Physics of Nanoscale Devices
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Lecture - 15
Fermi-Dirac Distribution

Hello everyone, today we will discuss the idea of Fermi Dirac Distribution which is essentially the way fermions are distributed in any system and there is a large number of fermions in a system. The way they are distributed is governed by the Fermi Dirac distribution. So, let me quickly review what we have done so far.

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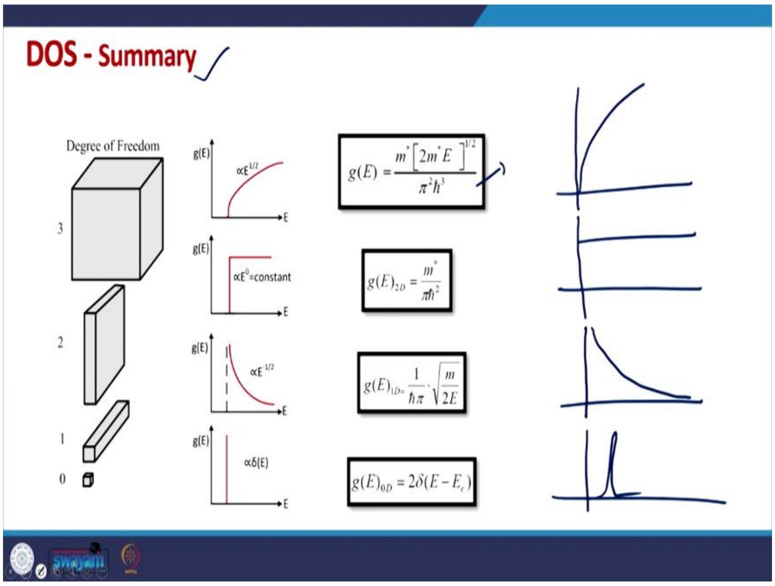

So, we concluded this important topic on density of states and we saw that as the dimensionality of the system changes, the form of the density of state changes fundamentally. So, for example, for 3D systems the density of states is an increasing function, for 2D systems it is a constant function, for 1D systems it is a decreasing function and for 0D systems it is a delta function in a way.

And along with this we also saw how do we sort of reformulate the density of states expressions in actual devices or in the conduction band of devices. Because, in most of the cases we are interested in the electronic population, the electron distribution in the conduction band because those are the electrons that contribute in conduction in devices and solids.

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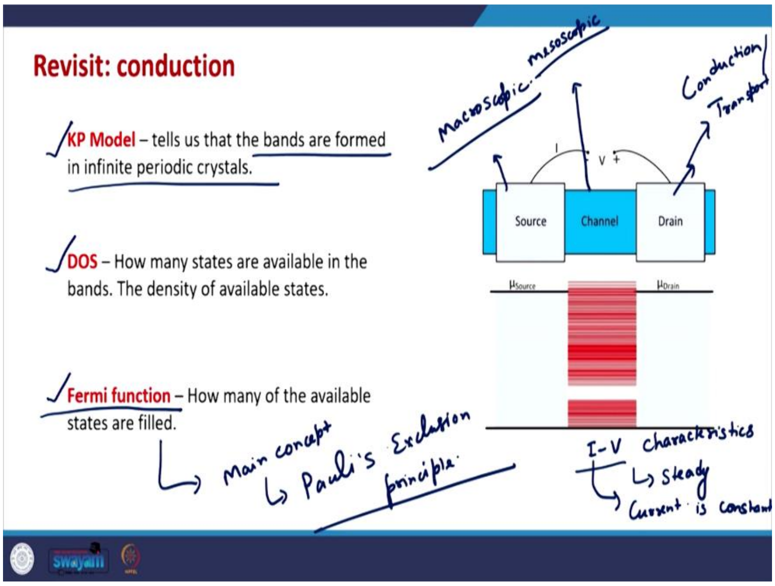

So, finally, we have this we have a two terminal device that we are trying to understand; we are trying to understand the mechanism of conduction or transport in a two terminal device and a two terminal device generally this middle region the channel region is taken to be a very small region the source and drain regions which are essentially like metals or which are like metallic contacts these are much bigger than the channel region.

