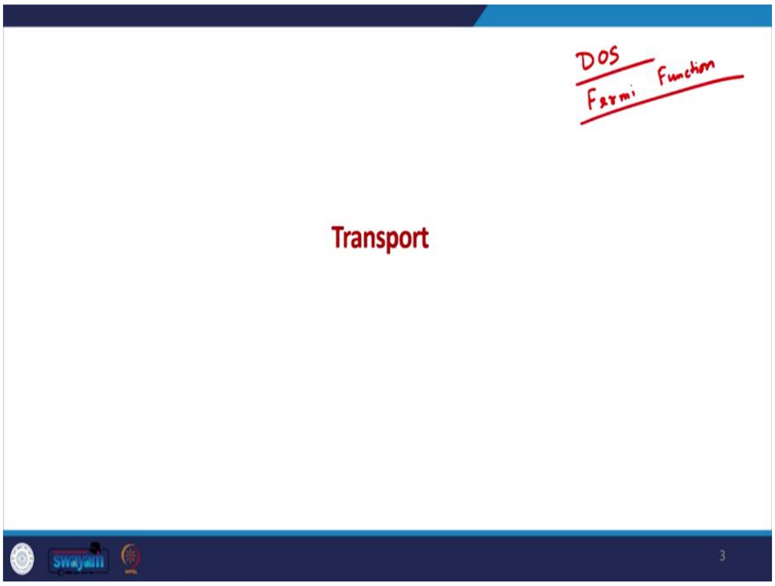

DOS

Fermi Function

Transport

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So, with this we will see how do we sort of start understanding the transport in electronic devices with our knowledge of density of states and the Fermi function ok.

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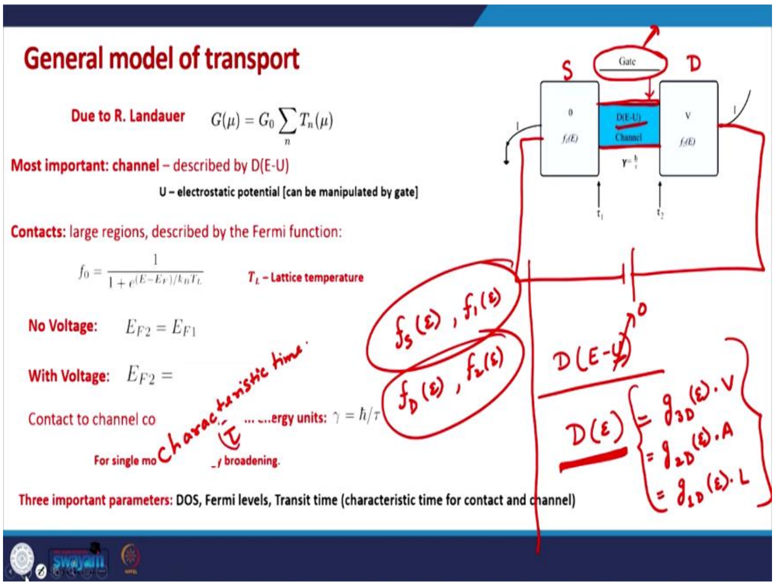

General model of transport

Due to R. Landauer $G(\mu) = G_0 \sum_n T_n(\mu)$

Most important: channel – described by D(E-U)

U – electrostatic potential [can be manipulated by gate]

Contacts: large regions, described by the Fermi function:

$f_0 = \frac{1}{1 + e^{(E-E_F)/k_B T_L}}$ T_L – Lattice temperature

No Voltage: $E_{F2} = E_{F1}$

With Voltage: $E_{F2} =$

Contact to channel co … …ergy units: $\gamma = \hbar/\tau$

For single mo… …y broadening.

Three important parameters: DOS, Fermi levels, Transit time (characteristic time for contact and channel)

Characteristic time.

τ

$f_S(\varepsilon)$, $f_1(\varepsilon)$

$f_D(\varepsilon)$, $f_2(\varepsilon)$

$D(E-\cancel{U}) \to 0$

$$D(\varepsilon) \begin{cases} = g_{3D}(\varepsilon) \cdot V \\ = g_{2D}(\varepsilon) \cdot A \\ = g_{1D}(\varepsilon) \cdot L \end{cases}$$

So, this is known as the general model of transport; this is generally applicable to the mesoscopic devices. Mesoscopic means few tens of nanometers in that range which is also the channel length in most of our modern transistors and in this case we will first start our discussion with a device like this which is a device two terminal device in which we have a source, we have a drain, we have a small channel in between, in principle there can be a third terminal as well known as the gate terminal.

Actually, all most of the practical devices are three terminal devices; all the transistors are three terminal devices. Just for the sort of sake of simplicity in understanding this formalism we will first understand the transport in two terminal devices and we will ignore the effect of the third terminal the gate terminal basically. So, let us see how this system can be understood we have a two terminal device system in which the source and the drain regions are large regions.

