Digital Protection of Power System

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Lecture 04

Fundamentals of Digital Relays

Hello friends. So, in the last class he have discussed regarding the different components of digital relays and we have discussed that there are different components available in digital or numerical relay. So, we have seen the block diagram of digital and numerical relay and we have seen that there are various components like the signal conditioning block is there, then conversions sub-system block is there, then digital input and digital output modules are also there algorithm module is also there and along with this various communication peripheral devices and the human machine interface block is also there.

So, and we have also discussed the importance of each and every component, why those components are required that also we have discussed. Now, in the today's class, we will discuss about the sampling. So, whenever we carry out the sampling, how what are the different rules we need to consider, then, we will also discuss the concept of aliasing.

So, if sampling is not proper, then what are the different errors that is to be introduced in the original or reconstructed signal. So, that also we will discuss and at last we will discuss what are the factors that is to be considered for sampling rate and we will also discuss the concept of sliding window.

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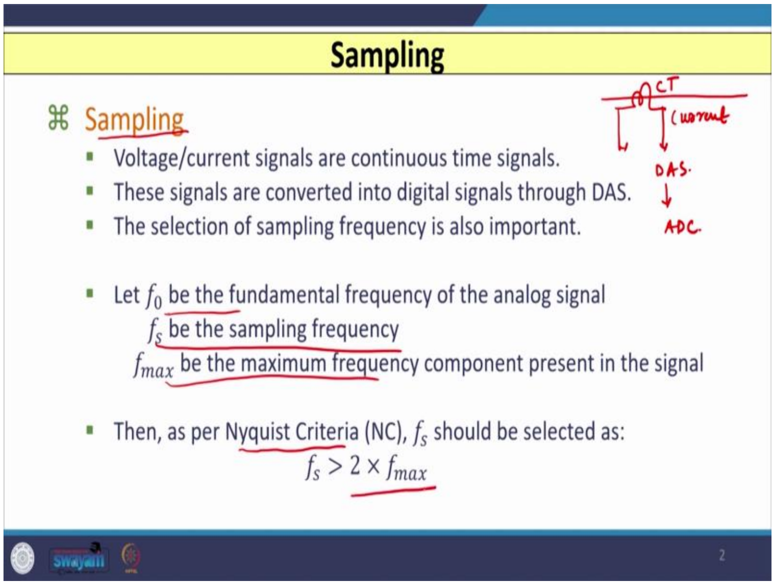

So, let us start with the first what is sampling. So, we know that the voltage and current are continuous signals. So, as I explained you in the last slide, when we have a feeder and when we are going to acquire the signals of let us say current from the secondary of the CT, this is our CT. So, then these current signals phase wise these signals are given or acquired by the data acquisition system and these signals are given to the analogue to digital converter block.

So, the function of analogue to digital converter block is to convert analogue signals which are acquired through CT secondary and PT secondary or CVT secondary into digital form. Now, whenever ADC is going to acquire the voltage or current or any other signals, then there must be some rule regarding the sampling frequency.

So, selection of sampling frequency plays an important role during data acquired from CT or PT secondary or both. So, let us consider, let us assume that we have the f_0 is the fundamental frequency of analogue signal, let us say the current signal which we acquired from secondary of CT and that is given to ADC, f_s be the sampling frequency that means, f_s is the frequency at which ADC is going to sample the data and f_{max} is the maximum frequency component present in the acquired signal these three are the points which we consider, then as per Nyquist criteria, sampling frequency of the acquired signal should be always greater than 2 times the maximum frequency component present in the acquired signal.

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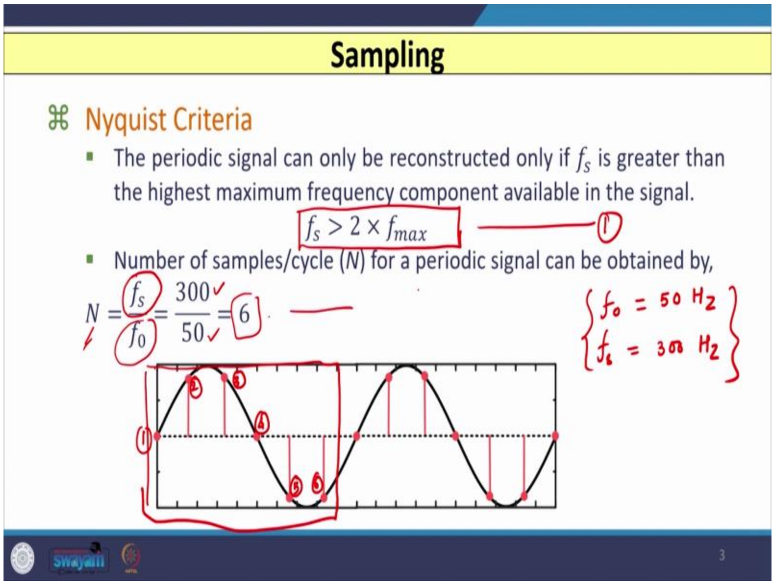

So, the periodic signal can only be reconstructed only and only if your sampling frequency is greater than 2 times the highest frequency component present in the signal. This is nothing but the Nyquist criteria and mathematically equation wise we can represent this using this equation, let us, say this is equation number 1, where f_s is the sampling frequency of the acquired signal at which we are going to sample and f_{max} is the maximum frequency component present in the acquired signal.

Let us, say we have a signal whose fundamental frequency let us say f_0 is the fundamental frequency of the signal which we are going to acquire, let us say it is 50 hertz and the sampling frequency which we are going to sample the signal let us say f_s that is 300 hertz. So, with this available data of fundamental frequency f_0 that is 50 hertz and fs that is 300 hertz, we can definitely find out number of samples required for a periodic signal in a cycle.

So, in one cycle, how many number of samples are there we need to acquire or ADC has to acquire that we can find out if these two values are known, that is f_0 and fs. Then, how we can calculate? We can calculate using the equation N which is nothing but the number of samples per cycle that is equal to the sampling frequency divided by the fundamental frequency.

So, if we do that, then if we put these values, we will get the 6 samples. So, you can see here in the waveform, this is the 1st sample, this is the 2nd sample, this is the 3rd sample, this is the 4th

sample, this is the 5^{th} sample and this is 6^{th} sample. So, in one cycle here you can see in one cycle, we are going to acquire 6 samples and this is how we are going to acquire.

Now, here you can see that your fundamental frequency f_0, this is normally constant maybe either in our Indian system 50 Hz or maybe some other countries 60 Hz. However, only variable think or parameter is the sampling frequency and that we can vary if we vary the sampling frequency then number of samples which we need to acquire in a cycle that may vary, so, it can be 6 or it can be higher value also or it can be lower value also.

