

**Design and Analysis of VLSI Subsystems**  
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**Lecture - 68**  
**DVFS**

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Dynamic Voltage Frequency Scaling - In the form of task completion

A	10/1sec	0.1
B	20/1sec	0.2
C	100/1sec	1

$E = CV_{dd}^2 N$  where  $N = \text{No. of switching in time interval } t$   
 $E = CV_{dd}^2 \left(\frac{N}{t}\right) t$   
 $E = CV_{dd}^2 (\text{rate}) t$

$E = CV_{dd}^2 [f_{\text{switching}} t]$   
 $\propto f_{\text{clock}}$

$E \propto V_{dd}^2$  &  $E \propto \text{rate}$

If  $V_{dd}$  & rate are decreased by  $\frac{1}{2}$ , Energy decreases to  $\left(\frac{1}{8}\right)^{\text{th}}$

Let us begin with this particular topic. Let us go to the next slide and also I will pick my pointer. This is something we had already seen in the last lecture, where I had defined energy in terms of,

$$E = CV_{dd}^2 N$$

Here N is representing the number of the switching in the time interval of t. What it is actually coming from the energy term which is nothing but

$$E = CV_{dd}^2 f_{\text{switching}} t$$

This particular factor of  $f_{\text{switching}} t$  gives us the number of the switching. This  $f_{\text{switching}}$  can also be rewritten in the form of  $\alpha f_{\text{clock}}$ , where  $\alpha$  is nothing but the activity factor multiplied by the  $f_{\text{clock}}$  and then the product of  $\alpha f_{\text{clock}}$  and the duration of interest will give me the number of switching. We want to define this particular energy term, energy delivered by the  $V_{dd}$  for a particular task.

What we are trying to pursue or read as an energy delivered for completing a particular task and then when we are talking about a particular task, we can always relate it to the number of the switching does that the task performs at the output nodes. It could be many output nodes or it could be a single output node in a large digital circuit design. But if I can characterize a particular task to be completed such number of the switching in the specific nodes or in all the nodes or in one particular node then I think we can actually define the energy delivered by the  $V_{dd}$  for a particular task. That is what we are going to look into this particular lecture where we will consider the task completion to see the impact of DVFS on the energy term.

Remember that DVFS is basically a technique to reduce the energy that is required for the completion of the task. We will try to evaluate how much of energy that has been reduced or it has been saved or how much of energy benefits we will get using the DVFS technique.

Coming back to this particular energy term which we have arrived from the  $f_{switching}t$ , we can also have it as a function of number of the switching in the time interval of  $t$ .

$$E = CV_{dd}^2 \frac{N}{t} t$$

$$E = CV_{dd}^2 \text{rate } t$$

The rate of the switching or for a particular task we can also define the rates. If it is the rate of 100% that means, that we need whatever number of switching in that particular time duration, that will say that it is a 100% of rate or in an absolute way we can also say that it is the 10/10 seconds, it is nothing but 1 switching per 1 second. So, that we can also define the rate in terms of the number of switching per seconds. But generally this rate is basically a normalized value.

For example, if I have a lot of task if I have a task A, if I have a task B, if I have a task C and then so on. Each of this task is defined by a rate in terms of the absolute value where I need 10 switching to be done in let us say 1 second and then 10 or rather I will say B task is 20 switching in 1 second and the C task will be nothing but 100 switching in 1 second and then so on. What I mean by the normalized is the normalized factor is one more column where we consider the maximum of this which turns out to be in this

particular case C task is having a 100 switching per second, and then use that to normalize all other tasks.

