Design and Analysis of VLSI Subsystems
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Lecture - 40
Assymetric Gates Analysis

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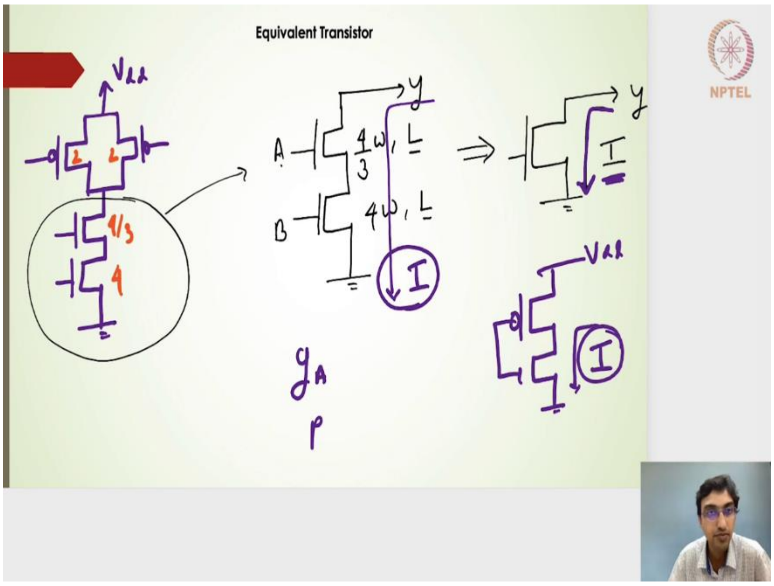

Hello students welcome back to this lecture. Let us continue where we had stopped, we had seen the Asymmetric Gates. Let me redraw that asymmetric gate of the two input NAND gate. The pull up side we had two of the PMOS transistors which are in parallel and then the pull down side we have two of the transistors which are in series.

I have now PMOS and then the NMOS transistor drawn and we had seen a size of 2 and 2 and then here we had a size of 4/3 and then a size of 4. I am going to consider this particular portion and then draw it as a two transistor series for the inputs A and B and this is my output y.

I am going to rewrite this in the form of 4/3 W and then 4W and then we will have a channel length of 1L, where L represents for that particular technology node, if it is for a 65nm technology node it will be 1L, where 1L represents 50nm.

Now, what we really need is an equivalent transistor, that the current from this equivalent transistor becomes easier for us to calculate and from this particular current I should be able to draw the 2:1 inverter.

What we really want is to find out for to estimate the logical effort of the transistor of the input A and B, we need to identify what is the current that will be flowing across this particular path, and that becomes much easier, if I have a single equivalent transistor and then it becomes easier to find out what is that particular current and if it is a current of I, I should be able to find a 2:1 inverter or any other size of inverter, where the current flow is the same current as what is flowing in this particular two series transistor, which is nothing but equivalent transistor of 1 and then if this current and then this current are same. Then I should be able to find out what is the logical effort what is the parasitic and then help to facilitate the linear delay model to estimate the overall delay of the critical path. What we had done previously is if I had the same sizes.

If it was the size of 4W and then 4W and then the channel length of L and L its equivalent transistor will be nothing but 4W, twice the channel length or 2L or we could have also said that it is nothing but 4W divided by 2L. Let me take one more slide here.