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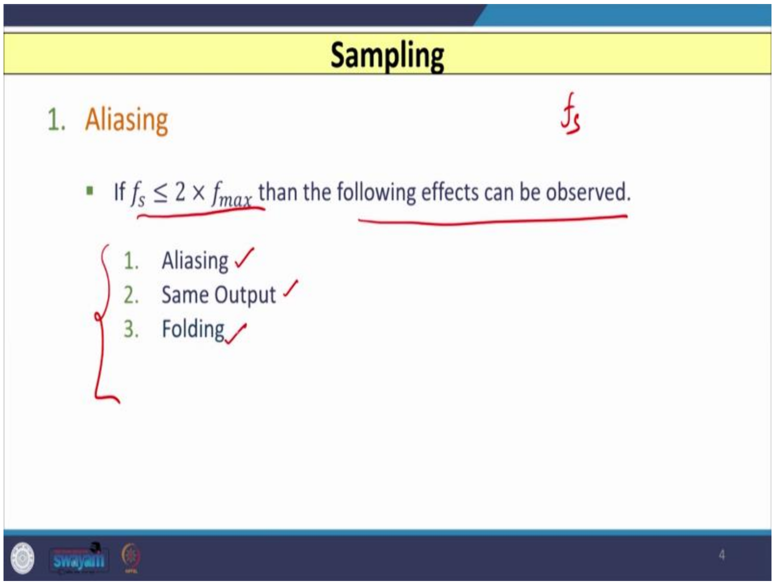

Now, let us consider a case where sampling frequency f_s, which at which we are going to acquire the signal is not going to follow the Nyquist criteria or Nyquist rule. So, f_s is let us say lower than or equal to 2 times the maximum frequency present in the acquired signal. If this is the case, then it is going to lead these three effects. The first effect is known as aliasing effect, the second effect is known as same output and third effect is known as folding phenomena. So, let us consider each and every effect in detail one by one.

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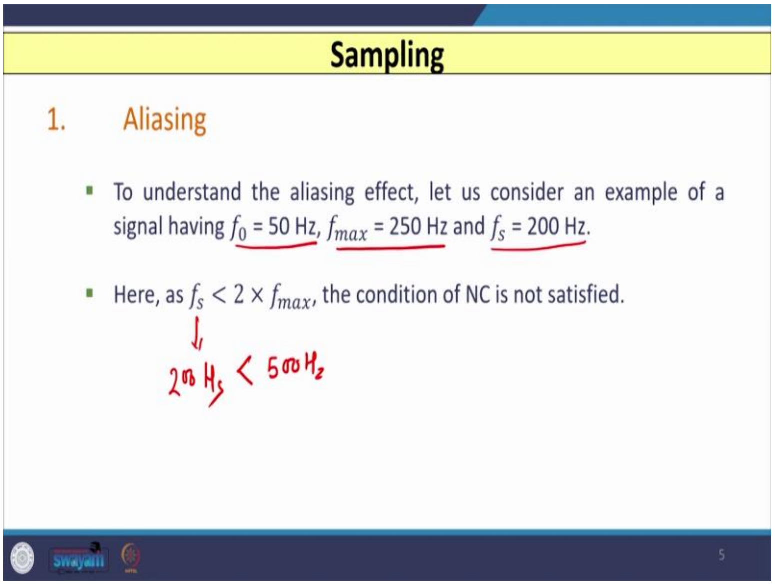

So, now, let us see what is aliasing. So, to understand the aliasing effect, let us consider one example of a signal who has 50 Hz fundamental frequency. So, your f_0 is 50 Hz and this signal is sampled at a sampling frequency of 200 Hz and the maximum frequency component present in this signal is let us say 250Hz.

So, according to Nyquist criteria, your fs which is 200 Hz in this case, this is 200 Hz that is lower than 2 times the f max f max is 250 Hz. So, 2 times f_{max} that is 500 Hz, so, 200 Hz is lower than 500 Hz. So, Nyquist rule or criteria is not followed and it may lead to the three different effects as I told you.

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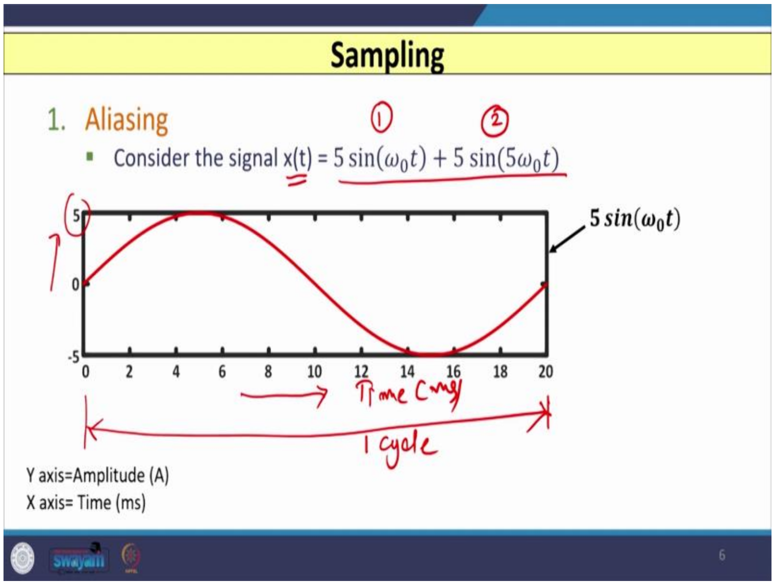

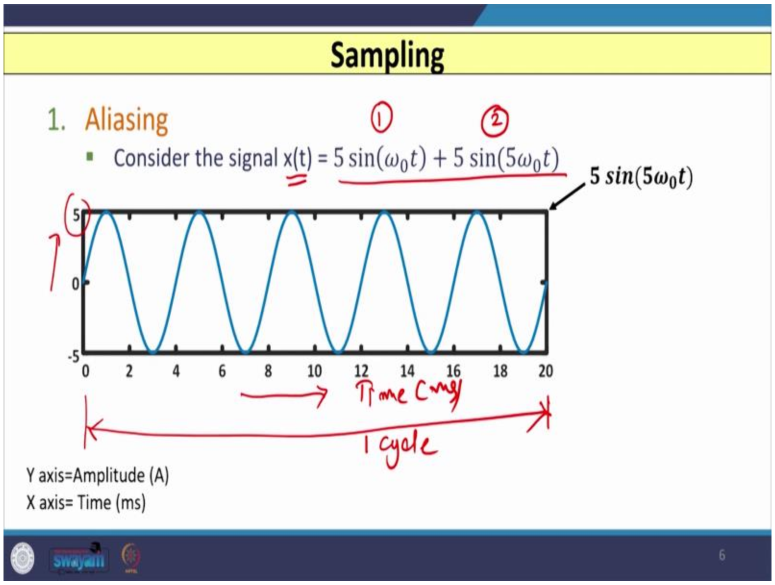

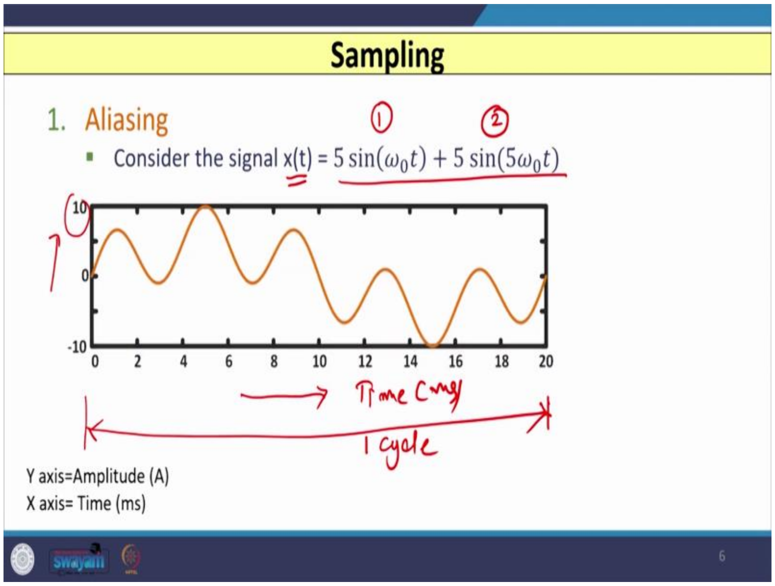

Now, to understand the aliasing in more details, let us consider a signal $x(t) = 5\sin(\omega_0 t) + 5\sin(5\omega_0 t)$ where $\omega_0 t$ is nothing but $2\pi f_0 t$, f_0 is the fundamental frequency and the second part that contains $5\sin(5\omega_0 t)$. So, if I plot this x(t) signal which contains two parts one and two, one is $5\sin(\omega_0 t)$ and other is $5\sin(5\omega_0 t)$, then these signals look like this.