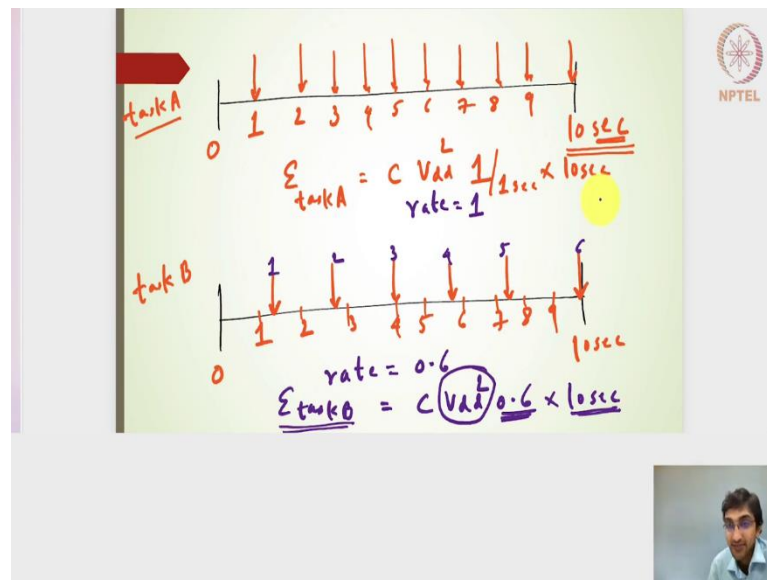
In this case the rate for the task C is 1, the rate for the task B is 0.2, the rate for the task A is 1, because I am actually dividing the 10 with that of the 100 the maximum value. I am basically having a normalized value here. A normalized value of 0.1, 0.2 and 1. Normally we will see that the rate will not go beyond 1, it will be there in between 0 to 1. In that sense it is a normalized value and then if I do a multiplication of  $t$  I should be able to get the energy factor. Finally, if I multiply that with the maximum the rate of switching for a given set of tasks, then I can find out the absolute energy value.

In this particular term of energy if you can notice that it is actually a function of or I have written it as a function of  $V_{dd}$ , the rail voltage, and it is a function of the rate. And remember that the rate is nothing but the number of switching per unit time and it is a function of basically  $f_{switching}$ , because  $n$  is actually defined as the product of the  $f_{switching}t$ .

The  $f_{switching}$  is nothing but it is related to the  $f_{clock}$ . Basically the rate is actually dependent on the  $f_{clock}$  the frequency of the clock frequency. What I mean is if  $V_{dd}$  and rate are decreased by half the energy naturally decreases by  $1/8^{th}$  because energy is directly proportional to the square of  $V_{dd}$ .

If the  $V_{dd}$  rail voltage is decreased by half instead of 1 volts if I have a rail voltage of 0.5, I will get the energy to be decreased by  $1/4$ . The rate if it is decreased by half that means, that a clock frequency somewhere it is kind of switching to a half the clock frequency and thereby my energy is going to decrease by half the time, due to the rate getting decreased by half. Put together if I actually decrease  $V_{dd}$  rail voltage as well as the rate by half I will actually get the energy to be decreased by  $1/8^{th}$ , which is a huge energy savings.

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Now, what do you mean by getting the rate to be lowered? let us say that I have a time duration of let us say 10 seconds. I have this as a 10 seconds and let us say for task A, I am going to say that task A requires 10 switching to be done in 10 seconds. This is starting from 0 second, this is ending at 10 seconds.

Generally, 10 seconds is too high in a VLSI design in today's technology node. It should be in terms of nanoseconds or even go to an extent of picoseconds. But just to understand make it very very simple I am using the seconds unit instead of the nanoseconds or picoseconds.

If I choose the task A and let us say that the task A needs 10 switching to be done in 10 seconds, in each of the 1 second, in each of the 1 second interval 3, 4, 5 and then 6, 7, 8, 9 and then 10 seconds it is going to do a switching at each of this time interval and then thereby it completes its task. The energy required for the task A here I am going to write it as  $CV_{dd}^2$ . Then the rate for 10 seconds, the rate is I will say that 1 switching per 1 second multiplied by the duration of interest is 10 seconds. This will be my energy that will be expended for completing the task A.

$$E_{taskA} = CV_{dd}^2 1/1sec \times 10sec$$

Now, let us say that for task B I have again the same 10 seconds window, but task B is requires let us say that it has a 10 seconds of time duration, but it requires only 6

switchings to be done. If I actually pick the interval 2, 3 and then 4 5, and then 6, 7, 8 and 9 and this is my 10 second. Although it is not divided linearly, but in that sense what I am saying is in each of this intervals or 1 seconds, the switching we require is only 6 switching. It can actually delay its switching or rather it can have a reduced rate. The switching can come here and then I will require only 6 switchings to be done and then finally, it arrives here.

