Physics of Nanoscale Devices
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Lecture - 16
Fermi Function, General Model of Transport

Hello everyone. Today we will conclude our discussion on Fermi function. Basically the Fermi Dirac distribution function and we will start a new topic which is generally called as the General Model of Transport which is popularly known as the general model of transport.

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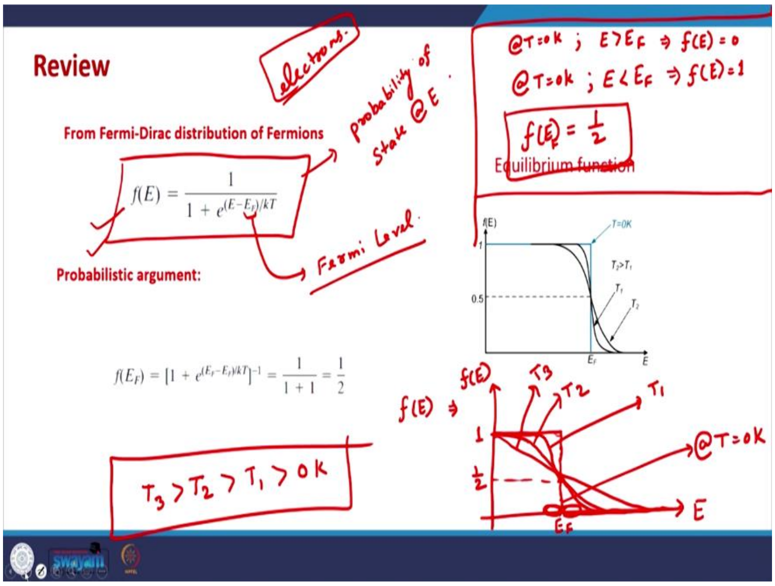

So, this is what we saw in the previous class that fermions essentially electrons because electrons are also fermions. Electrons are distributed according to a certain function that is known as the Fermi function or Fermi Dirac distribution function. This is a function of energy and this also tells us about the probability of a state of energy E or probability of the state being occupied by electron at a certain energy.

And it has an important constant that is known as the Fermi level and there are two ways to understand Fermi level; one is Fermi level is the level below which all electronic states are filled at 0 kelvin. Second definition which also comes from this formula is Fermi level is the level whose probability of being occupied by an electron is half basically.

And this is how if we plot this function Fermi Dirac distribution function this is how it would look like essentially. So, if we plot f(E) as a function of E. In our last class we also saw that at T equal to 0 kelvin if E is greater than E_F for energy is greater than Fermi level f (E) is 0 ok. And at T equal to 0 kelvin for E less than E_F f (E) is 1. So, f(E) essentially at if we plot f(E) versus E as a function of f (E) versus E at T equal to 0 kelvin if this is where the Fermi level is if this is E_F it will be 1 up to Fermi level.

So, up to this point this function will be 1 and after E_F it will drop to 0. So, this is how f(E) versus E will look like at 0 kelvin. So, this is at T equal to 0 kelvin. As soon as we increase the temperature what happens is that this distribution starts getting skewed. Skewed in the sense that now the probability of the states here just below E_F starts getting less and the probability of states just above E_F starts increasing. The probability of these states being occupied.

So, which means that some of the electrons from energy levels just below Fermi level will go to energy levels just above the Fermi level. So, what it will look like is now the probability now the Fermi function will be slightly curved here. And we also saw that the Fermi function is half at 0 half at E equal to E_F this is also what we saw f(E) is $f(E_F)$ is half.

Fermi function at E equal to E_F is half. So, this is half if we increase the temperature even more what would it mean is that. Now, more electrons will have a tendency to go to higher energies and the Fermi function will be even more broadened and this happens at higher temperature.