Where on X-axis we have the time in milliseconds and on Y-axis you have the amplitude let us say we are acquiring current quantity so, let us say ampere. So, this signal x(t) looks like this. Now, if I plot only the first part of this signal x(t) that is $5\sin(\omega_0 t)$, then this looks like this and you can see that you have time on X-axis, so, in one cycle that is 20 millisecond, we can see here this is one cycle, it will have only 1 sinusoidal signal and it is amplitude is that is 5

Now, if I plot second part of x(t), then second part of x(t) looks like this that is $5\sin(5\omega_0 t)$. So, again you can see on X-axis you have 5 sinusoidal waves are available in 20 millisecond compared to earlier case you have only 1 sinusoidal wave in 20 millisecond. Moreover, amplitude is also 5 and if I combined both part 1 and part 2 of x(t), then you will have the signal which looks like this.

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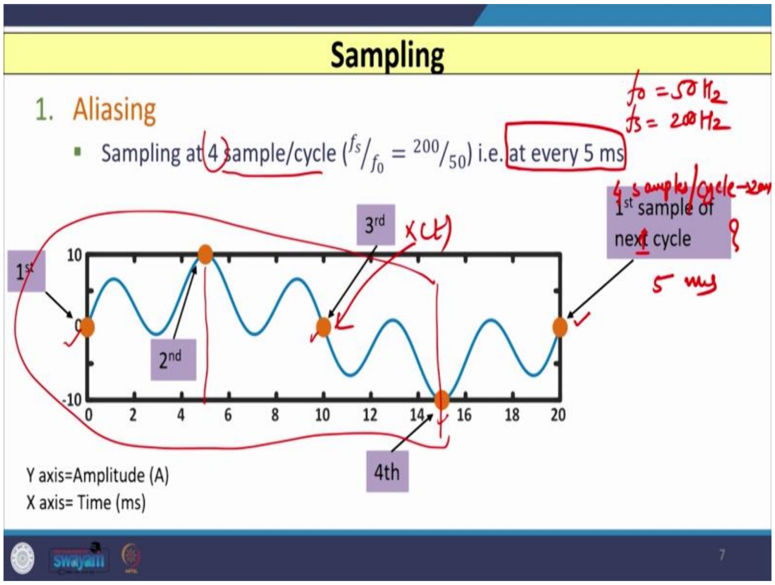

Now, if I consider let us say the we are sampling at 4 samples per cycle, so, as I told you my f_0 is 50 Hz and my sampling frequency we are sampling the signal at f_s is equal to 200 Hz, which is not going to follow the Nyquist criteria. So, if I take 200 by 50 then you will have number of samples in one cycle, so, that comes out to be 4.

So, in on time axis as I told you, we have 20 millisecond. So, we can say that for 4 samples, for 4 samples in a cycle, if we have 20 millisecond, then for 1 sample we have obviously, 5 millisecond. So, at every 5 milliseconds, we have to acquire the sample and total sample we have to acquire are 4 in one cycle.

So, if I just tell you the original signal x(t) this is our signal x(t). So, if I tell you this signal, we are going to acquire 4 samples at every 5 milliseconds, then this is your 1st sample, this one, the 2nd sample is acquired at 5 millisecond, the 3rd at 10 millisecond and 4th at 15 millisecond. So, this is going to complete your number of samples in a cycle that is 4. So, obviously, we are going to acquire these samples one by one and whenever this is the next sample, when it is available, again another 4 samples again another 4 samples like this.

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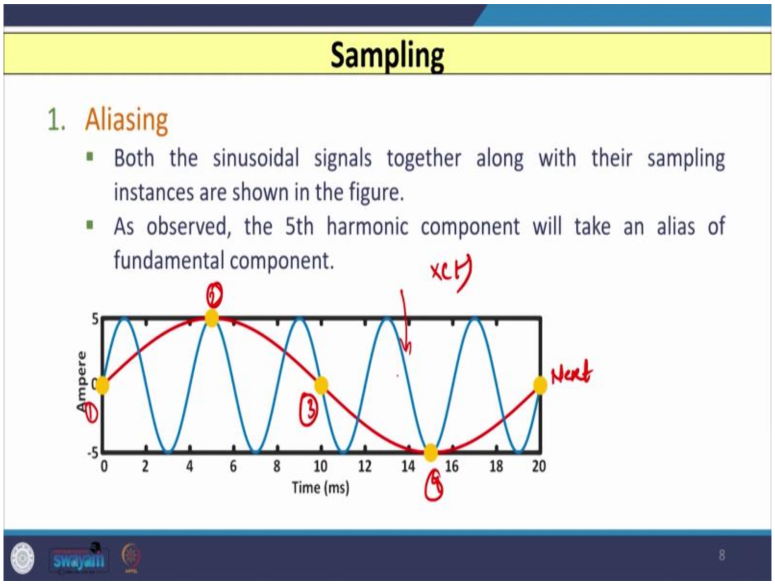

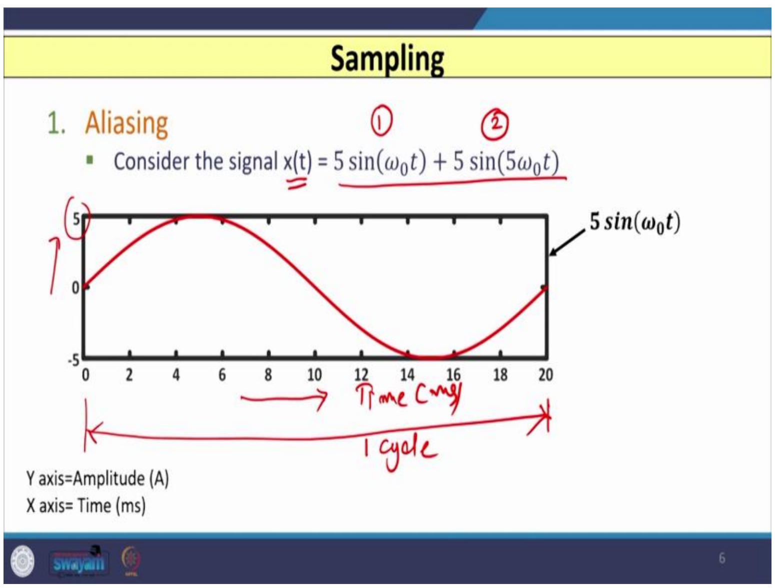

Now, you can see that if I plot both the parts of x(t) together the first term that is 5sin(ω_0t) and second part you can see that is 5sin(5ω_0t), then this both are superimposed together, and it looks like this. So, this is your red colour is your first part and blue colour is your second part of our signal x(t).