I have 6 switchings the switching number 1, the switching number 2, a switching number 3, 4, 5 and 6 in 10 seconds. My rate is now actually 0.6 as that of the previous one, here the rate I have considered it to be the maximum, the normalized value is actually 1. My energy for this particular task B will be nothing but,

$$E_{\text{task B}} = CV_{\text{dd}}^2 0.6 \times 10 \text{sec}$$

The 0.6 switching per second and then multiplied by 10 seconds, naturally my energy is going to decrease. What DVFS says is whenever I am changing the rate or whenever I am changing the clock, so as to adjust the rate, at the same time can I actually change the voltage rails. If I actually do instead of 0.6 rate as well as instead of only 0.6 rate as well as if I do a rail voltage change of 0.6 then I am going to have a drastic change in the energy. Hope this is clear. If I actually apply DVFS then I am going to have  $CV_{\text{dd}}^2 0.6$  into whatever the time  $t$ , it is going to give me  $0.216 CV_{\text{dd}}^2 t$  of 10 seconds. If I compare with this it is going to give me a naturally close to 80% of the energy savings.

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$E = C V_{\text{dd}}^L (\text{rate}) \times t$

→ 10 switching  
 → rate  $\times t$  is used to perform a task

→ For a task A, if rate = 1,  $\text{rate} \times t = 1t$   
 For a task A, if rate = 0.6 :  $0.6 t_1 = 1t$   
 $t_1 = \frac{1t}{0.6} = \frac{5t}{3}$

For a task A, if rate = 2 :  $2 t_1 = 1t$   
 $t_1 = 0.5t$

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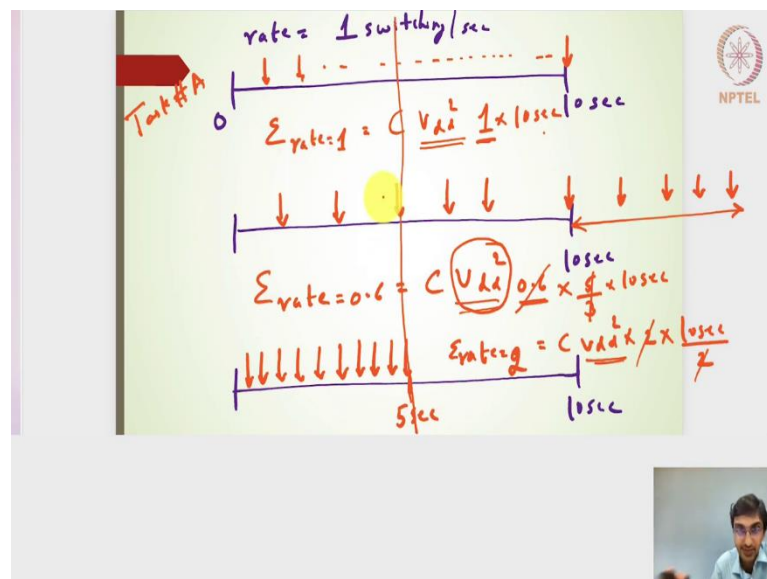
Moving forward. Let us take some few more instances where the rates are different and let us see what happens. As we know the energy is nothing but  $CV_{dd}^2 t$ , let us say for a task A we need to have the rate is configured to 1. The rate  $\times t$ , is the duration for which we are kind of probing the energy the overall energy. The 1 into t is what is required or rather the rate multiplied by  $t = 1 t$ . I will have an energy term of  $CV_{dd}^2 1 t$ .

Let us say that for a task A, for the same task A instead of rate to be configured as one if I actually do a configuration of 0.6. Then rate multiplied by  $t$  will be different in the sense if the task A requires 10 switching to be done in 10 seconds, and let us say that the rate is 1, switching per 1 second in this particular case. Then, it requires 10 seconds to complete the 10 switching.

Similarly, for the rate is 0.6 here. My rate  $\times t$  which will give me the 10 seconds or rather the 10 switchings here, the rate has been decreased now, this  $t$  has to be increased, that is what I am doing. I require  $1 \times t$  which will give me 10 switchings,  $0.6 \times t_1$  should give me the 10 seconds or rather the 10 switchings. My  $t_1$  should naturally increase if I am using a lower rate, just to complete the task A.