So, if this is temperature T1 this is temperature T2 and this is temperature T3; what we can say is that T3 is greater than T2 is greater than T1 that is greater than 0 kelvin. So, this is how a Fermi function will look like at different temperatures and this is what a few constraints that this Fermi function or Fermi levels; Fermi function around the Fermi level satisfies ok. So, these are some very simple things, but quite important things that we need to keep in mind.

So, essentially density of state tells us about how many allowed electronic states exist in a material exist in a device the Fermi function tells us about how many of those states are filled at equilibrium, ok. So, please keep this in mind that we mostly. So, this idea of Fermi function is applicable only at equilibrium, but now you might ask yourself that in devices generally the system is not in equilibrium, generally the system is in steady state as we also discussed in the last class.

So, in that case actually this idea of Fermi function cannot be used. But sometime, but most of the times in our analysis the system is not very far away from the equilibrium. So, what do I mean by that I mean that in equilibrium there is no external applied voltage.

So, if we apply a very small voltage the system will be out of equilibrium, but that will not be very far from equilibrium and that kind of situation is known as the near equilibrium condition, near equilibrium situation and the transport for that voltages is known as the near equilibrium transport.

So, we can apply the idea of Fermi function and Fermi levels in near equilibrium because it does not change drastically in near equilibrium situation when the equilibrium is slightly disturbed. But when the system is very far away from equilibrium then this idea is not directly applicable, ok. So, these are few things that we need to keep in mind.

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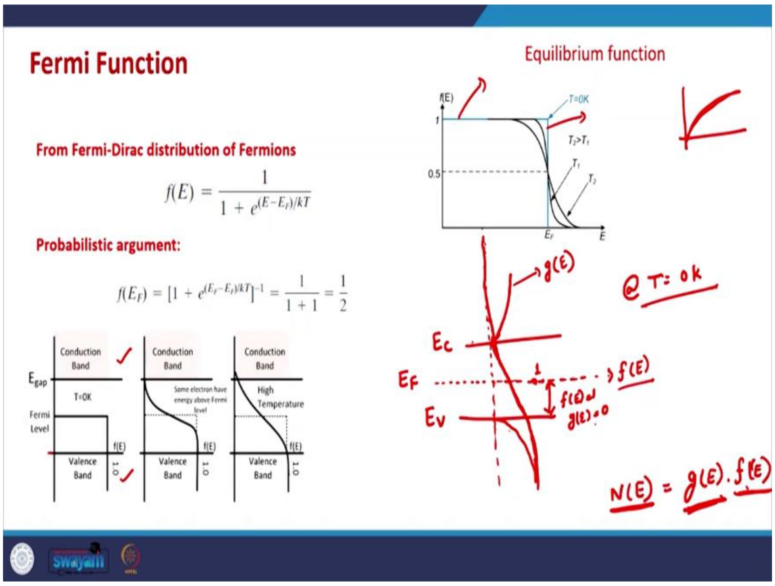

And one thing that is basically there is that this Fermi Dirac distribution function tells us about the occupancy of various states. So, if we consider essentially a semiconductor material this is how the conduction band and valence band in a semiconductor look like. So, this is so, we have a valence band we have a conduction band.

So, this is the top of the valence band this is the bottom of the conduction band and above the bottom of the conduction above Ec there is conduction band below Ev there is valence band. And when the semiconductor is not doped in that case the Fermi level sits in the middle of the band gap.

So, Fermi level sits in between of Ec and Ev. So, we have already seen how various density of states look like. So, if this is a 3D material the density of states in the conduction band will have this kind of relationship. So, the density of state as you might recall follow this kind of distribution and this is how they will be distributed in the conduction band. So, there will be some density of states here, but this number will not be very less.

But as we go away from the bottom of the conduction band the density of states will increase. So, this is the density of states in the conduction band. Similarly, the density of states in the valence band will look like this. So, if we only focus our discussion or our discussion in the conduction band the number of electrons because the density of states in valence band is generally used to calculate the number of holes in the system.