So, the channel region is in a way a mesoscopic system and the source and drains are microscopic. And that is why we need to understand the density of states in the channel region in order to see how many allowed electronic states are available at a given energy. And we saw that the conduction depends on the available electronic states between the source Fermi level and the drain Fermi level when we apply a voltage that is what we saw in the last class.

So, we have gone through the KP model which tells us that in infinite periodic crystals bands are naturally formed we have also understood the idea of density of states which basically tells us how many allowed electronic states are available as a function of energy, how many allowed electronic states per unit volume per unit energy.

Now, finally, what we need to see is how many of the available electronic states are filled with the electrons in equilibrium. So, when there is no transport there is nothing happening in the device in equilibrium situation how many of the available electronic states are filled and how many of them are empty.

And that is governed by this function which is known as the Fermi function or Fermi Dirac distribution function. This comes from quantum statistics and one of the main concepts that play role here is the Pauli's exclusion principle. So, please keep in mind this distribution is an equilibrium concept.

So, there are two kind of generally we talk about equilibrium and we talk about steady state, sometimes we also talk about the transient states actually. So, that is important difference between these three situations, in equilibrium as I told you in previous one of the previous classes that each and every process in the system is countered by the reverse process which is balanced by the reverse process.

So, if energy or if number of particles are flowing in one direction in equilibrium the equal amount of energy or number of particles will be flowing in the reverse direction. So, that ultimately each process is balanced by the counter process. So, resultantly nothing is happening in the system.

Second idea is the idea of steady states in steady state the system is. So, in steady state a process is happening in one direction as compared to the other direction. So, this is happening but the rate of change of variables is 0.

So, for example, if n number of particles are flowing from left to right in a device and n/2 number of particles are flowing from right to left in a device. So, resultantly n/2 number of particles will be flowing from left to right. So, this number of this resultant number this net number of particles will not change with time.

So, that is the idea of the steady state in steady state there is a net flow of energy particles or there is a net I would say a process happening in the system, but that process is constant with time that is not changing with time. So, generally our device analysis is done in steady state generally when we do I-V characteristics calculations generally we assume steady state. So, that electrons are flowing from one direction to other direction.

But this flow is constant, this is not changing with time which means the current is constant at an instant, although we can change the voltage which will change the current, but it will very soon come to the steady state and that is where we do the analysis.

When we suddenly change the voltage in a device it will suddenly change the current and that is the moment when this rate of change of current or this current will not be constant it will be changing with time and that is known as the transient state when the system variables are changing with time.

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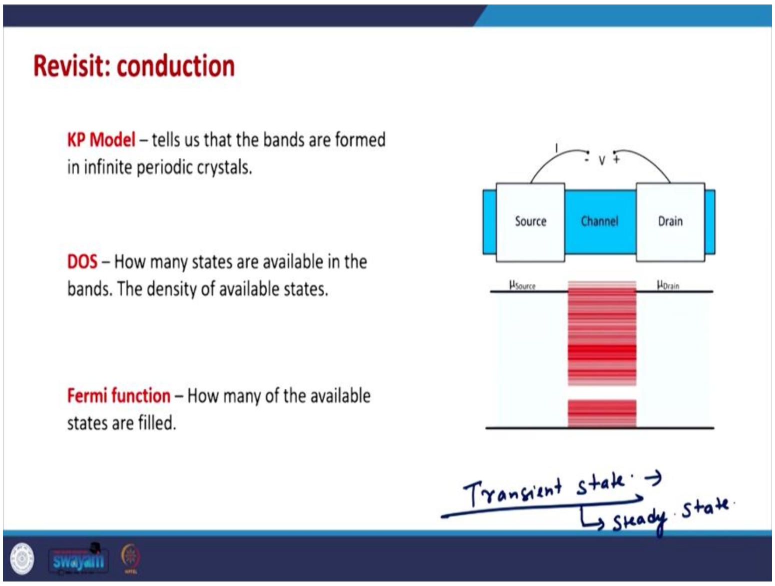

So, that is the transient states, state of the system generally happens when we suddenly change the force or suddenly change the action in the system, suddenly change the voltage applied on a system. Generally, in most of the system most of the stable systems they quickly go to steady state, they quickly attain a steady state specially in our solid state devices although this analysis is in detail done in control theory and control systems which is sort of out of scope of this course.