The source region is generally grounded the drain region is generally at a positive voltage with respect to the source this is how the battery is connected through this device. The channel region is a small region in between and the channel region is characterized by the density of states. So, the channel is characterized by the density of states which is represented by D(E).

If we have a third terminal as well the gate terminal and if we apply a voltage on the gate terminal that this third terminal this applied voltage because of the electrostatics in this direction this can alter the density of states and the effect is incorporated in this way. So, the third term or because. So, this U basically accounts for the potential energy of the electrons because of the third terminal.

Even in two terminal devices this U might come because of the interaction of the channel with various other sort of I would say the contacts or, but for a simple two terminal model system this U is taken to be 0 and generally we deal with the density of states in the channel and the density of states now is the total number of electronic states per unit energy.

So, for a 3D material this would be the density of states in 3D channel times the volume for a 2D material this would be the 2D density of states times area and for a 1D system this would be the 1D density of states into the length of the conductor.

So, generally this is how this D(E) is defined in the system ok; the source and the drain this is how the channel is characterized this is basically modeled by the total number of electronic states per unit energy this is known as the total number of electronic states or density of states multiplied by the volume or the area or the length the source and the drain regions they are modeled by the Fermi functions, the source Fermi function or the drain Fermi function.

Sometimes has f_1 and f_2 ok; now we have individually sort of represented the electronic distribution or the existence of electronic states in this entire device. So, we have modeled the source and drain contacts we have also modeled the electronic states we have modeled the source and drain contacts with the Fermi functions and we have modeled the channel with the density of states in this way.

But what happens when we connect the source to the channel? And that is an important part actually that is in transport this interaction of contacts with the device with the channel is of crucial importance. Quantum mechanically, it will lead to scattering in the system, it will lead to because the electronic states because of the interaction they might also get modified moreover the electrons in the channel they will now see a different environment when they reach to the source or to the drain terminal.

So, that will lead to the scattering mechanisms in the system. So, in this in our simple transport model this the contact and the channel or the source channel and the drain channel contact is modeled by a new parameter known as the characteristic time of the contact. So, this is essentially what we this is modeled this is sort of denoted by τ ok.

So, the way to define the characteristic time of a contact with respect to channel is it is the time that an electron will take from that contact to go through the device. So, it is the time that an electron in the.

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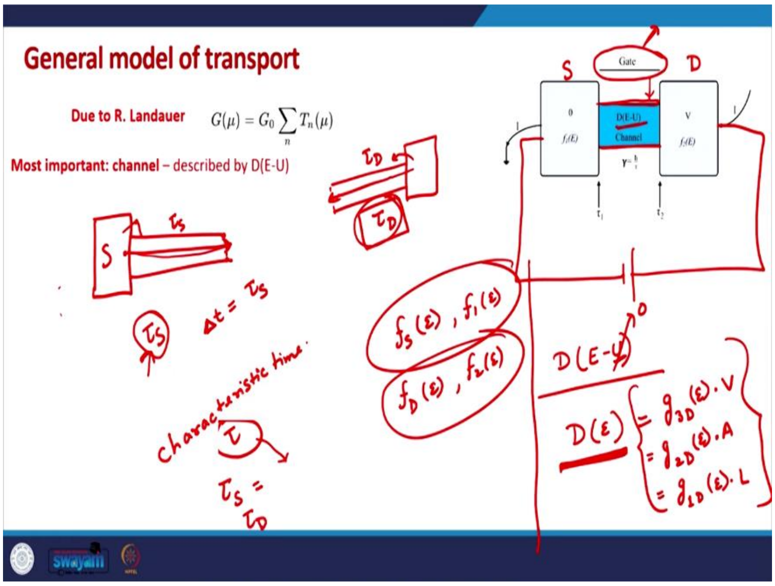

General model of transport

Due to R. Landauer $G(\mu) = G_0 \sum_n T_n(\mu)$

Most important: channel – described by D(E-U)

So, for example, if we are just considering the source contact and the channel at the moment we are not concerned about the drain contact. So, in this case the characteristic time of the source the τ_S will be the time that an electron takes from the source terminal to cross the channel to go to the other end of the channel.

So, it is the time that the electron takes to jump from the contact to the channel and to cross through the channel. So, this time tau the characteristic time essentially models the interaction between the contact and the channel also takes into account the travel time in the channel. So, in that way it is not just the characteristic time of the contact, it is the characteristic time of the channel plus contact channel plus that contact ok.

So, τ_S is the time that the electron will take from the source to go through the channel to the other end. Similarly, τ_D will be the time that an electron will take from the drain terminal to jump into this channel and cross through the channel. So, this will be τ_D this will be τ_S ok. So, the interaction between the contacts and the channel is modeled by the characteristic time of the contacts with that channel.