So, let us see how things go in this system this function tells us about how many electronic allowed electronic states are available in the conduction band and in the valence band. And the Fermi level of the system is sitting in the middle of Ec and Ev. So, at T equal to 0 kelvin if we see at T equal to 0 kelvin the Fermi function is 1 below $E_{F,}$ 0 above E_F this is what we saw from this plot basically this plot.

So, if we also plot the Fermi function on this energy axis this vertical axis here is the energy axis on x axis we have taken here the density of states and we can also take the Fermi function on the same axis so, that we can see how both of them collectively work basically. Because g(E) the density of state tells us about how many states are there f(E) tells us about how many of those states are occupied. So, the Fermi function will look something like this it will be 1.

So, an x axis if this is the Fermi function or it will be 1 below E_F and 0 above it will be 0 for these energy values. And the number of electrons in a system will be number of available states times the Fermi function. Density of states times the Fermi function it also needs to be multiplied by the volume for 3D materials by area for 2D materials and by length for 1D materials. So, the total number of electrons in the systems is this N(E) will be the total number of electrons in the system will be density of states times the Fermi function, ok.

So, what will be the total number of electrons in the conduction band here? In the conduction band this g(E) is nonzero, but f(E) is 0 at 0 kelvin. So, at 0 kelvin no electron will be present in the conduction band because this multiplication this factor of f(E) is 0. In the conduction band and it makes the entire expression 0 in the valence band this f(E) is 1 for all electronic states it means that all the states in valence band will be filled all these electrons will have all these possible E k values.

In the band gap f(E) is 1 from Ev to E_F in this region f(E) is 1, but g(E) is 0. So, no electron will exist in this because the product of g(E) and f (E) will be 0 in the band gap. So, at T equal to 0 kelvin all the electrons will be sitting in the valence band of the semiconductor ok, so, that we can in a straight forward way deduce from this E k plot. But the situation is different at non zero temperatures at non zero temperatures the Fermi function is not 1 below E_F always and it is also not 0 above E_F.

So, this is the density of states in conduction band this is density of states in valence band. Now, the Fermi function is 1 here it starts decreasing it becomes half it is 0 far away from the. So, this is how broadly the Fermi function will look like at T equal to non zero temperatures ok this is 1/2.

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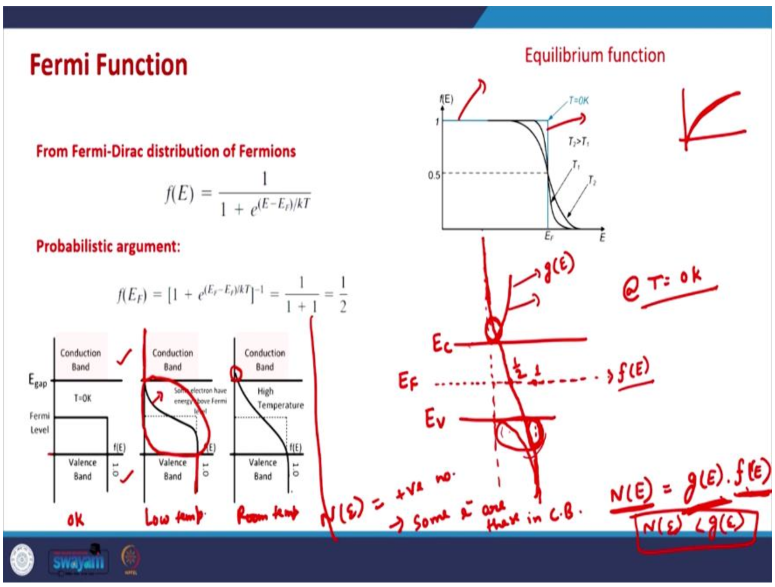

Now, this value is 1 here, but here it is not 1 here also this is I guess not very clear from this diagram, but this is the conduction band the Fermi level will be something like this. So, now in the valence band the value of the Fermi function here it drops below 1 in this region.