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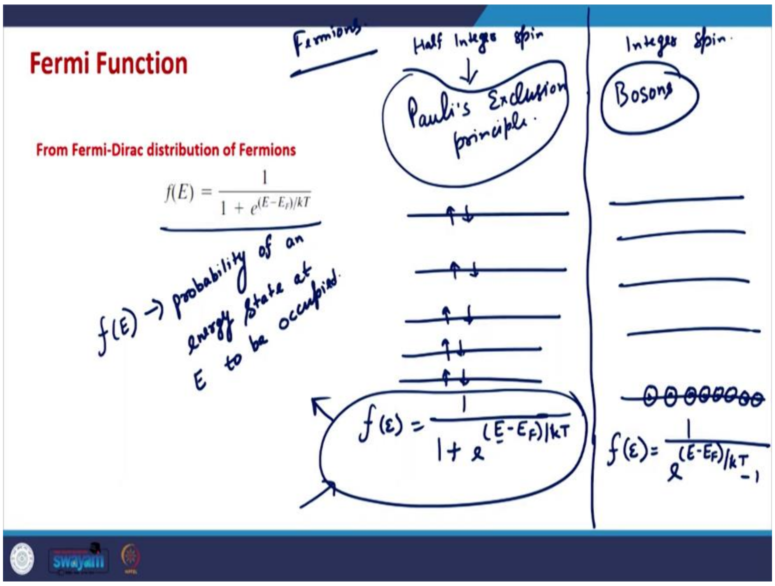

So, now, we will see how a Fermi function or Fermi Dirac distribution function governs the distribution of electrons in various energy states in the solids or in any system. So, this is essentially the Fermi Dirac distribution function we discussed in last class that fundamental particles can generally be categorized in two categories: One is fermion, second is bosons. Fermions have half integer spins so, fermions have half integer spin and bosons have integer spin including spin 0.

So, they follow the these are the fermions, fermions follow Pauli's exclusion principle. And what Pauli's exclusion principle says is that no two fermions can occupy the same quantum state they no two fermions can have the same set of quantum numbers what it means is that a single electronic state, single electronic level can only occupy two electrons of opposite spins.

So, that all the quantum numbers are not same for the electrons. So, if we have for example, in a system if we have these kind of energy levels discrete energy levels each of these can take 2 electrons of opposite spin each of these level cannot hold 2 electrons of the same spin because that is not allowed by Pauli's exclusion principle. Similarly, this level can also hold and this level can also hold 2 electrons of opposite spin.

However, in bosons if these are the discrete energy levels many particles can exist in single electronic state. There can be many particles in the single electronic state generally at room temperature some of these particles are in higher energy states.

So, the distribution of electrons according to Pauli's exclusion principle is given by the Fermi Dirac distribution function which essentially reads as f(E) is $\frac{1}{1+e^{\frac{E-E_F}{kT}}}$ where this k is a constant Boltzmann constant, T is the temperature, E is the energy at which we are concerned about the occupancy of electron at that energy level.

And the bosons distribution function is given by the Bose Einstein distribution function and this is how it basically reads, it has a minus 1 sign in the denominator. There is a subtle difference, but this difference is extremely important because this changes the entire statistics this changes entirely how various electronic states are filled in the system ok.

So, we will be mostly concerned about fermions, we are essentially mostly concerned about electrons and holes which are essentially electrons in with a certain E k relationship. So, our focus will be on Fermi Dirac distribution which has a probabilistic interpretation as well.