So, you can see that here I have shown both the parts of x(t) along with I have also shown the sampling instant this is your 1st sample, 2nd, 3rd and 4th and again next sample, this is your next sample. So, you can observe from these two diagrams with sampling instances that the 5th

harmonic component that is this blue one that will take an alias of the fundamental component. So, that means, that is going to hide behind the fundamental component.

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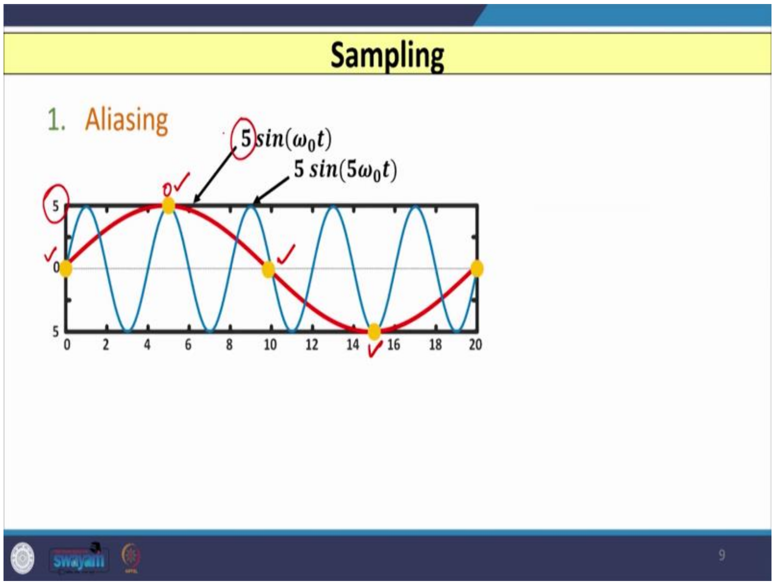

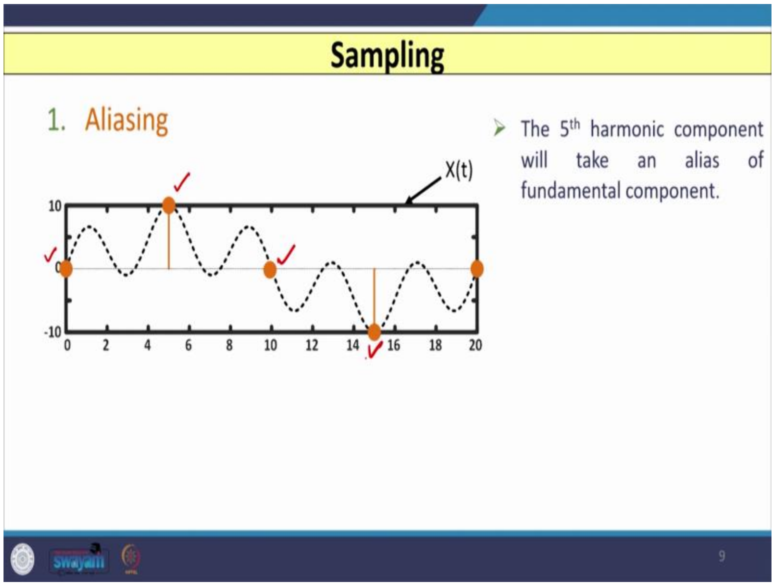

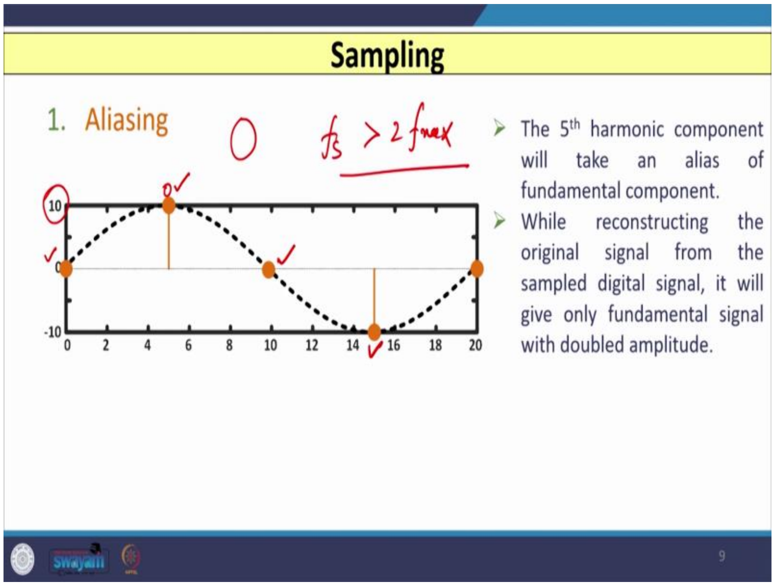

So, if I consider the same signal red one your first part $5\sin(\omega_0 t)$ and second blue one second part $5\sin(5\omega_0 t)$, then we can say that your 5^{th} harmonic component will take an alias of the fundamental component. So, you will see that you will find like this and when you reconstruct the original signal from your whatever available samples in digital form, then it will give you the fundamental component only with double amplitude.

So, you can see that amplitude of this that is 10 whereas our original signal again you can see its amplitude is 5 only. So, we can say that when we are going to reconstruct the signal in which Nyquist rule is not followed that means you are sampling frequency is not greater than 2 times f_{max} if this rule is not followed, then we will observe aliasing effect and whatever is the 5^{th} harmonic in this case that is going to take an alias of the fundamental component.

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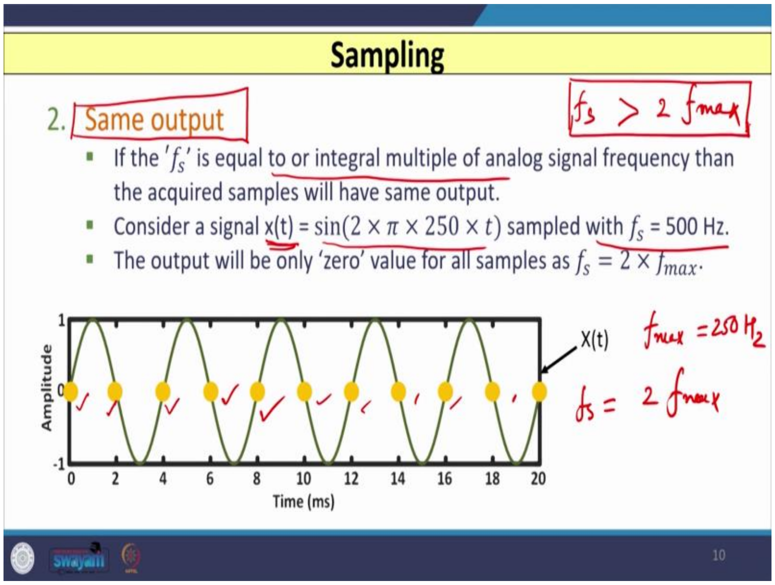

Now, second effect let us discuss that is known as same output. So, as the name suggests same output means, at every time we are going to have similar output same value this is happened when your sampling frequency is equal to or integer multiple of analogue signal frequency then your acquired signals will have the similar output. So, to understand this, let us consider another signal x(t) = $sin\ (2 \times \pi \times 250 \times t)$ and this signal let us assume that it is sampled at f_s is equal to 500 Hz, you can see that here the f max that is the maximum frequency component present in this signal x(t) that is 250Hz.