So, some of these states are now not fully occupied because this product in this product f(E) is no longer 1. So, if g(E) is the number of available states f(E) is less than 1 it means that N(E) will be less than g(E) which means that some of the states in this system will now be empty.

Similarly, here g(E) is non zero and f(E) is non zero very close to the bottom of the conduction band here, which is also visible here in this diagram basically ok. So, in this regime where in this regime of conduction band where f(E) is non zero and g(E) is also non zero N(E) will also be nonzero. So, N(E) will be a positive number, which means now some electrons are present are there in conduction band.

So, at 0 temperature all the electrons are sitting in the valence band at non zero temperatures above a certain value of temperature as soon as electrons get enough energy to jump from valence band to the conduction band. Some of these electrons jump to the some of the valence band electrons jump to the conduction band basically and that is also evident from the this plot.

So, at very at non zero temperatures, but not very high temperatures this f(E) is still 1 in almost all the valence band and f(E) is 0 in almost all the conduction band although it changes from 0 to 1 between the valence band and conduction band, but since in the band gap g(E) is 0. So, no electron would exist at low temperatures. So, this is 0 kelvin case, this is very low temperature case and this is let us say room temperature case ok. So, this is the situation when the semiconductor is not doped.

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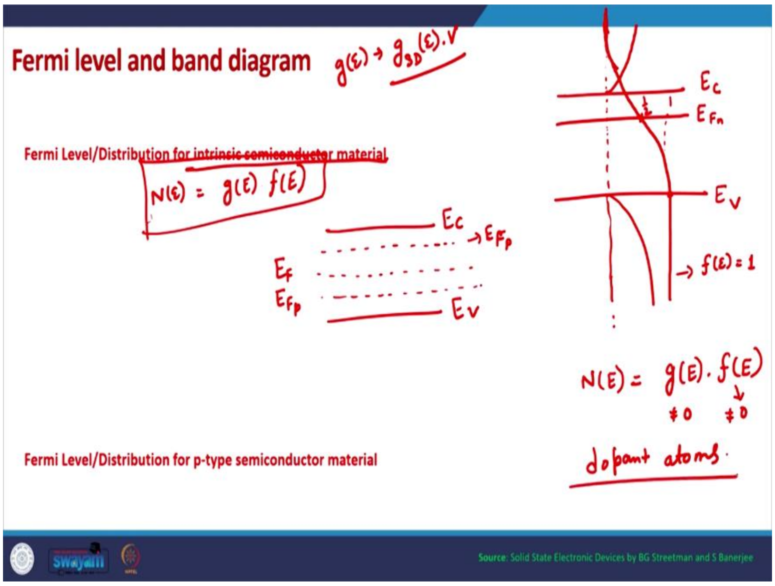

Now, if we dope the semiconductor. So, when the semiconductor is undoped which means intrinsic semiconductor the number of electrons is given by N(E) equals g(E) times Fermi Dirac distribution function, where g(E) now is for 3D material let us say g_{3D}(E) times volume total number of density of states into volume which basically gives us total number of electronic states per unit energy in the entire volume of the system.

And this case we have already seen in this plot here. This is the intrinsic semiconductor case and this is how electrons will be distributed in an intrinsic semiconductor at T equal to 0 and above T equal to 0 kelvin. But as soon as we start doping the material what happens is that doping basically changes the Fermi level ok. So, p type doping brings the Fermi level down n type doping pulls or pushes the Fermi level up.

So, if these are the edges of conduction band and valence band and this is the intrinsic Fermi level of the system this will be the Fermi level. Typically this will be the Fermi level when we have a p type doping and typically this might be the Fermi level when we have n type doping. Now, in these cases by doing our analysis of this expression g(E) times f(E) giving to be to N(E) we can deduce the number of energy states in the system basically, ok.

So, for example, in an n type material when we have the Fermi level sitting very close to conduction band this is the Fermi level of n type material this is the valence band a edge. Now, the if we plot g(E) the density of state in conduction band and valence band this is how it looks like. And if we now plot Fermi function around the Fermi level in this case it will be now the Fermi level is very close to the conduction band and at nonzero temperatures this is how it would look like.