So, the Fermi Dirac distribution function f(E) basically intuitively means the probability of an energy state at E energy to be occupied. So, this is the probability with which an energy state will be occupied by the electrons at a given temperature.

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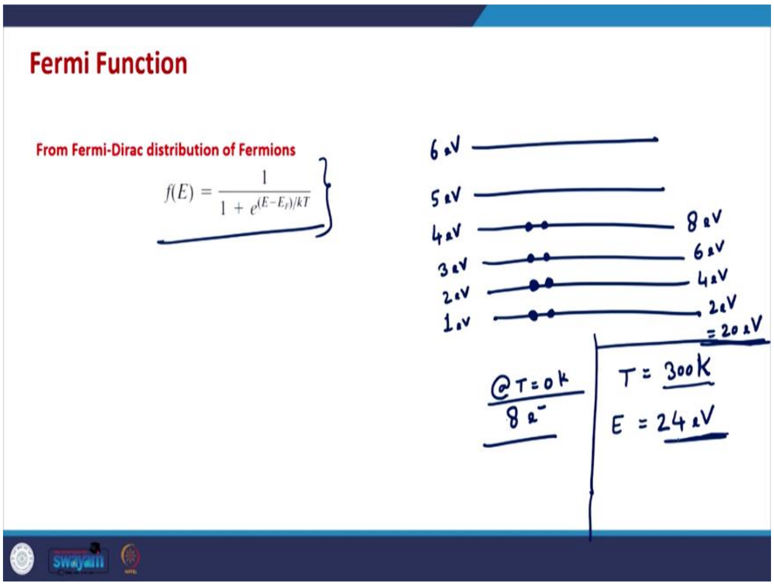

So, there is another way of understanding this let us say we have a system there is an actually this probabilistic argument of Fermi Dirac distribution function needs to be understood properly because we need to see where this probabilistic interpretation basically arises from.

So, if we have a system in which there are various energy levels let us say there are 6 energy levels and let us say the energy of each level is 1eV the energy of the ground energy level the lower most energy level is 1 electron volt, the second energy levels energy is 2 electron volt, third one is at 3 electron volt, fourth one is at 4 electron volt, and fifth one 5 and six one 6 e V.

Generally, this is not how energy levels are distributed in solids, generally the distance between energy levels increases gradually as we go up in actual atoms or solids, but this is just to understand why how electrons will be distributed in a system. So, now, if we have 8 electrons in this system then at T equal to 0 temperature, let us say we have 8 electrons in this system at T equal to 0 temperature when all the electrons are in their ground states.

So, to say they do not have any thermal energy all the electrons will be at the lower possible, lower most possible energy state. So, two of these electrons will occupy the ground state 1eV state, two of these electrons will occupy the 2eV state similarly two will occupy 3eV and two electrons will occupy 4eV state.

So, that is how electrons will be distributed in the in various energy levels in the solid at T equal to 0 kelvin when there is no thermal energy in the system and all the electrons are sitting in their lower most energy configurations. And for these two energy levels I have also shown the spin levels, so that the Pauli's exclusion principle is not violated, but let me remove the spin label from these electrons.

So, this is how electrons will be distributed at T equal to 0 kelvin, but now let us say that we are at room temperature at or let us say at 300 kelvin temperature. So, at 300 kelvin temperature electrons will have some thermal energy as well they might, they will be having this additional amount of kT energy coming from the thermal energy and let us say so, in at T equal to 0 kelvin what is the total energy of electrons.

So, one electron 1 eV for this, one for this, 2 eV, two for this, two for this, 4eV 3 3 6 4 4 8, so 8 eV here, 6 eV here, 4 eV here and 2 eV here. So, in total electrons will have 20 eV of energy at T equal to 0 kelvin ok. Now at room temperature let us say the energy of electrons now is let us say 30 eV just for an example just its not the exact figure this at a certain temperature let us say the energy is increased by 30 eV.

So, now, what can happen is. So, with 30 eV energy how many electronic configurations are possible? So, one of the possible configurations is that, so, if 2 electrons are here, 2 are here, 2 are here and let us say these 2 electrons or just for since we have less number of energy level.