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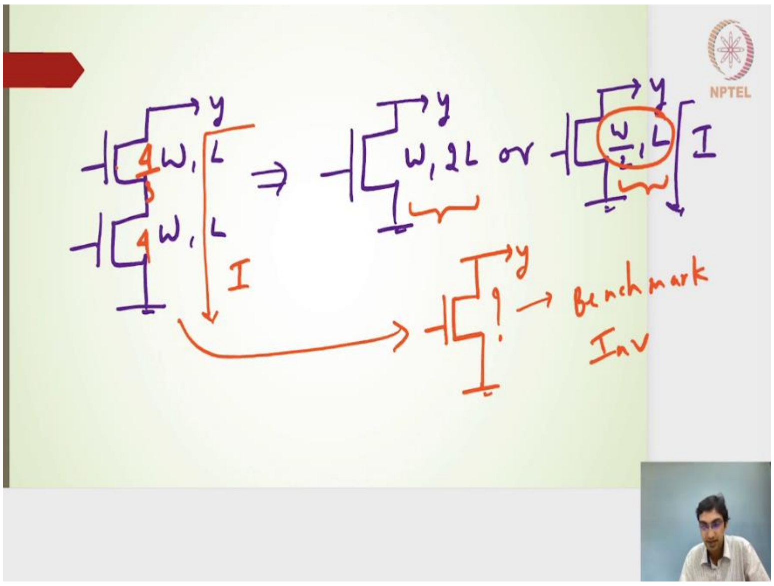

What I meant was if I had two transistors of the W size and then the channel length of L, its equivalent transistor turned out to be nothing but W, 2L this is what we had seen or we could also say that this is nothing but W/2L because both the current equations will match with that of this particular current equation and that is why we said that this is my size or this is my equivalent transistor.

Now, suppose if it is 4/3 here and then 4W here what should be my size here what should be the equivalent transistors sizing? that is the question that we need to answer, if I can find out that size that will be very useful in estimating what is the benchmark inverter. I have to consider for estimating the logical effort and then the parasitic because I think 4/3 and 4 it is kind of very easy because we know that the falling resistance is nothing but r and 2:1.

But if it is of some different ratio where I will not get that particular falling ratio and then more over on top of it if we use instead of a long channel current model instead of that if we use a short channel current model then I think this particular estimation will become little bit tedious.

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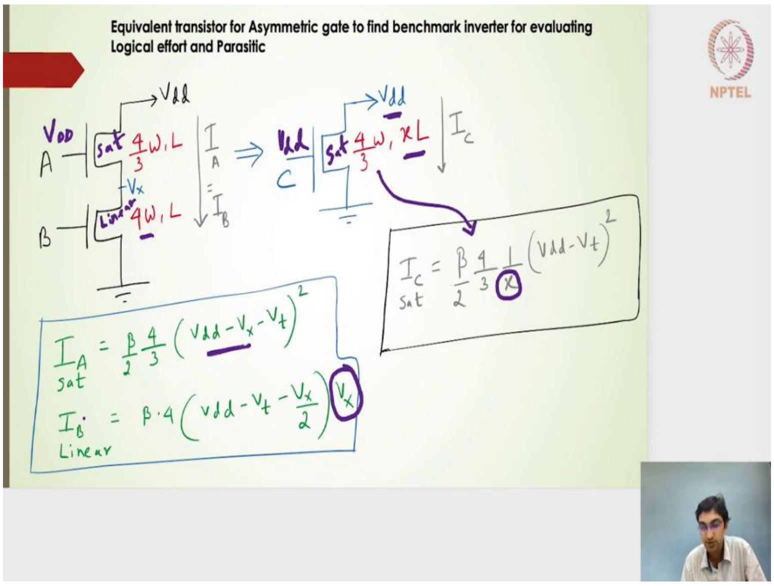

What we are going to do is moving ahead. I have this 4/3 and 4W or transistors we want to find out the equivalent transistor in the form of equivalent single transistors where I can get this current to be same.

Now, suppose that the A transistor has 4/3 W and then the B transistor has 4W and then the same channel length of L and let us say that its equivalent C single transistor is 4/3 W width and then x length, that means, I need to find out what is x.

I am saying that A, B transistor in series is equivalent to be a single C transistor with x length and 4/3 W and 4W is been equivalent to that of a single transistor of 4/3W. That means, that this current here IC current it should be equal to that of this current that is flowing along the 2 series transistors and whenever we have these two series transistors, we know that the transistor A will be in saturation and then B will be in linear.

My saturation current and the linear current should be same as that of this single equivalent transistor current I_C. If we consider V_{dd} here and then C being 1 here there is logic level 1, that means V_{dd} is passed here.