Similarly for task A if rate has been increased then the number of switching can be accommodated in a lower amount of duration of time. If it is increased to 2, then my  $t_1$  the duration for the task A to complete 10 switchings will be nothing but  $0.5 \times t$ .

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Let us try to have a look at it one by one. What I meant in the previous slide is let us say I have the 10 seconds of duration, 0 to 10 seconds for the task A which is now configured with a rate of let us say 1 that means, that 1 switching per second. I will have 1 switching to be done at each of the seconds and let us say that it requires 10 switchings should be done, it will complete the task A will in 10 seconds.

The energy for the task A. All these are task A here in this particular case. The energy that required to complete the task A with a rate of 1 will be nothing but,

$$E_{\text{rate}=1} = CV_{\text{dd}}^2 1 \times 10\text{sec}.$$

Now, if I actually decrease the rate for the 10 seconds, now decreasing the rate implies that I need to have in fact, the number of switching or rather the rate of switching is going to be lower. It will have only 6 switchings 1, 2, 3, 4, 5 and then the 6 switchings to be done in 10 seconds. But I require 10 switchings to be done, it will have to extend beyond this and whenever it does complete 10 switching. This will be the 10 switching, it will be extended up till here and then the way it will extend is it will be 5/3 times extension of the 10 seconds. My energy for the rate for completing the task A,

$$E_{\text{rate}=0.6} = CV_{\text{dd}}^2 0.6 \frac{5}{3} 10\text{sec}$$

$$E_{\text{rate}=0.6} = CV_{\text{dd}}^2 \times 10\text{sec}$$

Again, if I have one more case where the rate is doubled and then let us say that this is 10 seconds, the rate is doubled in the sense the rate is 2 now, that means that I will have now more switching. I will have a switching of 10 switching as 3, 4, 5, 6, 7, 8, 9 and then 10 and this is the task A is completed exactly in the 5 second.

Now, the energy that is estimated,

$$E_{\text{rate}=0.6} = CV_{\text{dd}}^2 2 \times \frac{10\text{sec}}{2}$$

$$E_{\text{rate}=0.6} = CV_{\text{dd}}^2 2 \times 10\text{sec}$$

What this example says is if task A is to be completed within this particular duration I can actually increase the rate decrease the rate and based on the number of the switching it is going to complete. The energy if I do not change the rails at all based on the change in the rates then I am not going to get any kind of benefits. But for the same thing whenever the rate is 1, if I change the  $V_{dd}$  here, if I change the  $V_{dd}$  here for a rate of 1, I will get  $1V_{dd}$ . I will get  $CV_{dd}^2 \times 10$  seconds. But here if I actually change because the rate is now changed to 0.6 and if the  $V_{dd}$  is also changed to 0.6, I am actually getting 80% of benefits of the energy as that of the first case.

Similarly, if I actually change the rate if it is 2 times then I will have to have a  $2V_{dd}^2$  then it is going to have more energy than that of whatever is seen here. Then if I want to compare, then I will need to compare that with respect to the time interval of the same time interval, whichever is ending first I will have to use the same time interval. In this case, I will have to compare in the 5 seconds window what is the energy that has been compensated here, energy compensated here, energy compensated here and then try to extract or try to see what is the energy at that particular time window of 5 seconds.

Naturally, whenever the rate is lower and if we can actually have a lower rail voltage naturally that particular energy component will give me lot of energy benefits.

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If  $\text{rate} \propto V_{dd}$

change in rate needs a change in  $V_{dd}$

For a task #B, rate of 0.6 is needed to complete

rate  $\times t = 0.6t$

Energy =  $C \times (0.6V_{dd})^2 \times (0.6 \times t)$

$E = 0.216 C V_{dd}^2 t$  achieves 78.4% reduction in energy

$C V_{dd}^2 t$

Going forward. What we are saying is if I have only the change in rates it will not give you that much of benefits in the energy, but somehow if the rate  $\propto V_{dd}$ , if the change in