Let us say we take the energy to be 20. So, at T equal to 0 kelvin the energy was 20 eV now let us say the energy is 24 eV. So, one of the configurations in this situation will be please try to guess it what could be one of the configurations.

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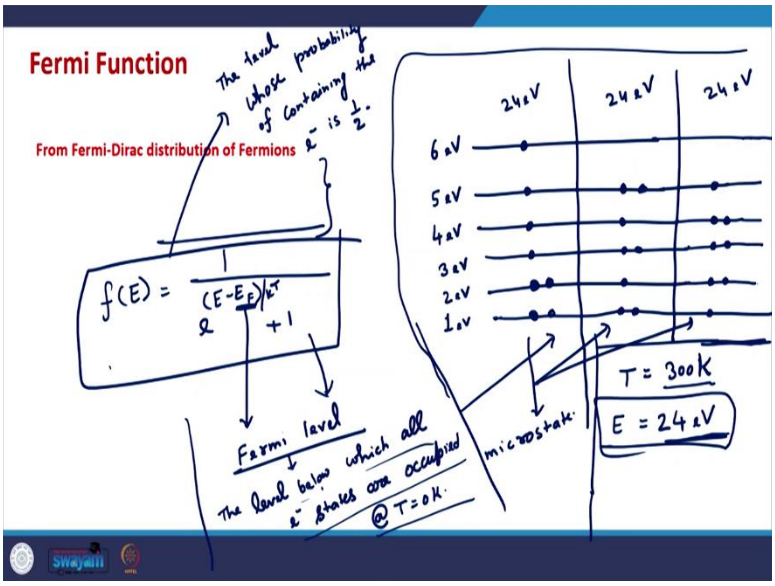

So, one of the configurations would be that this electron which was earlier sitting here in this energy level, this might go here, this might go here. So, the energy is now, so the energy. So, one of the electronic configuration could be this, in this case the energy of the system would be 6 plus 5 plus 4 15 plus 3 18 plus 2 plus 2 22 plus 1 plus 1 24 eV.

So, at a certain temperature if the energy is increased because of the thermal energy then one of the configurations that the electrons can take is this, another configuration that the electrons can take is. So, it is an interesting exercise to find out how many electronic configurations are possible at 24 eV energy I would recommend you to trying doing that.

So, at 24 eV energy one another configuration could be that now 2 electrons in the ground state, 1 electron here, 2 electrons here, 1 here, 1 here. So, that will make it 2 plus 2 4 plus 6 10 plus 4 14 plus 5 19 and so, 2 electrons here. So, this configuration will also have 24 eV energy. So, this is 24 eV configuration of electron this is also 24 eV configuration of electrons.

Similarly, what could be the third possible configuration? It could be 2 electrons here, 2 here that would make energy 2 plus 4 6 plus 4 10 plus 6 16 or plus 6 16 2 plus 4 6 plus 6 12 plus 4 16 plus 6 22, now we need 2 eV energy. So, not that this electron can go slightly let us say now this electron is making a jump let us say in the ground state we only have 1 electron.

So, here now the energy will be 1 plus 4 5 plus 6 11 plus 4 15 plus 5 20 plus 4 24. So, this could be another configuration of electrons in the energy levels ok. So, 8 electrons might have this configuration. So, this is a very simple example what I basically intend to show here is that at room temperatures or at non zero temperatures electrons gain additional energy and with additional energy there are multiple configurations of electrons possible in a system or in an atom.

So, each of these configuration in thermodynamics is known as a microstate of the system. So, what it means is that now the system can have multiple microstates at non zero temperatures the system can have multiple microstates ok, with same energy.

So, essentially when we consider all of them together there will be a probability that certain energy level is occupied or unoccupied because in some configurations that energy level may be occupied in another micro states or another configurations that energy level might be might not be occupied ok.

So, at non zero temperatures there is a probability corresponding to each state that the state is occupied and that probability is given by the Fermi Dirac distribution function or the simply the Fermi function which is written as this essentially this ok.

