Design and Analysis of VLSI Subsystems
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Lecture - 59
Energy and delay analysis for interconnect with repeaters

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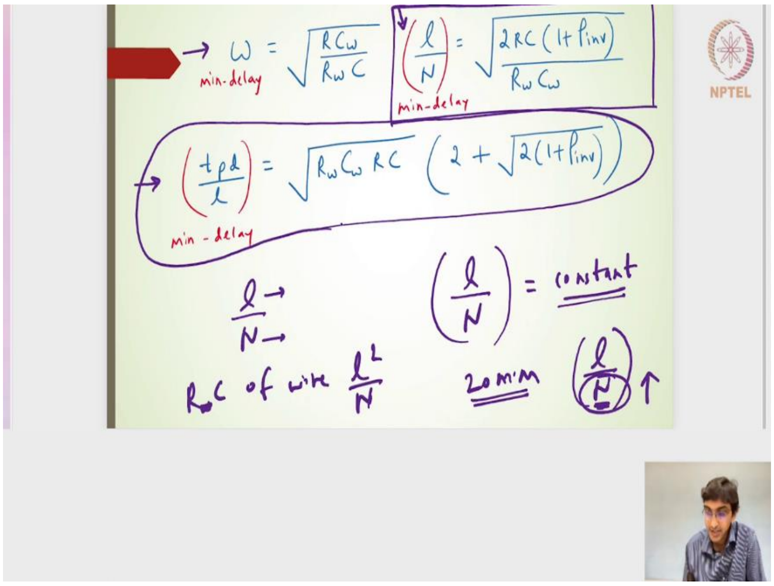

Moving on, what we have learned in the previous sections is the optimum W. The optimum W here which will give me the minimum delay and then this particular optimum W is the scaling factor for the repeaters. It is nothing to do with the size of the length of the wire or such its only the scaling factor or the width of that particular repeater and that too the inverter we had used in the form of the repeater.

The scaling factor of the inverter which has to be optimized, so as to get the overall delay of the wire along with the repeater to be minimum. To get the minimum delay this is the overall width of the inverter that we have to use for n minus one inverters.

This is the overall width parameter and then the overall l/N, where N is the number of segments and l is the overall length of the wire connecting between the driver and then the receiver. The l/N the minimum expression, so that we will get the minimum delay. This is the again the optimum W and optimum l/N values.

Now what we had earlier stated was, if I do the n such segments I should be able to find the best l/N and now if I want this will be a constant value or this will be for obtaining the minimum delay, this will be my constant value. Say if tomorrow if I am going to use instead of 10 mm wire, if I am going to use 20 mm of wire.

This constant will remain the same. My l/N is remains the same. In that sense if it is 20 mm of wire my N value or the number of segments that I will create that will be more or rather the number of repeaters which I will add at each of this segments will be more, that is what we had started earlier.

We had the overall the wire and then its RC, the RC delay of the wire was nothing but $\frac{l^2}{N}$. Now, in this particular case if I make N segments and then characterize by l/N, then in that sense I will actually have the l/N value as a constant and if I have the l/N as a constant value and if I keep increasing the N l length of the wire then the number of segments will be increased.

The l/N is a constant value and l will be a directly proportional to the number of segments of the wire. If I have the l/N as a constant value the earlier notion of l/N being constant and then the overall length of the wire, if it increases number of repeaters will increase proportionately.

In that sense I will have this particular factor to be held constant and my overall length of the wire and its delay will now be directly proportional to only the length of the wire, it will be a linear relationship. That is what we had started earlier and that is where we are now with this optimized w and l/N parameters. The overall delay per unit length of the wire will be nothing but given by this particular expression.

$$\left(\frac{t_{pd}}{l}\right)_{min,delay} = \sqrt{R_w C_w RC}\left(\sqrt{2(1+\rho_{inv})} + 2\right)$$