the rate will have an change in the  $V_{dd}$  it is a linear relationship that means, if the rate is changed, then the  $V_{dd}$  also changes then I will have a larger energy savings a larger impact on the energy benefits. The change in the rate needs a change in the  $V_{dd}$ . If we can configure that way then for the example task B a rate of 0.6 is needed to complete, what it really means is I need 6 switching in 10 seconds let us say, that is an example. Then I can have this,

$$\text{rate} \times t = 0.6t$$

My overall energy in this particular case for duration of 10 seconds,

$$\text{Energy} = C (0.6V_{dd})^2 \times (0.6xt)$$

Which will give me a 78.4% reduction in the energy if it was the  $CV_{dd}^2t$ . If I was actually doing the task B with a rate of 1 and then going up to 10 seconds we will give me the energy of  $CV_{dd}^2t$ . In that sense, it is going to give me an reduction of energy in 78.4%.

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The slide content is as follows:

**Dithered levels**  
 $\text{rate} \propto V_{dd}$   
 Continuously changing  $V_{dd}$  for the rate requires prediction of the tasks parameters, and changing the clock frequency via PLL. Extensive hardware design.

If 2-dithered levels  $\Rightarrow$   $0.5 V_{dd}$   $\leftarrow$   $0.75 V_{dd}$   
 $\text{rate} = 0.5$   $\leftarrow$   $\text{rate} = 0.75$

$(\text{rate} = 0.6) t = 0.5 t_1 + 0.75 t_2$  — ①  
 $t = t_1 + t_2$  — ②

$t = 0.4 t$   
 $t = 0.6 t$

Diagram: A horizontal timeline from 0 to  $t = 10 \text{ sec}$ . The first interval  $t_1$  is labeled "0.5 rate" and the second interval  $t_2$  is labeled "0.75 rate".

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Going to the next one, moving ahead. What we are saying is the rate, whenever we want to plan our rate the switching rates for completing a task, we will also change the  $V_{dd}$  in a linear relationship. If the rate tomorrow for task A if it is 0.39 the rate is required, then if the rate of 0.39 is required, then my  $V_{dd}$  should also be 0.39 volts and if day after tomorrow if I want to change this 0.39 let us say 0.49 my  $V_{dd}$  value should also be 0.49

volts. But remember that the rail voltages in a digital design we actually cannot get so much of rail voltages available. The different rail voltages are available in the form of a voltage divider, but we do not want to have a lot of rail voltages available, otherwise it will occupy its own space. We will have space occupied by the rail voltages itself and it will not have enough space to design our circuits.

In that sense what we do not want is a continuously analog values of the rail voltage going from 0 to 1 volts. But what we can do is we can have some specific rail voltages 2 or 3 levels of  $V_{dd}$ s  $0.5V_{dd}$  or  $0.75V_{dd}$  or maybe a  $1V_{dd}$  or  $1.2V_{dd}$  or very few select  $V_{dd}$  rails could be made available. We can try to accommodate the task completion by using those specific  $V_{dd}$  rails and then configuring the rate according to the  $V_{dd}$  rails that are available. If I have a  $V_{dd}$  rail of 0.8 then why not configure the task completion with 0.8 of the rate, if I have a  $V_{dd}$  of 0.5 then why not accommodate the rate of 0.5.

The other disadvantages is continuously changing the  $V_{dd}$  for the rate requires the prediction of the task parameters. Let us say the task A is there and after that task B is there and after that task C and then so on those tasks are listed. What we need to do is, if I know predefined parameters of the tasks for task A, B, C and D. So, if I know that task A needs a switching rate of 1 per second and task B requires a 0.6 per second task, C will require 0.69 per second and I need to have a characterization on a prediction of the task parameters. Then I need to change the rate, changing the rate will require a continuous changing of the clock frequencies.

Remember that the rate was a function of the clock frequencies and the clock frequency is changing we have to do using the phase lock loops PLL. We will require an extensive hardware design and this again occupies the space and you know it having this extensive hardware design is kind of a big overhead. Instead of doing that why not have a dithered levels, dithered levels means having a specific levels, having a specific rail levels and then configuring the rate according to those specific rail voltages that are available.