So, according to Nyquist criteria your sampling frequency should be greater than 2 times f_{max}. However, here we have f_s that is same as 2 times f max, because your f_s is 500 Hz and f_{max} is 250Hz. So, 2 times f_{max} that is also 500 Hz. So, if this is the case, then you can see from this waveform, you will have the 0 value for all samples. So, this is also an important phenomenon and that is why we need to consider this Nyquist criteria while selecting the sampling frequency of the acquired signal.

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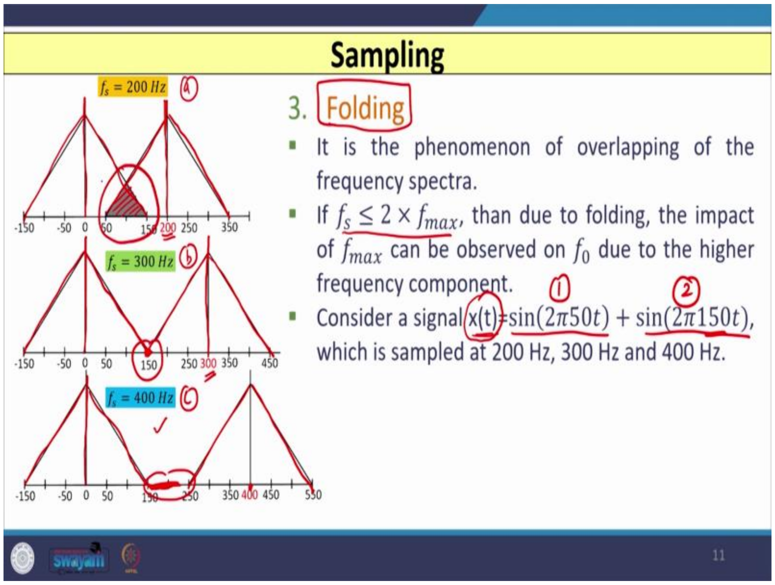

Now, the third important effect is known as folding phenomena. So, folding phenomena is nothing but overlapping of frequency spectra. So, this is going to happen when your sampling frequency is lower or equal to 2 times the maximum frequency component present in the acquired signal.

So, in this case, to understand the folding phenomena, let us consider the x(t) one signal which is given by x(t) =$\sin(2\pi 50t) + \sin\,(2\pi 150t)$. So, x(t) contains two parts the one part you have sin(2π50t) and second part you have sin(2π150t) and let us assume three different sampling frequencies that this signal x(t) is going to acquire at three different sampling frequency the first it is let us say 200 Hz sampling frequency, second let us say it is 300 Hz and third let us say it is going to acquire at 400 Hz.

Now, if we consider the same x(t) which is acquired at 200 hertz sampling frequency so f_s is 200 Hz, then if I consider the x(t), then you can see that we can start from this 0 part here and then we have the component 150. So, you can go from this let us say up to 150. So, you can see 0, 50, 100 and 150. So, on both the side you can go up to 150.

Then the next part you can see your sampling frequency is 200 Hz. So, I have marked here 200 Hz. So, another line is again at 200 hertz and from 200 Hz you have to go both the sides minus 150 so, 200 minus 150 it will be available here 50 and on this side 200 plus 150 that is 350. So, you can see there is an overlapping of this.

Let us, say we sample this signal x(t), which has two components sin(2π50t) and sin(2π150t) at sampling frequency that is 300 Hz. So, in that case you have the first part here and you have the 150 on this side and another 150 on this side and when you write the sampling frequency here that is 300 Hz, then you can plot on both sides again 150 and again 150. So, 150 and 450.

So, then you will have this point, so, you will get same output, second type of error and third if we acquire the same x(t) signal at fs is equal to 400 Hz. So, here the Nyquist criteria is followed, then you can see there is no overlapping here. So, you can start from this 0 and you can go both the side 150 and then again you have from 0 to 400 here. So, again 400 minus 150 that is 250 and 400 plus 150 that is 550. So, there is no overlapping here you can see here and here. So, this is nothing but the folding phenomena.

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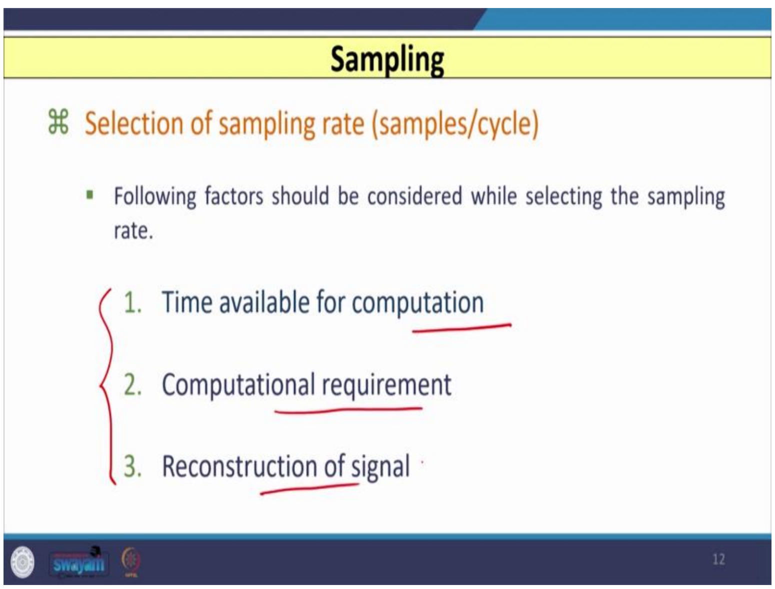

Now, let us see when we are going to select the sampling rate that is what are the number of samples in a cycle we need to have or consider, then we need to consider three important factors. So, while deciding how many number of cycles I have means let us say we are acquiring one current signal.

So, then, whether we will go for 6 samples in one cycle or 12 samples or 20 samples or 40 samples in a cycle that is decided by these three important factors. These factors are the first time available for computation, the second is the computational requirement and third that is the reconstruction of signal.