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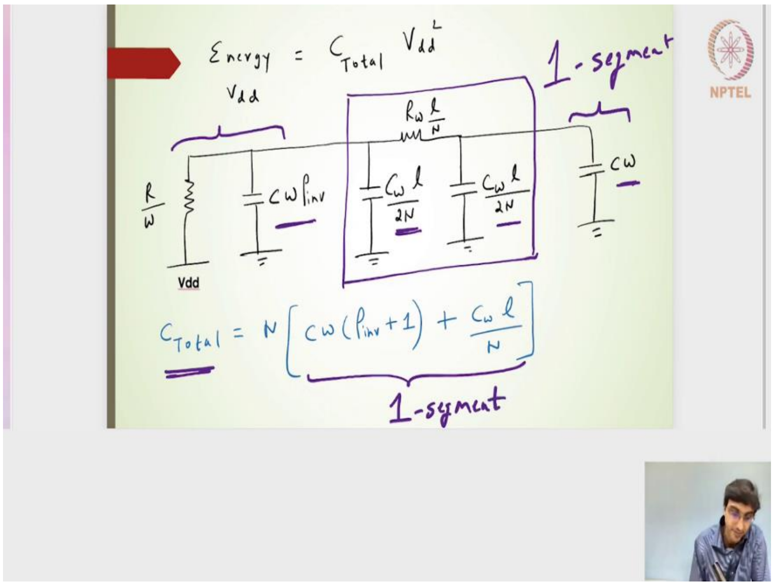

Now, let this is a circuit representation for 1 such segment where we had started with the repeater one. This particular portion is for the repeater 1 this is the input capacitance of the repeater 2 and then this particular in between model is nothing but the π model of the 1 segment of the wire.

In this particular one segment of the wire what should be the energy that is to be delivered by the V_{dd}. If I have the inverters connected to the V_{dd} rail and then this particular repeater also connected to the V_{dd} rail, what is that particular Vdd the power or the energy that has to be delivered by the V_{dd} rail for across all these particular capacitances of course, we will have the resistances.

We know that if I have an RC circuit which is connected to the V_{dd}, then the overall energy that is being delivered by the V_{dd} will be nothing but $CV_{dd}{}^2$In this sense if I can actually find out the total capacitances. There are four such capacitances one is this one, one is this one, one is this one and then this one and then multiplied by V_{dd}^2 that will be the energy delivered by the V_{dd} rail.

The overall total capacitances is nothing but the four of these capacitances and that will be for one segment and if I actually multiplied by N times, I will get N such and this is the overall capacitances for the overall length of the wire, where the overall length of the wire is nothing but segmented into N segments. This is for 1 segment and multiplied by N will give me for N segments the total capacitances.

$$C_{Total} = N\left[CW(\rho_{inv} + 1) + \frac{C_w l}{N}\right]$$

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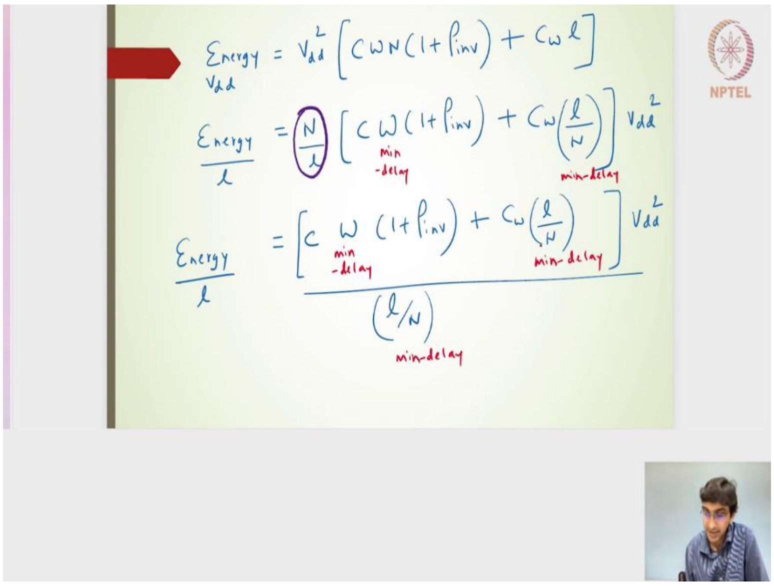