So, this is 1 half this is let us say 1. So, what it means is now the Fermi function f(E) is 1 in the valence band. It is which means that all the states in the valence band are occupied, but the Fermi function is also nonzero in the conduction band. Because now the Fermi level is shifted towards the conduction band edge and that would result in some electronic population in the conduction band as well, because now this is non zero this is also non zero, ok.

So, that would result in some electronic population in the conduction band alright. But you might wonder where are these electrons coming from in the conduction band. Because now the valence band all the electrons are there in the valence band all the electronic states are filled in the valence band. So, these electrons are essentially coming from the dopant atoms.

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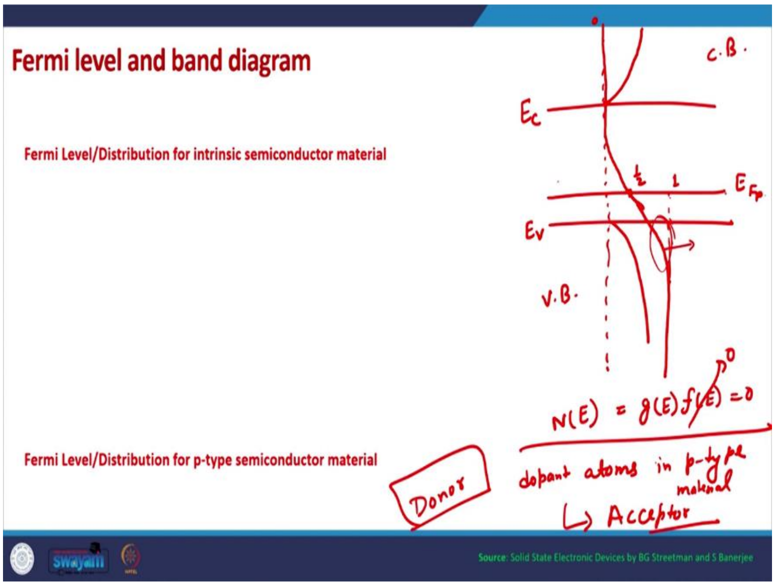

In an n type material the Fermi level will sit close to the valence band edge. Here it is sitting. And now if we plot both density of states this is how density of states would look like in a 3D material. And if we now plot the Fermi function here the Fermi function would be 0 far away from the Fermi level as soon as it approaches Fermi level it starts increasing and it becomes 1 far away from the far below from the Fermi level again.

So, this is 1, this is 0, this is 1 half. Now, the Fermi function is 0 in the conduction band this is the conduction band, this is the valence band, Fermi function is 0 in the conduction band which means number of electrons in conduction band will be 0, because now the this f(E) is 0. So, that is why the number of electrons in the conduction band will be 0.

Additionally you see you can see here that this f(E) is less than 1 for some states here. Some states close to the top of the valence band. So, some of these states are now empty which means that the probability of these states being occupied is less than 1. So, that means, that some of these states are empty, but you might wonder again wonder where these electrons are going from here essentially these electrons are jumping or these electrons are being taken by the dopant atoms in p type material.

These dopants in p types are also known as acceptor atoms and dopants in n type materials are known as donor atoms. And as the name suggests the donors and the acceptors donate and accepts electrons respectively. And that is why this kind of Fermi Dirac distribution applies to n type and p type materials ok.