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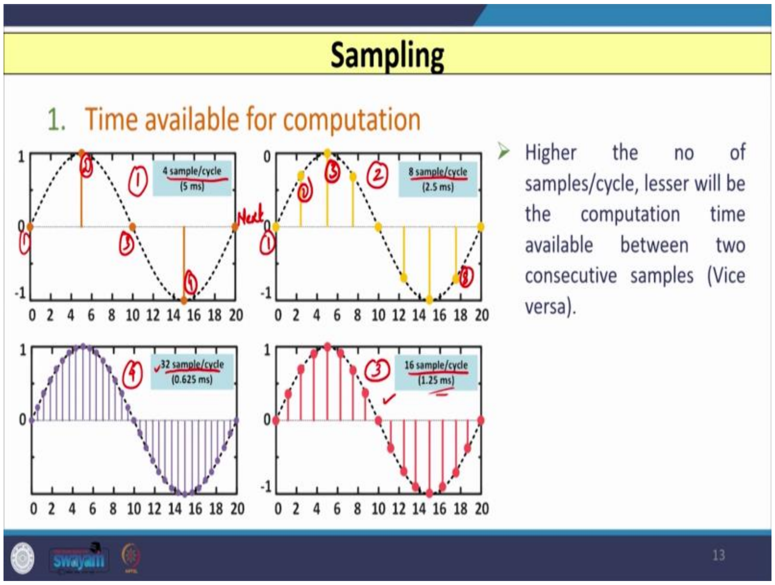

So, let us discuss each factor one by one, the first factor is the time available for computation. So, here I have shown 4 graphs and in each graph, you can see I have used 4 samples per cycle in 1st graph. In 2nd graph, I have used 8 samples per cycle. In 3rd graph, I have used 16 samples per cycle and in 4th graph, I have used 32 samples per cycle.

So, I have doubled the number of samples in a cycle from 4 to 8, 8 to 16 and 16 to 32. So, here you can see that when I have 4 samples per cycle in the 1st graph, we have to acquire the samples at this point, 3, 4, and then this is your next sample. Similarly, when you have 8 samples, that means when you have 4 samples in a cycle, you have to acquire the sample at every 5 millisecond, when you have 2nd graph when you have 8 samples in a cycle, then you have to start here and you have to end here 1 to 8.

So, 1st, 2nd, 3rd like that. So, you have to acquire the sample in the second graph at every 2.5 millisecond, this time further reduces if you go the 3rd waveform, where you have 16 samples you have to acquire and that each at 1.25 milliseconds and in 4th graph you have to acquire 32 samples each at 0.625 millisecond. So, you can see that higher the number of samples in a cycle you acquire lesser will be the computation time available between two consecutive samples and this is very important point.

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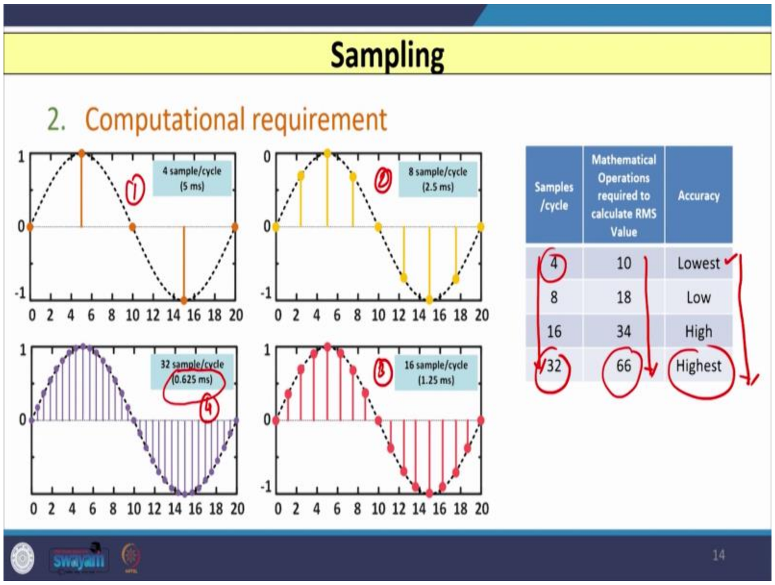

The second is the computational requirement. So, again same 4 graph I have shown 1, 2, 3 and 4 and in 1, 2, 3 and 4 we have only difference is the number of samples in a cycle 4, 8, 16 and 32 in graph number 1, 2, 3, 4. So, as you increase the number of samples in a cycle, then whatever mathematical operations you need to perform to calculate a particular quantity which is required by the algorithm, let us say RMS value or some phasor value or any other thing that is also going to increase and at the same time as you increase the number of samples in a cycle accuracy is also going to increase.

In one case, where 4 samples per cycle is there accuracies lowest and in 4th case where a number of samples in cycles are 32 where the accuracy is the highest but the mathematical operations required to calculate the RMS value that increases and the time between each sample that is also very small, that is 0.625 millisecond compared to 1st graph where you have 5 milliseconds.

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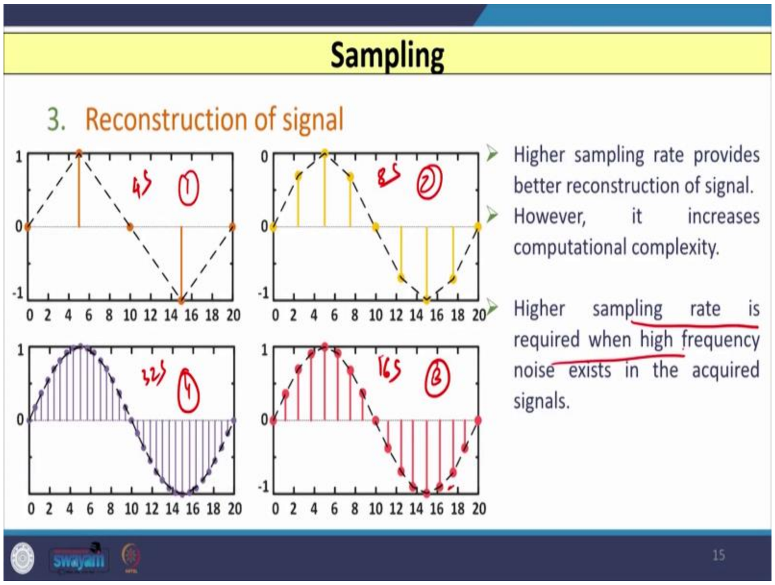

The 3rd thing that is the reconstruction of signal. So, again, you can see this is your 1st, 2nd, 3rd and 4th graph where you have 4 samples, you have 8 samples, you have 16 samples and you have 32 samples. So, here you can see that when you reconstruct the signal, this signal is not sinusoidal wave, this 8 samples is also not perfectly sinusoidal, 16 you will find few deviations at few points, but 32 it looks like perfect sinusoidal.

So, higher the sampling rate provides better reconstruction of the signal that we can see. So, as we increase the number of samples in a cycle, then reconstruction is better. However, if we have higher number of samples in a cycle, then it also increases computational complexity. So, higher sampling rate however, it is required when high frequency noise exist in the acquired signal.

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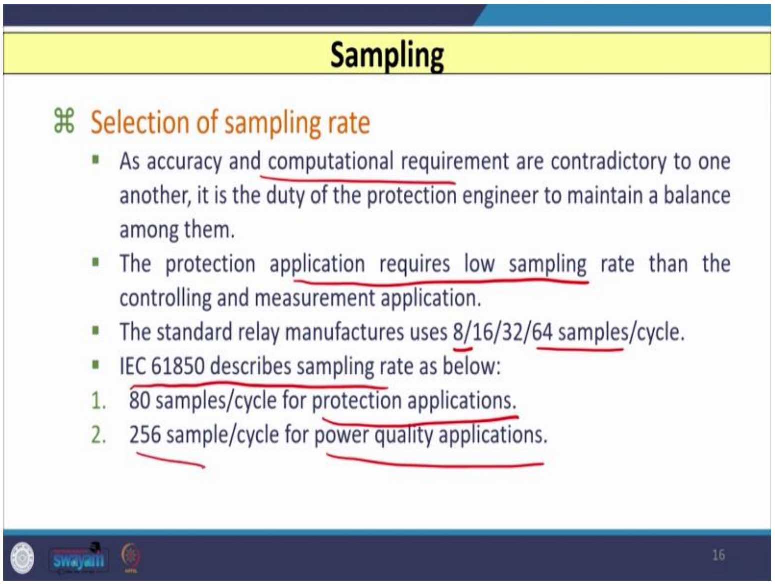

So, when we select the sampling rate, we have to consider the three important parameters which we have discussed, however, as accuracy and computational requirement both are contradictory in nature. So, it is the duty of protection engineer to have a balance or maintain a balance among these two things.