This particular transistor will always be in the saturation mode and again A and B it has to operate only when we give a logic level of 1, that means V_{dd} is been supplied to the transistor A input and V_{dd} is supplied to the transistor B input. If V_{dd} is the case here for the gate side of the transistor A, $V_{dd} - V_x$ will be nothing but V_{gs} and $V_{ds} = V_{dd} - V_x$.

$V_{ds} > V_{gs} - V_t$, that is why this will be always in saturation and then this will be in linear region and then this will be in saturation because we have the V_{gs} to be nothing but V_{dd} which will always this V_{ds} is nothing but V_{dd}. Which will $V_{dd} - V_{ds} > V_{gs} - V_t$.

If I have this particular current equation I_C current equation it will be nothing but,

$$I_{c_{sat}} = \frac{\beta}{2}\frac{4}{3}\frac{1}{x}(V_{dd} - V_t)^2$$

The saturation current for the transistor A is can be written as,

$$I_{A_{sat}} = \frac{\beta}{2}\frac{4}{3}(V_{dd} - V_x - V_t)^2$$

The linear current for the B transistor here is nothing but,

$$I_{B_{sat}} = \beta \cdot 4 \cdot (V_{dd} - V_t - V_x/2)V_X$$

If I equate this 3 currents, what we are likely to get is what should be the x value, but we have 2 variables what is x and what is V_X and we have 2 equations basically and 2 variables. If $I_A = I_C$ I will get one equation, if $I_A = I_B$ I will get another equation and I have 2 variables. I should be able to find out the x value.

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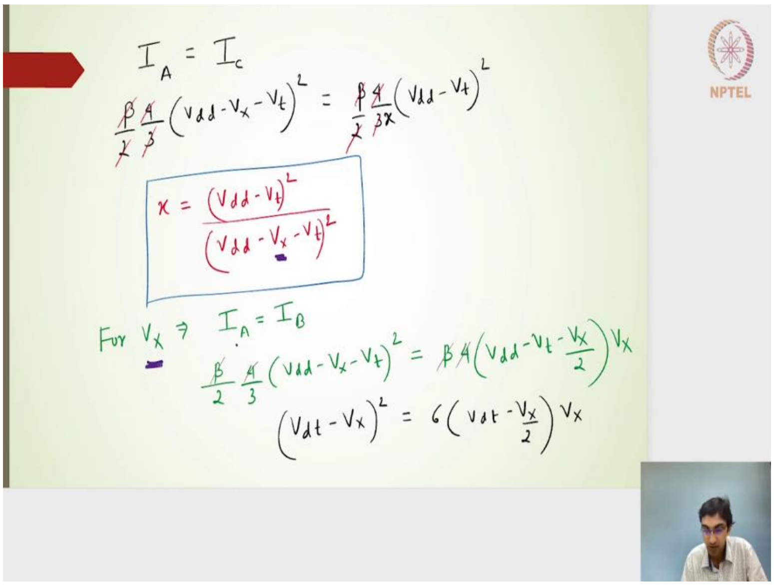

If I equate I_A and I_C I should be able to find out what is x in terms of V_X.

$$I_A = I_C$$

$$\frac{\beta}{2}\frac{4}{3}(V_{dd} - V_x - V_t)^2 = \frac{\beta}{2}\frac{4}{3}\frac{1}{x}(V_{dd} - V_t)^2$$

$$x = \frac{(V_{dd} - V_t)^2}{(V_{dd} - V_x - V_t)^2}$$

If I equate I_A and I_B I should be able to find out what is V_x,

$$I_A = I_B$$

$$\frac{\beta}{2}\frac{4}{3}(V_{dd} - V_x - V_t)^2 = \beta \,.\, 4 \,.\, (V_{dd} - V_t - V_x/2\,)V_X$$

$$(V_{dd} - V_t)^2 = 6(V_{dd} - V_x/2\,)V_X$$

$$V_{dt}^2 + V_x^2 - 2V_{dt}V_x = 6V_{dt}V_x - 3V_x^2$$

$$4V_x^2 - 8V_{dt}V_x + V_{dt}^2 = 0$$

$$V_x = \frac{8V_{dt} \pm \sqrt{64V_{dt}^2 - 16V_{dt}^2}}{2\,.\,4}$$

$$V_X = V_{dt} \pm \frac{4}{8} V_{dt}\sqrt{3}$$

$$V_X = V_{dt}\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$V_X = 0.1339V_{dt}$$