So, you got this idea at T equal to 0 kelvin all the electrons assume the lowest possible energy and there is only 1 microstate possible for the system, but at non zero temperatures because of the additional energy now multiple micro states are possible. So, we need to see the probability that a certain state is occupied and that probability is given by the Fermi Dirac distribution function or the Fermi function.

So, that is essentially the idea of the Fermi Dirac distribution function or the Fermi function. So, here this E_F which seems to be a constant here this is known as the Fermi level of that system Fermi level of the material and this is also known as electro chemical potential. The way it is defined is that it is the level below which all the states are occupied at T equal to 0 kelvin.

So, it is the level below which all electronic states are occupied at T equal to 0 kelvin. So, this is one way of defining this please keep in mind this is a very important concept in solid state physics.

We are essentially in a way compressing the entire quantum statistics or quantum statistical ideas into the Fermi level and the Fermi Dirac distribution function or the Fermi function. So, we are taking these ideas and we are not going into the details of distribution of electrons in solids or the probability with which various energy levels would be occupied.

So, this is a very important idea specially in condensed matter physics and solid state physics. So, one way of defining the Fermi level is it is the level below which all electronic states will be occupied at T equal to 0 kelvin. It also has a factor of kT another definition of Fermi level is, it is the level whose probability of containing the electron is half. So, we will see both of these things. So, it is the level whose probability of occupancy is half basically ok.

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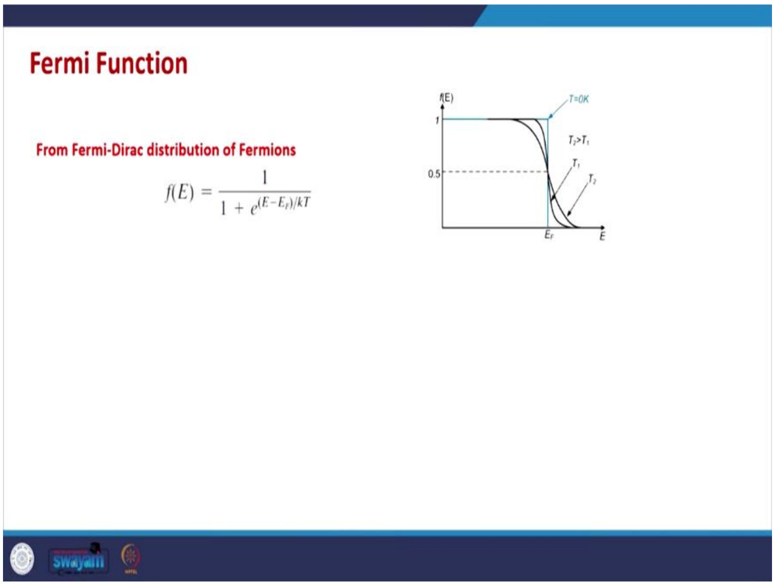

So, now we will explicitly see what happens, how this Fermi level looks like Fermi function looks like or in other words the Fermi function looks like.

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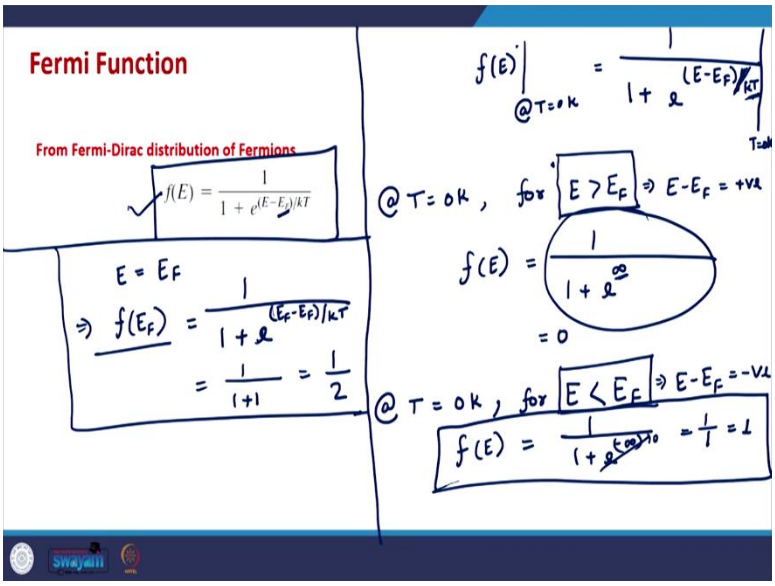