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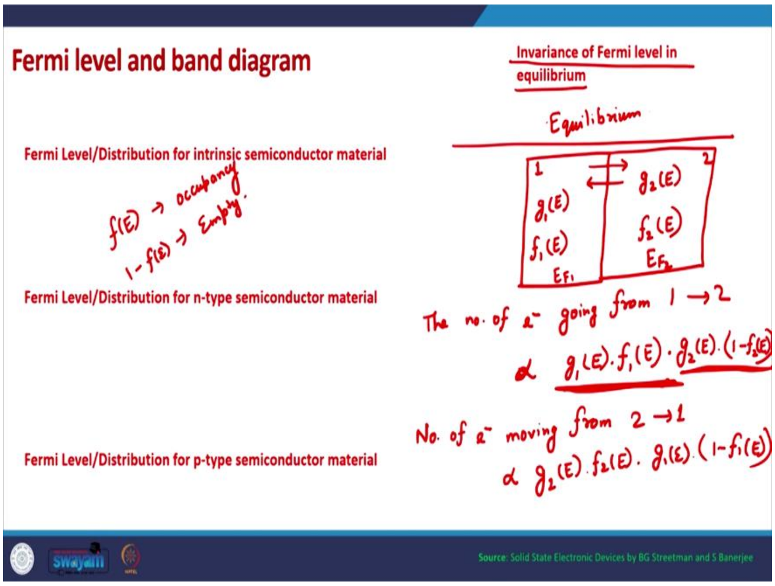

So, I hope this idea is now clear. We are now at the last sort of point here on the Fermi function and which essentially says that the Fermi level is invariant in equilibrium. So, what it means is that when the system is in equilibrium the Fermi level across the entire system, across the entire device, across the entire semiconductor is invariant.

It is a single energy level it is not bending, it is not sort of changing its value; the Fermi level is invariant across a semiconductor if the semiconductor is in equilibrium. So, this is also quite important specially in MOSFET analysis and even in diode n-p junction, junction analysis. And we will also quickly prove this let us say that there are two semiconductor materials in close proximity to each other; material 1, material 2 and this material has density of states to be $g_1(E)$ it has Fermi function to be $f_1(E)$.

Similarly, this material 2 has density of states to be $g_2(E)$ and Fermi function to be $f_2(E)$. Now, what we say is that this entire system is in equilibrium. And by definition in equilibrium every process is balanced by a counter process. So, what it means is if there is a flow of particles from this side to this side from material 1 to material 2. The same amount of particles will flow in the opposite direction from material 2 to material 1 as well.

So, we will see let us say here the Fermi level is E_{F1} here the Fermi level is E_{F2} basically. Now, the number of electrons so, we let us see how electrons balance to each other the flow of electrons balance from left to right from 1 to 2 to 1 to 1; the number of electrons going from 1 to 2 would be proportional to I would say proportional to the number of filled or number of electrons in the material 1, which will be given by $g_1(E)$ times $f_1(E)$.

So, at a certain energy let us say at energy E if there is 1 electron present in the left side and on the right side there is no electron in the same energy. So, then this electron will have a tendency to go to the right side from material 1 to material 2. So, if a level is filled on left side and it is empty on the right side electron will move from left to right from 1 to 2. Similarly, if a level is filled in 2 and is empty in 1 the electron will move from 2 to 1.

So, the number of electrons going from 1 to 2 will be directly proportional to we are not integrating here actually we need to integrate number of filled states. So, to say into number of empty states on the right side and number of empty states on the right side will be $g_2(E)$ times $1 - f_2(E)$.

So, f (E) tells us about the probability of a state being occupied and 1 - f (E) will tell us about the probability that a state is empty. So, this tells us about that a state is filled with electrons and this tells us about that this state is empty on the right side. So, the flow of electrons from material 1 to material 2 will be directly proportional to the states filled on the left side in material 1 to states empty in material 2. Similarly, the number of electrons moving from 2 to 1 will be directly proportional to states filled in material 2 into states empty in material 1 using the same logic essentially.