What I mean is if I have a specific rail of  $0.5V_{dd}$  and  $0.75V_{dd}$ , then for any task whether it is task A, B, C, D and whatever up till Z task, if I can configure the rate of switching to 0.5 match with that of the  $V_{dd}$ , and then the for a  $0.75V_{dd}$  if I can have configure the rate of switching to be 0.75. That it matches with that of  $0.75V_{dd}$  and then try to accomplish that particular tasks.

In this particular example, what I have done is we want to achieve let us say for task A if you want to achieve 0.6 of the switching rate that to be completed in a time duration of  $t$ . Now, the question it be achieved by these two dithered levels, these two levels of 0.5 rail and 0.5 rate, and 0.75 voltage and then the rate of 0.75 and a rail voltage of 0.75. Let me draw a diagram here. What we need to achieve is I am taking the same example of 0 to  $t$  is equal to 10 seconds.

Ideally, we wanted 6 switchings to be done in the 10 seconds, but let us say that the  $0.6V_{dd}$  is not available in our design. But we have  $0.5V_{dd}$  and  $0.75V_{dd}$ , that means, that I can actually choose the switching rate of 0.5 and I can choose a switching rate of 0.75 because it matches it gets aligned with that of  $0.5V_{dd}$  rail and  $0.75V_{dd}$  rail.

In that case what we are saying is if I need 6 switchings to be done in 10 seconds why cannot I use the 0.5 rate. The 0.5 rate means it is pretty slow, slower than that of the 0.6. I will have one switching here, I will have one switching here, I will have one switching here and then I will switch in to 0.75 switching rate. I need 6 switching, what I will do is I will have a faster switching now. This will be 0.75 and then this will be configured to 0.5 switching rate. I am going to write it as 0.5 rate and then 0.75 rate, because we need to achieve 0.6 rate overall in 10 seconds. That means, that I need to complete 6 number of switchings in 10 seconds. Why cannot I use a slower rate, a slower rate initially and then a faster rate finally, or the otherwise, I can also have 0.75 rate initially and then go slow to 0.5 rate and then complete the 6 number of switchings in 10 seconds.

What it means is the number of switching by using these two dithered levels, by using these two rates it still should match with the total number of switching. The total number of switching is,

$$(\text{rate} = 0.6)t = 0.5t_1 + 0.75t_2 \quad \text{---(1)}$$

This  $t_1$  and  $t_2$  is our the time on the time domain. How much of amount of time the 0.5 rate is configured and how much amount of time the 0.75 rate is configured. If in that sense I will have this particular equation,  $0.5t_1 + 0.75t_2$  should give me the total number of switching.

$$t = t_1 + t_2 \quad \text{-----(2)}$$

We know that this  $t_1 + t_2$  should be equal to that of the 10 seconds in this case it is  $t$ . We have two equations, we have two variables or unknowns  $t_1$  and  $t_2$ . I should be able to get this  $t_1$  and  $t_2$  in terms of the value  $t$ . I will get

$$t_2 = 0.4t$$

$$t_1 = 0.6t$$

What we have done in this particular slide is we want to configure the 0.6 rate for a particular task, but because 0.6 the rate configuration and its rail voltages are not available. We will use 0.5 rate, configure it the task for 0.5 for some amount of time  $t_1$  and then switch to 0.75 rate, for the  $t_2$  interval which will sufficiently give me the 6 number of switchings in 10 seconds.

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The slide content is as follows:

	Computation time	rate	V <sub>dd</sub>
$\epsilon_1 \rightarrow t_1 = 0.6t$		0.50	0.5 V <sub>dd</sub>
$\epsilon_2 \rightarrow t_2 = 0.4t$		0.75	0.75 V <sub>dd</sub>

DVFS - dithered

$$E = c(0.5V_{dd})^2(0.5 \times 0.6t) + c(0.75V_{dd})^2(0.4t)$$

$$\rightarrow E = 0.24375 C V_{dd}^2 t$$

achieves 75.62% reduction in energy

Gives 96.45% benefits of DVFS

$E_{DVS} = 0.216 C V_{dd}^2 t$

78.4%

$\frac{75.62\%}{78.4\%} = 96.45\%$

1 V<sub>dd</sub>, rate = 1

What I have done is I have written a table here the computation time, the rate and then the V<sub>dd</sub> rail. If I am choosing the 0.5 rate here, I need to choose 0.5V<sub>dd</sub> and 0.75 rate here, then I will have to choose the 0.75V<sub>dd</sub> because this will give me the DVFS, the voltage as well as the rate or the frequency switching or the dynamic voltage frequency switching.