The energy delivered by the V_{dd} will be nothing but,

$$\underset{Vdd}{\text{Energy}} = V_{dd}^2[CWN(\rho_{inv} + 1) + C_w l]$$

If I want to find out the energy per unit length. It will I will actually divide on both the sides by l and this is what I have.

$$\frac{\text{Energy}}{l} = \frac{N}{l}[CW_{min,delay}(\rho_{inv} + 1) + C_w \left(\frac{l}{N}\right)_{min,delay}]\, V_{dd}^2$$

$$\frac{\text{Energy}}{l} = \frac{\left[CW_{min,delay}(\rho_{inv} + 1) + C_w \left(\frac{l}{N}\right)_{min,delay}\right] V_{dd}^2}{\left(\frac{l}{N}\right)_{min,delay}}$$

If I actually do this if I want to put this particular optimized parameter of the W and optimize parameters of l/N for the minimum delay expression. I should be able to find out what is the overall energy per unit length of the wire.

For the parameters which we have estimated for the optimized w and then l/N for the achieving the minimum delay. We are trying to estimate what is the energy per unit length for achieving the minimum delay, putting all these things together. This N/l will come as a denominator into l/N here for the minimum delay and then W as the minimum delay and then l/N as the minimum delay.

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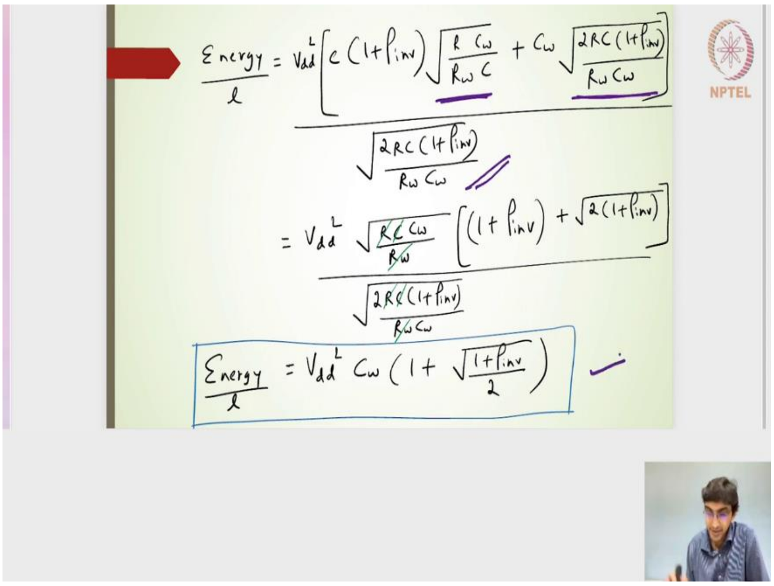

The overall energy per unit length if I put this w optimum, l/N optimum value here. I will get the energy per unit length is nothing but,

$$\frac{\text{Energy}}{l} = \frac{V_{dd}^2\left[C(1+\rho_{inv})\sqrt{\frac{RC_w}{R_w C}} + C_w\sqrt{\frac{2RC(1+\rho_{inv})}{R_w C_w}}\right.}{\sqrt{\frac{2RC(1+\rho_{inv})}{R_w C_w}}}$$

$$= \frac{V_{dd}^2\sqrt{\frac{RCC_w}{R_w}}\left[(1+\rho_{inv}) + \sqrt{2(1+\rho_{inv})}\right]}{\sqrt{\frac{2RC(1+\rho_{inv})}{R_w C_w}}}$$

$$\frac{\text{Energy}}{l} = V_{dd}^2 C_w \left(1 + \frac{\sqrt{(1+\rho_{inv})}}{2}\right)$$