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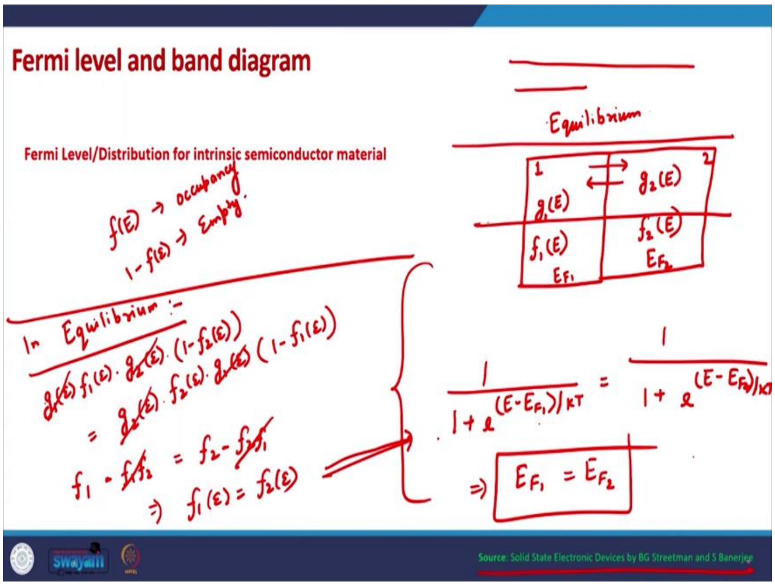

So, what it means is that in equilibrium by definition the number of electrons going from 1 to 2 should be equal to number of electrons going from 2 to 1. So, this $g_1(E)$ times $f_1(E)$ $g_2(E)$ times 1 - $f_2(E)$ is equal to $g_2(E)$ times $f_2(E)$ into $g_1(E)$ times 1 - $f_1(E)$. So, now, we can see that $g_1(E)$ $g_1(E)$ cancels on the left side we are left with f_1 minus f_1 f_2 on the right side we are left with f_2 minus f_1 f_2 .

So, even this cancels. So, what it says is $f_1(E)$ should be equal to $f_2(E)$. And since everybody is at the same temperature what it implies is that this basically means $\frac{1}{1+e^{\frac{E-E_{F1}}{kT}}}$ is equal to $\frac{1}{1+e^{\frac{E-E_{F2}}{kT}}}$. So, which means that E_{F1} is equal to E_{F2}.

So, the Fermi level across the entire system across the two materials is same in equilibrium. So, they will both have Fermi level at the same energy value across the entire system. A good reference to read about this is this. So, today with this we basically conclude our discussion of Fermi function and Fermi level.

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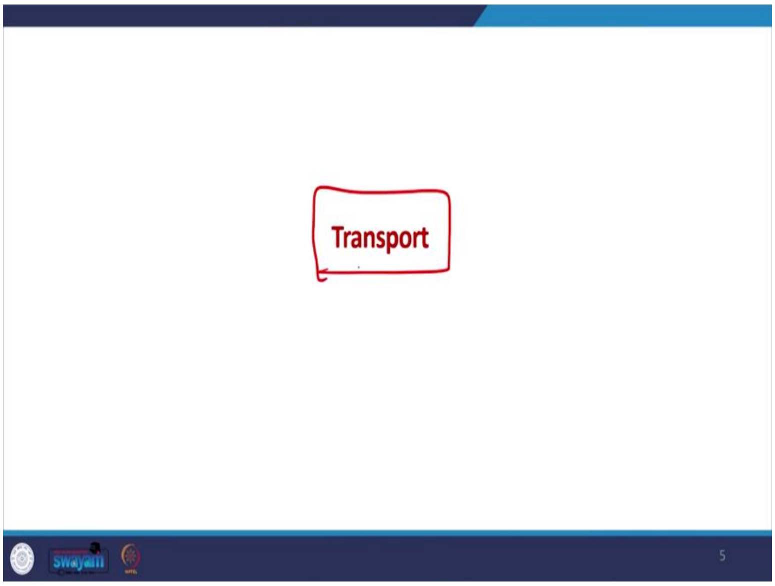

In the next lecture we will start our discussion of the theory of transport in solids specially in nano devices. We could not start this in today's class. So, let me not take more time and let us start this in the next class.

Thank you for your attention; see you in the next class.