So, if we plot the Fermi function or if we try to sort of see how the Fermi function at T equal to 0 kelvin looks like. So, it will be $\frac{1}{1+e^{\frac{E-E_F}{kT}}}$ at T equal to 0 kelvin.

So, as we put T equal to 0 here as T approaches 0 here then the denominator becomes 0 start approaches 0 as well which means now the numerator will blow up, numerator if numerator is positive it will be it will go to positive infinity if numerator is negative it will go to negative infinity.

So, which means at T equal to 0 Kelvin's kelvin temperature for energy values greater than E_F for energy levels higher than Fermi energy levels E -E_F would be positive which means this f(E) will be 1 divided by 1 plus this quantity here will tend to infinity.

Generally, we do not write it like this, but just to show you. So, this denominator of this entire function will be tending to infinity. So, it will be 0. So, what it means is that all energy levels above E_F are empty at T equal to 0 kelvin. So, which is essentially this is why we define Fermi level as the energy level below which all levels are occupied at T equal to 0 kelvin that basically comes from here. Now let us see what happens to energy levels below E_F.

So, at T equal to 0 kelvin for energy levels less than E_F for all energy levels that have energy less than E_F, E - E_F will be negative and the Fermi function will be 1 by 1 plus exponential of minus infinity this exponent here will tend to minus infinity which will essentially make this 0. So, this will be 1 by 1 equal to 1.

So, which means that the probability of all energy levels above Fermi level for all energy levels sorry below Fermi, all energy levels whose energy is below Fermi level will be 1. So, all the energy levels below Fermi level will be occupied at T equal to 0 kelvin and that becomes the defining feature of the Fermi level as well. The second definition that I told you was that Fermi level is the energy level whose probability of being occupied by electron is half let us see how that comes about.

So, this is the Fermi function and in this Fermi function if we replace E by E_F; that means, the left hand side will become f (E_F) the probability of E_F being occupied and that is equal to 1 divided by E_F - E_F by kT. E_F is 0 exponential 0 is 1. So, essentially it becomes 1 plus 1 it becomes 1 by 2.

So, Fermi level is the energy level whose probability of being occupied at any temperature is half, at T equal to 0 temperature this situation becomes slightly complicated because now we have 0 in numerator, 0 in denominator and this analysis needs lot more maths. But generally this is the case at or at least above 0 kelvin temperature Fermi level is the level whose probability of being occupied is half basically and these 2 are the defining feature of the Fermi level.

So, today we saw how in a very intuitive way how Fermi Dirac distribution function or the Fermi functions idea comes about and we also try to understand the idea of Fermi level. So, this idea of Fermi Dirac distribution function comes from the quantum statistics and that is because according to Pauli's exclusion principle no two fermions can occupy the same energy state, same level same quantum numbers.

So, 1 energy level can hold 2 electrons of different spins because different spins can have different spins will have different quantum numbers and in presence of magnetic field their energy will also be different because that will cause splitting. So, that is why 2 electrons can occupy the same energy levels, but the 2 electrons need to be of opposite spin.

Because of that principle at non zero temperatures generally there are many micro states possible for a system and that is why we need to see or we define the probability of a certain state to be occupied by electrons and that is given by the Fermi Dirac distribution function or Fermi function.

So, in the next class we will discuss some more ideas about or some more or maybe application of Fermi Dirac distribution function and then we will start with the discussion of theory of transport in devices. See you in the next class.

Thank you for your attention.