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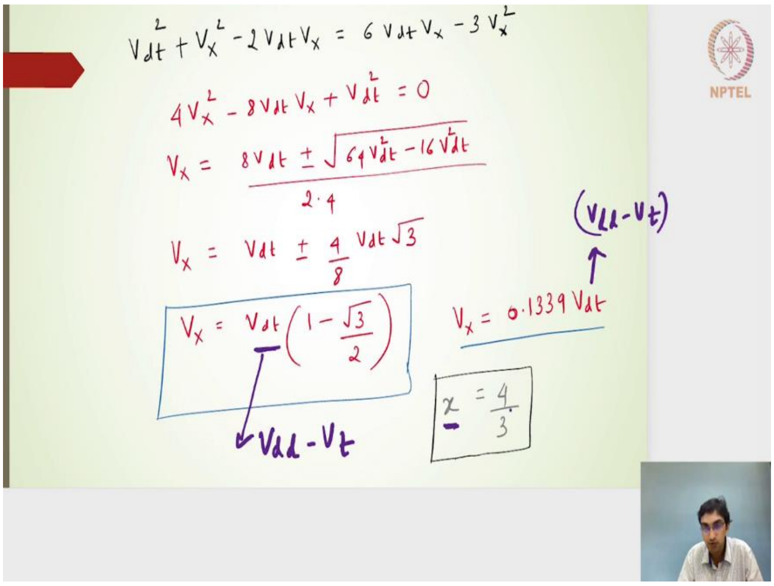

$$x = \frac{4}{3}$$

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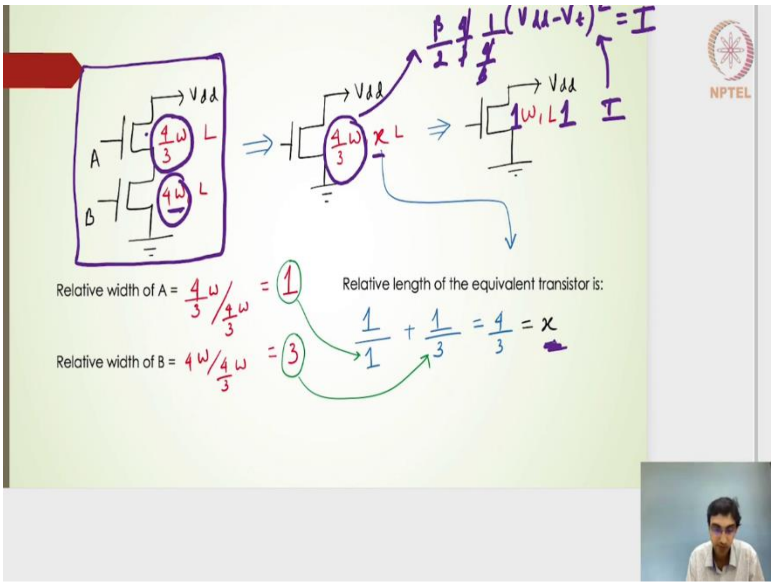

If I put this particular value of x here which is nothing but 4/3. 4/3 W comma 4/3 x and if I rewrite that particular current equation, this is the current equation for this particular transistor 4/3 W and 4/3 x will be nothing but saturation current beta by 2 width is scaled by 4/3, length is scaled by 4/3.

I have to write it in the denominator of 4/3, this gets cancelled what we have is $(V_{dd} - V_t)^2$. Now, this will be my current equation for the transistor whose width is scaled by 4/3 and x is also 4/3 turns out to be nothing but same as that of 1W and then 1L.