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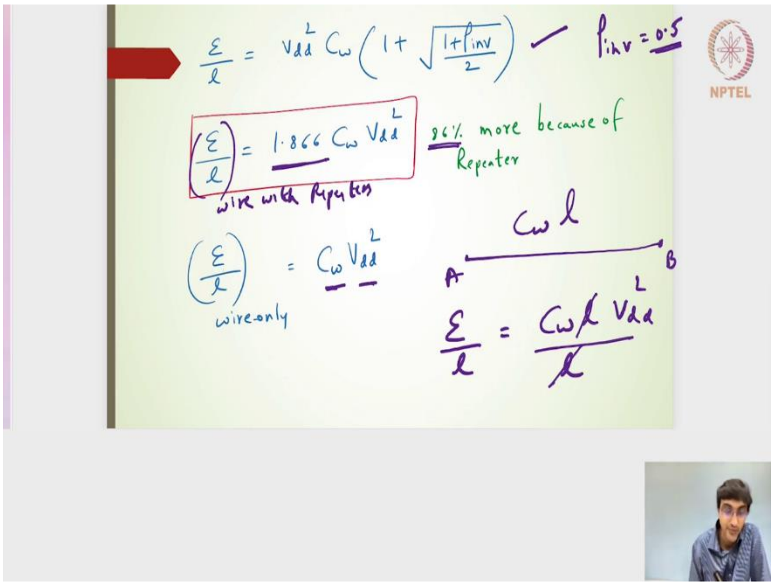

The energy per unit length if I use this particular expression and then put ρ_{inv} value using the folded design technology. I will get over the $\sqrt{1+0.5/2} = \sqrt{\frac{1.5}{2}} = 0.866.$

$$\frac{E}{l} = 1.866 C_w V_{dd}^2$$

It is actually 86% more because of the repeater. It is this particular energy per unit length it turns out to be $1.866 C_w V_{dd}^2$. If I consider the energy per unit length of the wire alone and not having any kind of repeater I will get,

$$\left(\frac{E}{l}\right)_{wire,only} = C_w V_{dd}^2$$

If I consider only the wire from A to B and no such repeaters at all it will be $C_w l$. My energy per unit length will be nothing but,

$$\frac{E}{l} = \frac{C_w l V_{dd}^2}{l}$$

We will have $C_w V_{dd}^2$. The energy per unit length of the wire for wire with repeater. This is with repeaters and the number of repeaters is nothing but N - 1 turns out to be 86% more because of the repeater, hope this is clear.

The energy per unit length of the wire alone will be $C_wV_{dd}^2$ and then energy per unit length of the wire with the repeaters is $1.866C_wV_{dd}^2$ which is 86% more than that of the wire alone.

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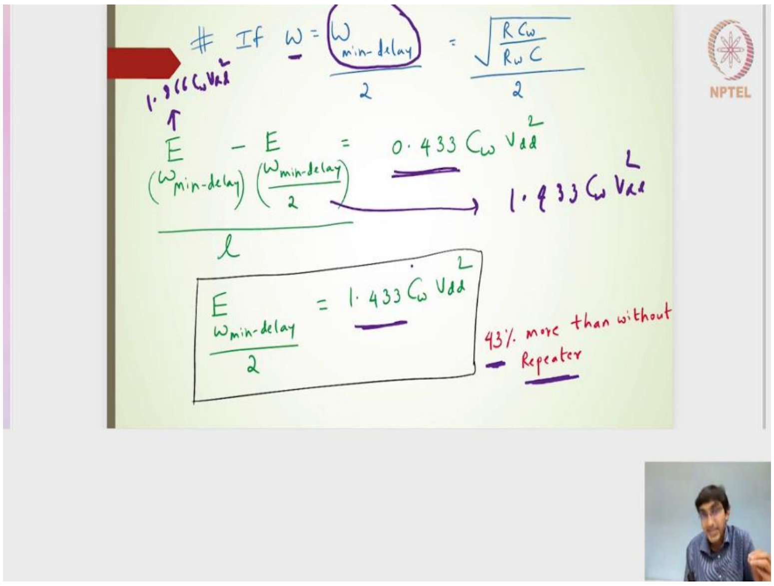

The very important parameter, what if I make the width of the repeater or the inverter in this particular case to be half that of the optimum.

$$W = \frac{W_{min-delay}}{2} = \frac{\sqrt{\frac{RC_w}{R_wC}}}{2}$$

Notice that the optimum width which we have calculated here, W of minimum delay it is for obtaining the minimum delay. If I make any changes in the width of the repeater or the inverter right my delay is likely to increase from the minimum value, but how does it estimate or how does it figure out in terms of the energy?