Of course, we will this is called as a dithered dithering, dithered level DVFS. The reason it is called dithering is because we do not have all the rail voltages available with us. We

have only specific levels of  $V_{dd}$  and that is why we say that it is kind of truncated or it is a dithered DVFS.

If I am using a rate of 0.5, I will require the time interval of  $0.6t$  and if I use 0.75 after that then the time interval is  $0.4t$ . What it really means is to achieve a rate of 0.6 for a duration of  $t$ , we will have to use 0.5 rate for  $0.6t$  duration and 0.75 rate for  $0.4t$ . My overall energy now is nothing but the addition it is nothing but the cumulation of this particular energy. Which is nothing but,

$$E = C(0.5V_{dd})^2(0.5 \times 0.6t) + C(0.75V_{dd})^2(0.75 \times 0.4t)$$

$$E = 0.24375CV_{dd}^2t$$

If I actually compare with that of the 0.6, it used to give me  $0.216 CV_{dd}^2$ . This particular term is not at all a bad term or bad value because it is very very close to 0.216. If I really look into this with respect to a task A which was doing actually  $CV_{dd}^2$ , one which used a rail voltage of 1 and then a rate of 1, it actually gave me 75.62% reduction in energy when compared to a rail voltage of  $1V_{dd}$  and then a rate of 1. If I use the DVFS, this will be my 0.216 because I had the rate of 0.6. The  $V_{dd}$  has to be 0.6, it gave me 0.216. With respect to 0.216 this 0.24375, actually gives us the benefit of 96.45% benefits of the DVFS. Benefits in the sense, if I calculate this E energy of DVFS it is  $0.216 CV_{dd}^2t$ . When compared to the task A of  $1V_{dd}$  and 1 rate, it was  $CV_{dd}^2$ . I had a benefit of how much? 78.

Let me have a quick look at it, it gave me 78.4% and this one is giving me 75.62. If I actually compare 75.62% with that of 78.4% it is actually giving me 96%. Although this is a dithered DVFS technique, this particular energy term is giving the benefit of almost 96.45% benefits of what we can achieve in DVFS. But with DVFS if you use an analog values like whatever is the rate we will use the same  $V_{dd}$ , the problem there is we have to have an extra hardware or an hardware overhead. That was one of the issues and that is why we went into the dithered levels which is very very suitable for our digital design in terms of saving the space as well as giving almost close to 96.45% benefits of the DVFS technique, hope it is clear.

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2 dithered levels -  $0.5V_{dd}$  &  $1V_{dd}$

$$0.5t_1 + 1t_2 = 0.6t$$

$$t_1 + t_2 = t$$


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$$t_1 = 0.8t$$

$$t_2 = 0.2t$$

	Computation time	rate	$V_{dd}$
$\epsilon_1$	$t_1 = 0.8t$	0.5	$0.5V_{dd}$
$\epsilon_2$	$t_2 = 0.2t$	1	$1V_{dd}$

$$E = \epsilon_1 + \epsilon_2$$

Diagram: A horizontal timeline from 0 to t. A red arrow labeled  $t_1$  starts at 0 and ends at a point. A red arrow labeled  $t_2$  starts at that point and ends at t. Above the timeline, there are six vertical arrows pointing down, representing switching events. The first three are under the  $t_1$  interval and the last three are under the  $t_2$  interval. The text '10 sec' is written at the end of the timeline.

Moving on to the next one. Let us say that the two dithered levels we have is  $0.5V_{dd}$  and  $1V_{dd}$  instead of  $0.75V_{dd}$  and we have to achieve  $0.6$  rate  $\times t$ . That means, that the same example of 6 switching per in 10 seconds. I have to use the  $0.5V_{dd}$  rail that means, that I have to use a rate of 0.5 and then  $1V_{dd}$  then I have to use a rate of 1. What it means is here in from 0 to 10 seconds, I can choose to have a 0.5 rate. I will have a very slow switching. It could be whatever the number of switching and then finally, we will have a very fast switching because a rate of 1 is there. This particular rate will come at the end and then this particular 0.5 rate will come in the beginning or vice versa.