$\frac{\beta}{2}(V_{dd} - V_t)^2$ is nothing but this particular current is also the same. Coming back to this 4/3 W and then 4/3 x. What it really means is if I have these two transistors in series, which is having a width of 4/3 W, L, I mean L being the channel length width is 4/3.

Another transistor in series is 4, we can consider it to be an equivalent transistor of 4 by 3W,xL, where x I need to estimate, I can do the estimation using the current equations, write and then find out what is the x value or else a simplified form will be nothing but measure the relative width of A here, relative width of B and then the reciprocal of this relative widths will give me the relative length of the equivalent transistor.

What it means is let us try to calculate what is the relative width of A. The relative width of A is nothing but this transistor sizing 4/3 sizing divided by the 4/3, I have taken in the equivalent single transistor.

4/3 W divided by 4/3 will be nothing but 1 relative width of B will be nothing but 4W divided by the 4/3 W of the single equivalent transistors. It will be nothing but 3 the reciprocal of this value of 1 and 3, the reciprocal of that will be nothing but 4/3 and that is what the x will be.

In fact, this particular simplified form is actually coming from our current equations, we have 3 current equations, 2 of them equated, we will get 2 equations and then 2 variables, we will be able to evaluate the x value, this particular simplified form is actually coming from there.

But just as a handy calculation if I Am trying to find out the single equivalent transistor. I can have this as 4/3 W, xW, where I can find this x as nothing but the reciprocal sum, the sum of the reciprocal of the relative widths. I will be able to get the x value. I can consider a different width in the equivalent transistor.

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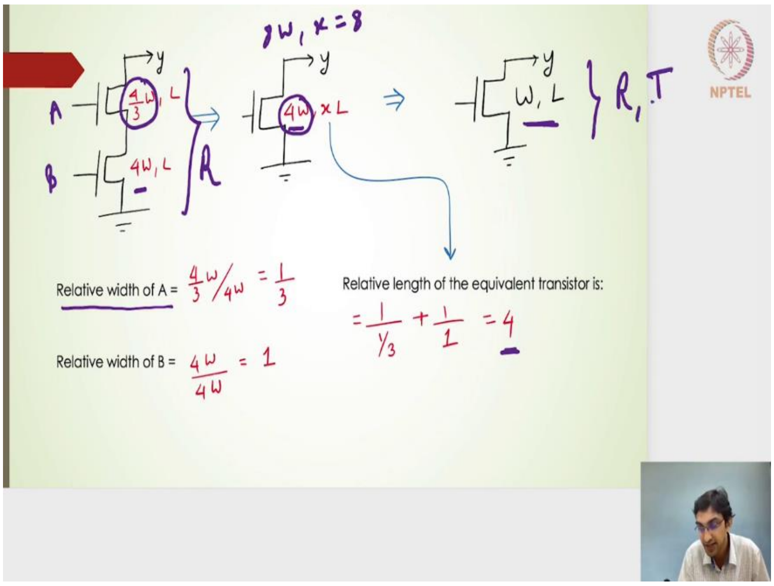

Instead of 4/3 W, I can consider 4W and find out the x using the same method. Find out the relative width of A=$\frac{\frac{4}{3}W}{\frac{4}{3}W}$. This is now 4 instead of 4/3, now I have to chosen or selected a 4 width here in the equivalent single transistor.

4/3 will be this 1 because this the relative width of A I am trying to find out which will be 1/3 and the relative width of B here will be nothing but 4W/4W will be 1 and x value is nothing but the relative length of the equivalent transistor will be nothing but the sum of the reciprocal of this relative width.

The reciprocal will be 1/3, reciprocal will be 3 and 1s reciprocal will be 1, it will be 4. I will get x a 4 if I consider 4W as the equivalent single transistor width. Finally, I will get a W/L, because the current of 4W,4L will be same as that of the current of W,L.

In fact, I can actually choose a different width here for the equivalent transistor. In fact, I can chosen 8W here, where x turns out to be 8 and if I choose a 10W there I will get an x as 10, because I have chosen 4/3 and 4, such that my equivalent falling resistances R and that is the reason why I am getting the overall falling resistances R coming from the current of I.