The energy with the optimum W parameter and then the energy with a different w parameter especially in this case.

$$E_{W_{min-delay}} - E_{\frac{W_{min-delay}}{2}} = 0.433C_wV_{dd}^2$$

What it means is the energy with a reduced width is likely to give me $0.433C_wV_{dd}^2$or rather this one was with the minimum delay it was $1.866C_wV_{dd}^2$.

If I do this one, the energy for the optimum W and if I optimum W/2 if the repeaters width are optimum W/2, I will actually get this value as $1.433C_wV_{dd}^2$. That is what I have written here the energy for the optimum W/2 will be nothing but 1.433 which is less than 1.866. This is 43% although it is 43% more than that of the wire without the repeater. Because without the repeater the energy is nothing but $C_wV_{dd}^2$. It is just a parameter saying that the energy could be reduced if I use the W value less than that of the optimum W.

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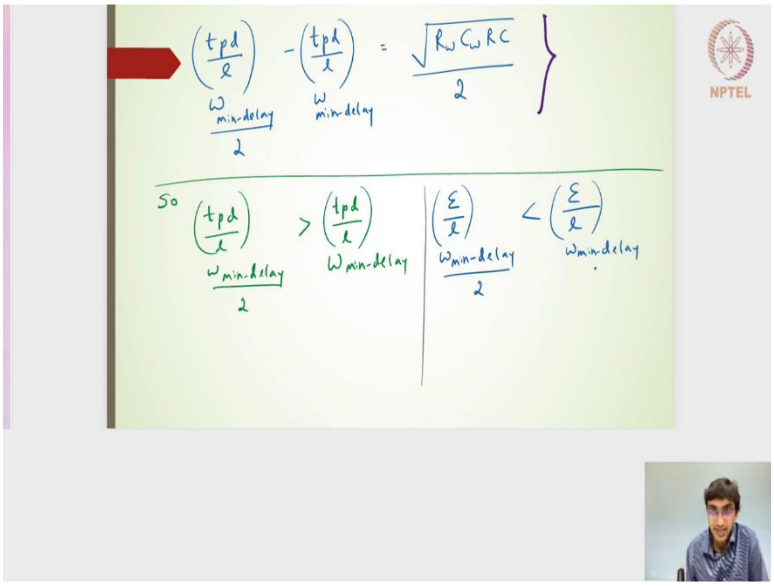

But what happens to the delay? The delay per unit length of the wire for two different W,

$$\left(\frac{t_{pd}}{l}\right)_{\frac{W_{min-delay}}{2}} - \left(\frac{t_{pd}}{l}\right)_{W_{min-delay}} = \frac{\sqrt{R_wC_wRC}}{2}$$

This particular parameter if it is greater that means, that this particular parameter if I increase if I decrease or if I have a W value which is more than or less than the optimum W the delay is naturally going to be increase. This will be increase, but the energy here will be decreased.