In this particular thing and this will be  $t_1$  and then this one will be  $t_2$ . I need to find out  $t_1$  and  $t_2$ . I have two equations

$$0.5t_1 + 1t_2 = 0.6t$$

$$t_1 + t_2 = t$$

I should be able to find out what is  $t_1$  and what is  $t_2$ , turns out to be 0.8 and 0.2. If I put it in a table of rate of 0.5 and  $0.5V_{dd}$ , a rate of one having using a  $1V_{dd}$  rail, this time interval is  $0.8t$  and  $0.2t$ . If I try to find out this is the energy 1 and energy 2, this is the total energy for completing the task whatever the task with to achieve the number of switching as 6 per 10 seconds that means, to achieve a rate of 0.6 I will have this energy  $E_1 + E_2$ .

$$\text{Energy}_{0.5V_{dd}+1V_{dd}} = (0.5V_{dd})^2(0.5)(0.8t)C + (1V_{dd})^2(1)(0.2t)C$$

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Energy  $= (0.5V_{dd})^2(0.5)(0.8t)C + (1V_{dd})^2(1)(0.2t)C$

$E = 0.3V_{dd}^2 Ct$  achieves 70% reduction in energy  
Gives 90% benefits of DVFS

Task A 0.6  
B 0.5  
C 0.75  
Z 0.75

0.216 CV<sub>dd</sub><sup>2</sup>t

0.6 V<sub>dd</sub> 0.75 V<sub>dd</sub>

$E = 0.3V_{dd}^2 Ct$

$$E = 0.3V_{dd}^2 Ct$$

Which is actually achieve 70% with respect to our overall savings in the energy, if we were to use  $1V_{dd}$  rail and a rate of 1. Even for this task which requires a rate of 0.6. In addition to that it gives actually 90% benefits of the DVFS. The 90% benefits means with respect to this if  $0.216 CV_{dd}^2 t$ . We are getting 78.4% of benefits here. Here we are getting 70%,  $70\% / 78.4\%$  is like is going to give me 90%. We are actually using a dithered DVFS with the two rails of  $0.5V_{dd}$  and  $1V_{dd}$  and that is actually giving us 90% of benefits of DVFS.

Hope you are able to understand this. I have given two examples having the dithered DVFS, dynamic voltage frequency scaling, where rate is directly proportional to the rail voltage. If I want to configure a particular task, for a particular rate switching rate then I will also choose that particular  $V_{dd}$  rail. Now, the question is how do we choose that particular or if we as a designer we have an ability to select what particular or we have an a designer we have an ability to design a particular rails, now the designing of the rails actually comes from the characteristics.

For a particular design if we have n number of tasks task A, B, C and so on and then if we have a rates to be designed. Let us say 0.6 is the rate for a task A, 0.5 is the rate for

task B,  $0.75$  and then so on. We will have to choose the particular rail voltage it could be either  $0.6 V_{dd}$  and  $0.75V_{dd}$ . It could be those 2 rails or 3 rails or maybe another set of rails which will give me a maximum energy benefits for performing all the tasks.

The rail voltage design is actually coming from the characteristics of this  $n$  number of task which is going to use the VLSI design, which is going to use our design, whatever design we have done on one particular part of the chip. For completing those tasks and if we are using that particular design then we need to calibrate or characterize those tasks, and what should be the rate for that, based on that rate information the switching rate information we should be able to design the optimal design of the rail voltages. Then this could be applied for all the tasks, hope this is clear.

What we have learnt in this particular lecture is the dynamic voltage frequency scaling. What it really implies is if you are changing the rate to achieve or to complete a particular task then the rail voltage should also be changed to get a maximum energy benefits.

But having different values of  $V_{dd}$  is not possible in a large higher order digital design because we will not have multiple rail voltages. What we will have a specific rail voltage and that is where the dithered DVFS technique is very very useful which is also giving us the benefits, very very close to the benefit of the natural DVFS technique.