For all protection applications normally it requires low sampling rate compared to the control and measurement application. Most of the standard relay manufacturers they use 8 samples to 64 samples in a cycle. So, they will give the range 8, 16, 32, 64, however, as per IEC 61850 that is substation and automation protocols for protection applications 80 samples per cycle is standard whereas, for power quality applications 250 samples per cycle is also standard.

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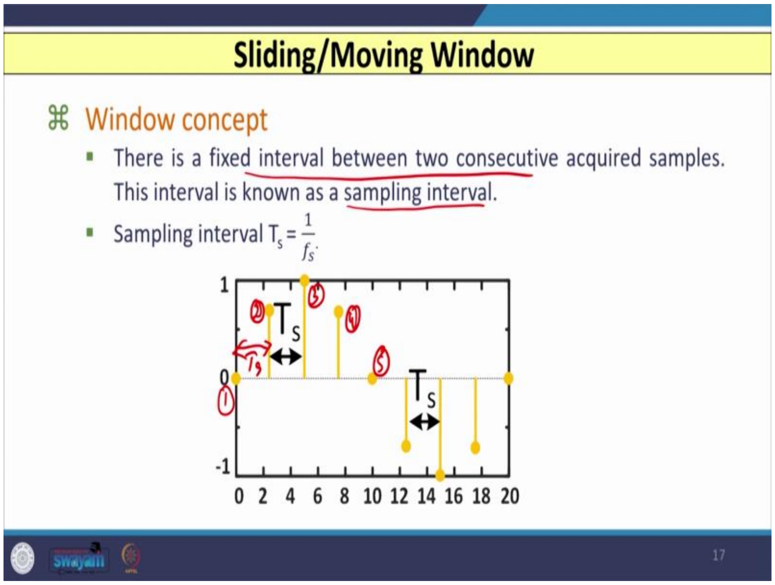

Now, let us see the sliding or moving window concept. So, when we consider the sliding or moving window concept, let us first consider one simple wave where I am going to acquire the sample at some fixed time interval. So, fixed time interval between two consecutive samples play an important role and this fixed time interval between two consecutive samples that is known as sampling interval.

So, here this is my 1st sample, this is 2nd, 3rd, 4th, 5th and so on. So, you can see the interval between 1st and 2nd that is also T_S between 2nd and 3rd that is also T_s and so on. So, that is fixed. How do you obtain the sampling interval T_S? It is simply reciprocal of sampling frequency f_s.

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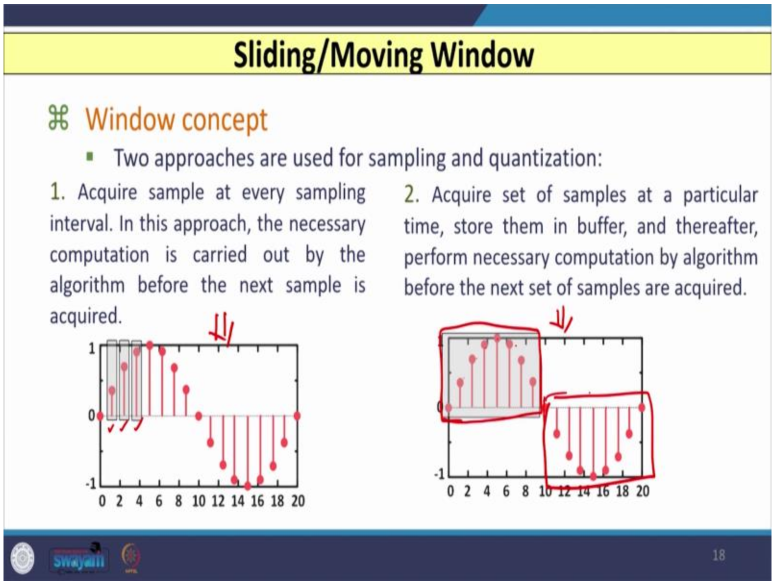

Now, when we consider the sampling and quantization then two approaches are normally used. The first is you can see this waveform where you can acquire the sample at every sampling interval. So, this is 2nd, 3rd when you acquire the sample, you acquire the sample and whatever necessary computation you perform before the next sample is available or acquired.

So, this is one approach. The second approach is you acquire a set of samples, let us say this window is given. So, you acquire let us say 1, 2, 3, 4, 5, 6, 7, 8 samples, let us say set of samples, let us say 8, store them in buffer and there after you perform the computation by the algorithm depending upon the logic and then you acquire next set of samples, let us say that is also 8 another approach is also possible.

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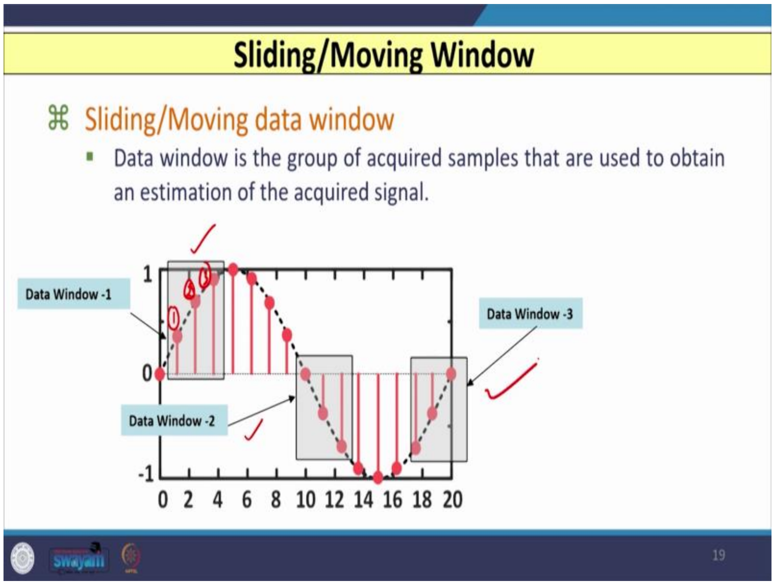

Now, the concept of sliding or moving window. So, here you can see that when what is data window or moving window or sliding window, data window is nothing but a group of acquired samples where you need to perform some computation or estimation. So, here you can see I have shown this data window 1 which contains 3 samples, let us say 1, 2, and 3. Similarly, you can see another data window that also contains 3 samples data window 3 that also contains 3 samples.