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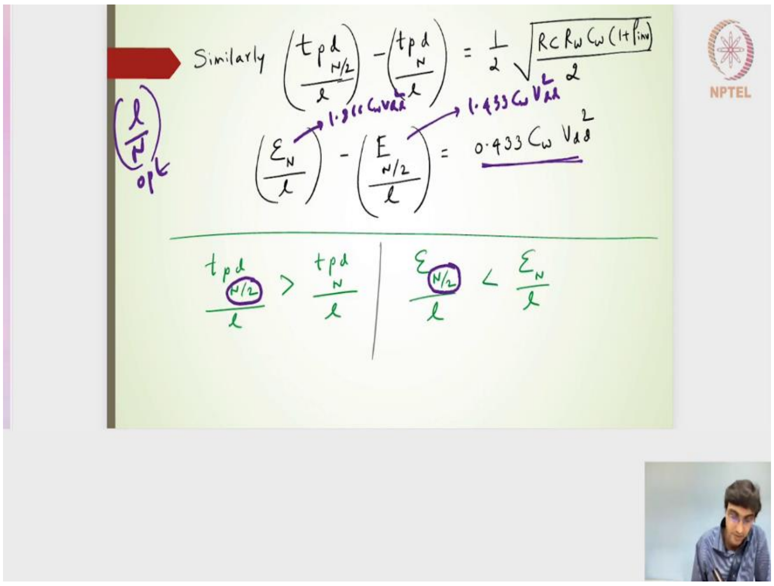

$$\left(\frac{t_{pd\,\frac{N}{2}}}{l}\right) - \left(\frac{t_{pd\,N}}{l}\right) = \frac{1}{2}\sqrt{\frac{R_wC_wRC(1+\rho_{inv})}{2}}$$

Similarly, if I actually reduce the N by 2 that means, that l by N is the optimum l by N we have and if I do some changes here l/N. In fact, if I take N/2 the number of repeaters here is divided by half. I have less number of repeaters for the same l and with respect to if I compare with the optimum number of repeaters for the same l, the delay turns out to be actually larger than that of the optimum l/N or the optimum number of repeaters.

$$\left(\frac{E_N}{l}\right) - \left(\frac{E_{N/2}}{l}\right) = 0.433C_wV_{dd}^2$$

In terms of energy if I actually reduce the number of repeaters, naturally the energy will reduce and for especially for N/2 repeaters, here it turns out to be again $1.433C_wV_{dd}^2$ when compared to the l/N, the optimum value or the end repeaters as an optimum number of repeaters I will get 1.866. This one will be $1.866C_wV_{dd}^2$ then this one turns out to be $1.433C_wV_{dd}^2$.

The overall energy if I change the optimum number of repeaters here the energy will decrease N/2 will ensure that the energy decreases, but the overall delay here increases than with respect to the optimum number of repeaters.

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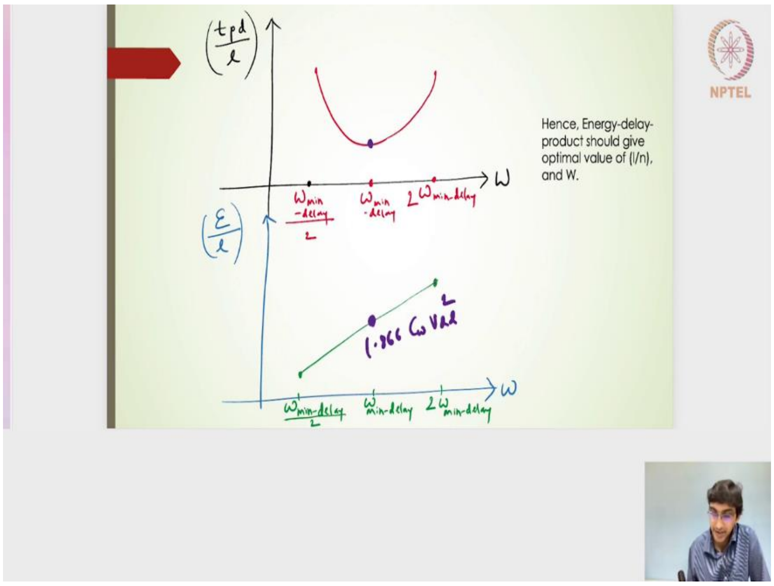

The overall delay per unit length turns out to be in this particular form although I have slightly magnified it possibly on this particular size. It has a minimum value here for an optimum W and then with respect to a different W it will be higher and with respect to the energy per unit length, if the width of the repeaters or the inverters in this particular case is reduced, I will have a lower energy per unit length, while it will be some particular value 1.866 for the minimum W and then if I have a higher size of 2W twice the optimum width, I will have a increased energy.