In this case τ_S and τ_D. Now, since we are talking about an electron and electrons have discrete energy levels quantum, in quantum mechanics electrons have discrete energy levels and in quantum mechanically if the energy of the electron is precisely defined its time evolution cannot be precisely defined because of the Heisenberg's uncertainty principle and if the position is precisely defined the momentum cannot be defined ok.

So, there is a relative uncertainty between the energy time and time between the momentum and position and there are other such conjugate set of variables as well in quantum mechanics. Now, when the electron jumps from the contact into the channel for example, when the electron jumps from the source into the channel and cross through the device cross through the channel, go to the drain it stays only for a finite time in the channel or the time for which it engages with the channel is τ_S .

And the electron that jumps from the drain into the channel and cross through the channel go to the other end. This electron engages with the channel for a time τ_D or it stays in the energy levels of the channel for time τ_S and τ_D. So, characteristic time is also the time for which the electron stays in the energy levels of the channel during transport. So, generally it happens that the drain is at positive voltage.

So, electron jumps from the source into the channel and to the drain generally the vice versa does not happen, but sometimes when the source is at positive voltage the electron can also jump from the drain to the channel and go to the source ok, but coming back to the main point since the electron is staying for a finite time in the channel it means that Δt is τ_S in this case or it means that there is a time frame of τ_S in which electron is there in the channel.

We do not know where it is it might be at any location this is wave function. So, the electron is a wave function. So, we cannot say where it is until we measure it and if we measure it, it can change the position as well, but what it implies is that because of the finite time in the channel there would be an uncertainty in the energy of the electron and that uncertainty in energy will be determined by the Heisenberg's uncertainty principle ok.

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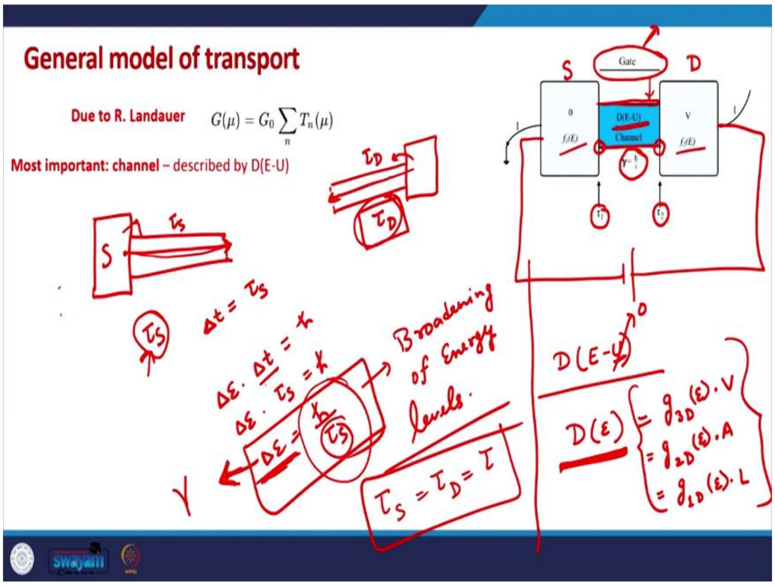

General model of transport

Due to R. Landauer $G(\mu) = G_0 \sum_n T_n(\mu)$

Most important: channel – described by D(E-U)

So, what it means is ΔE times Δt let us take the lower limit of the inequality is $\hbar$ and if this is τ_S, ΔE is $\hbar$. So, ΔE is $\hbar/\tau_S$. So, it means that if the electron stays in an energy level for only a time of τ_S, the uncertainty in the energy of the electron is ΔE. So, it leads to the broadening of the energy levels ok; because of the finite time of the electron in the channel the energy levels in the channels channel region are broadened by this value $\hbar/\tau$.

So, the electron is coming from the source it is $\hbar/\tau_S$; if it is coming from the drain it is $\hbar/\tau_D$. Generally, the source and the drain are made of the same material same exactly replica of each other. So, this τ_S is equal to τ_D is equal to τ. So, now, we have in a way defined the channel by the density of states, we have defined the electronic distribution in the source and drain by their Fermi functions and we have defined the connection between the source and the channel by τ_S or τ_1 and the connection between the channel and the drain by τ_D or τ_2.

So, now, we have everything with us we have I would say all parameters with us this energy and there is an important concept that emerges from this notion, from the quantum mechanical nature of electrons is that because in during transport electrons stay only for finite time in the channel that leads to the broadening of energy levels in the channel and this broadening is represented as gamma this parameter gamma.

This ΔE is represented as ϒ and this is given as $\hbar/\tau$. So, this is all what we have and now based on this we will see what is the current according to this model of transport. So, this model was given by Rolf Landauer and originally he gave this formula of the conductance of a 2D conductor where this $G(\mu)$ is the conductance, G_o is a constant and T is the transmission eigen values.

So, to say this the theory given by Landauer was simplified and recontextualized by Supriyo Datta and later on Mark Lundstrom and that is what we will study in the next class.

Thank you for your attention, see you in the next class.