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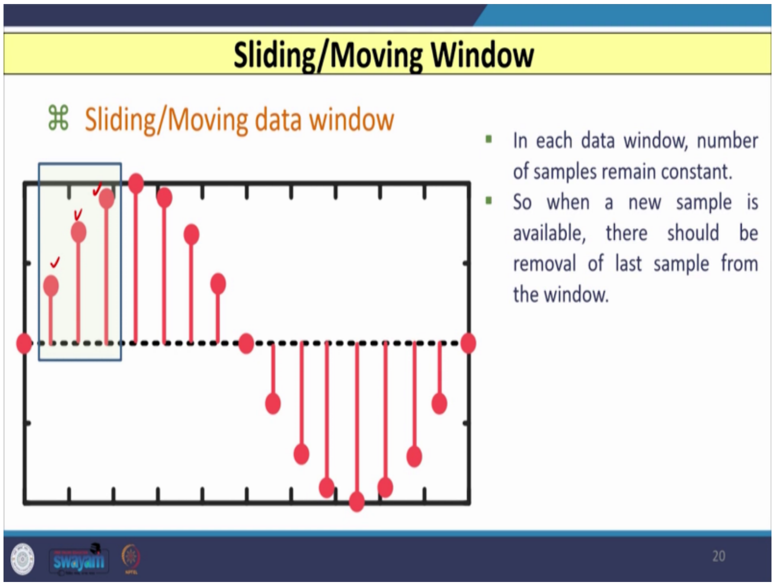
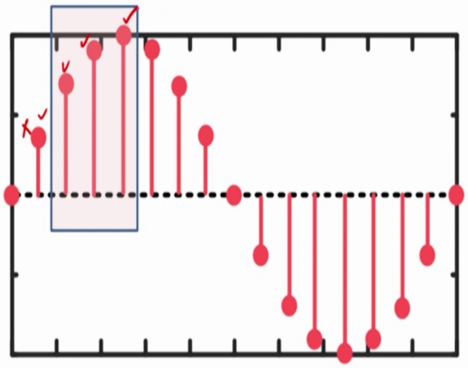


Sliding/Moving Window

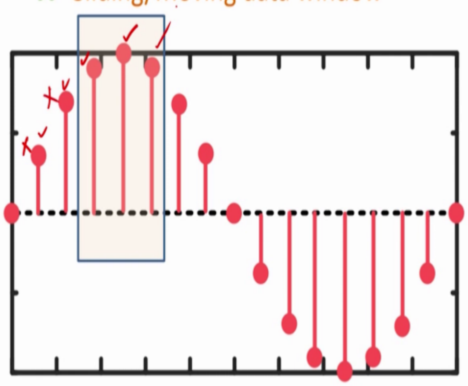
⌘ Sliding/Moving data window



- In each data window, number of samples remain constant.
- So when a new sample is available, there should be removal of last sample from the window.

Sliding/Moving Window

⌘ Sliding/Moving data window



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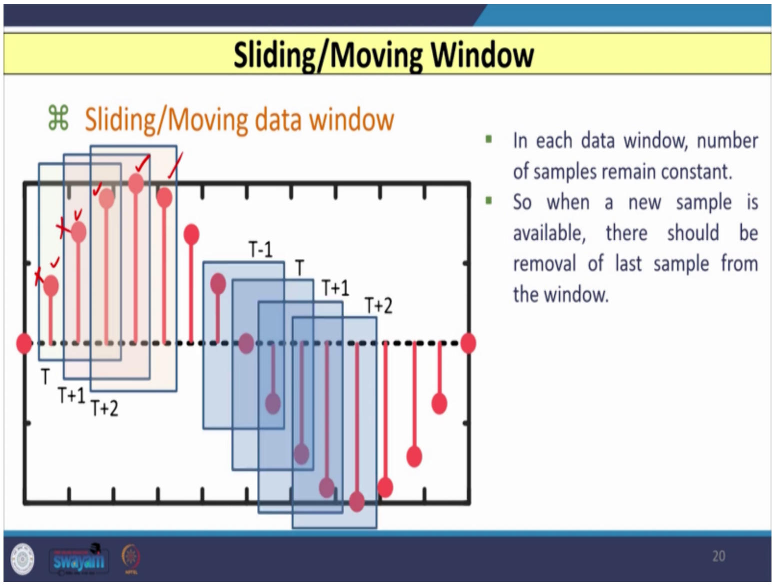

So, in each data window, a number of samples that remain constant. So, if I consider this you can see there are 3 samples. Now, whenever a next window I consider let us say this, then you will see this sample is discarded and this sample is again moved into the data window. So, you can see again I repeat, 3 samples, when I take next window, then number of samples in a window that remains constant that is 3 here. However, this sample is discarded and this is included.

Similarly, if I go further, then this sample is discarded and this is included like that you can proceed further. So, whenever new sample is available, the previous sample is discarded and the next sample is included. But keep in mind number of samples in a window that remain always constant.

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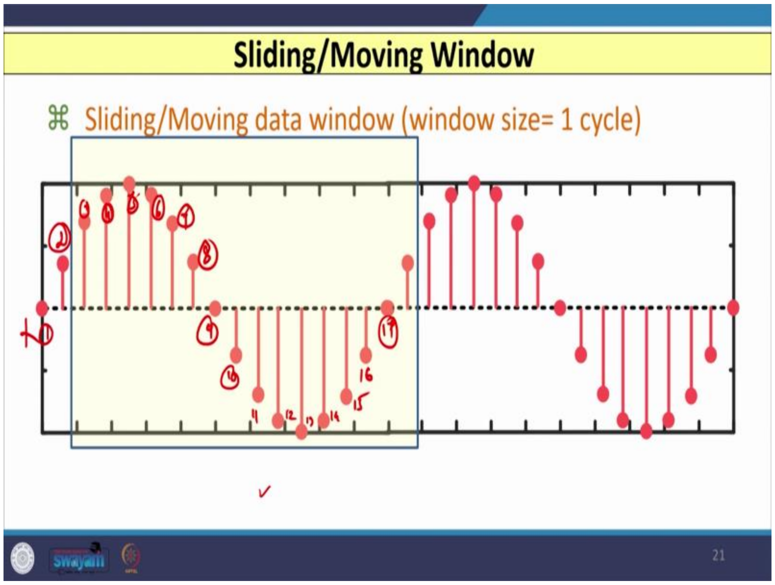

Now, if I consider a window with size of, let us say 1 cycle here, 2 cycles are shown, but let us say one cycle. So then you can see, this is 1 data window that contains the let us say samples from 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, 6^{th}, 7, 8, 9, 10, 11 12, 13 14, 15 and 16. So, 16 samples we have considered in 1 window. So in 1 cycle we have considered 16, samples are available. So, when I take next samples, the 1st sample is discarded and the next sample, 17 sample that is included. Similarly, when you further go then, 18 number sample is included and second numbers that is discarded.

So, this window is moving or sliding. So, that is why this is known as sliding or moving window concept. So, in this lecture, we started our discussion with the sampling and we have discussed that we have to follow the Nyquist criteria that is sampling frequency should be greater than the two times the maximum frequency present in the acquire signal, if it is not followed then three important effects are observed.

The first is the aliasing, second is the same output and third is the folding. Then we have discussed the concept of data window and along with that, we have also discussed the sliding window concept and with the sampling we have discussed what are the factors to be considered when we decide the sampling rate. Thank you.