Networks and Systems

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Lecture-65
Many Facets of the System Function (Contd)

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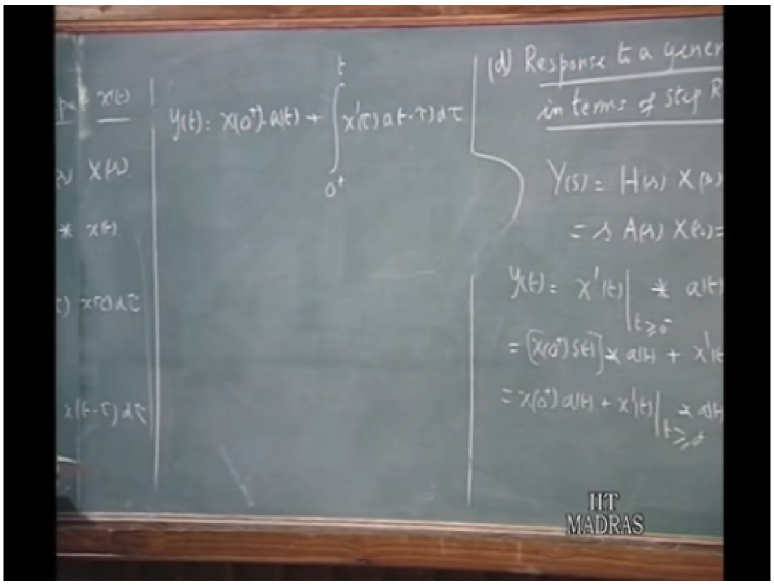

Now, using this step response, let us see how we can find out the response to a general input x t in terms of step response. Now, if x of t is the general input, y s will be h of s x of s, is a fundamental rule that we are having, but h of s is s times y of s. So, I can write this as s times a of s times x of s; that is what we are having. I can write this as s times x of s times a of s.

Now, y of s therefore, is the product of 2 Laplace transform; s times x of s and a of s. the time function corresponding to this is the indicial response at. What is the time function corresponding to this, is d x by d t, the derivative of this, taking into account that starting from t equals 0 minus. Therefore, y t can be written as x prime t.

When you take the derivative of x prime t, starting from t 0 minus; that is if x t is like this, this is x 0 minus, this is x 0 plus. When you taking the derivative you must take this transition also into account, and that is in meaning of this x prime t consider from t equals 0 minus onwards, convolved with a of t which is the inverse Laplace transform of that.

Now, you note that x prime t, if this is the variation x of t this is x t, x prime t can be written as in the 0 minus 0, because we always talk about causal inputs. So, the causal input 0 jump to 0 plus; therefore, the derivative you get x 0 plus times delta t plus the rest of the derivative taking starting from t equals 0 onwards 0 plus onwards.

So, x prime t starting from t equals 0 minus, can be thought of as an impulse function, which is comes because of this jump plus derivative from t equals 0 onwards. So, I can write this further as x 0 plus times delta t. this convolve with at plus x prime time which will considered only from 0 plus onwards, convolved with a; that is what we are having.

And we earlier observed that any function of time convolved with delta t, means is equal to the same function itself. Convolving with delta t does not change the function; therefore, I can write the as x 0 plus times a t. This you can also visualize easily, because the Laplace transform of delta t is 1.

Laplace transform of this is a of s, then the convolution as the Laplace transform 1 times a of s the must Laplace transform gives back as a of t plus x prime t t greater than equal to 0 plus convolved with a t. So, this is the result that we get. Now, let me put this down in the form of integral more explicitly.

So, what we are here is, y t therefore, is x 0 plus times a t plus this one I will put in the form of the integral staring from 0 plus to t x prime tau a t minus tau d tau; that is what you have; that is the meaning of that; that means, you response to any arbitrary input x t can be obtained if you know the indicial response for this step response, you can find it out other this comes from of course from this Laplace transform s a s times x of s.
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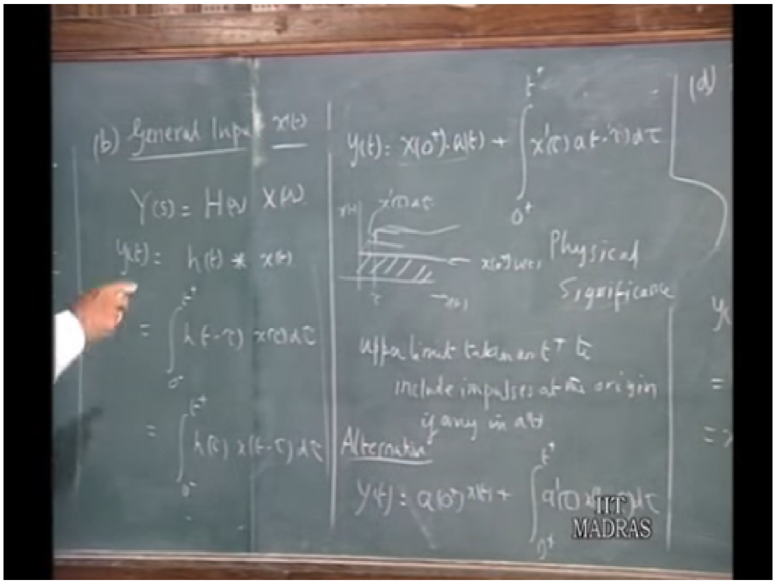

Now, one can also alternately later have some. First of all before that physical meaning of that, I will just briefly outline without finding too much time on that. Suppose this is x of t you can thing of this x of t as a step function, to start with this is x 0 plus times u of t plus a number of small steps occurring later on values of time, and each of this steps will have something related to x tau delta tau delta tau, something like it will be there.

So, we have the given x of t can be decomposed, as a number of steps starting with x 0 plus u t, and subsequent steps you have a height, depending upon the derivative and the interval that you are taking. So, each one of the x prime t delta tau, is the height of the step, and that occurs the point tau; therefore, it means for delayed step response.

So, summation of all those things will be representing by this integral, and the response to the initial step is represented by this term. So, that is the physical significance of this particular formulation of the output in terms of the indicial response. So, this is the physical significance of this.

Now, there are impulse as in the step response as the origin; suppose a of t has the impulse and origin. To take that in to account input as t plus, t plus to take into account impulses. May be I will write this separately, because instead of clipping the picture you can say, upper limit taken as t plus to include impulses at the origin if any in a t.

So, it is possible the step response may included at the impulse at the origin to take that complete impulse in the origin, to take that impulse into account the upper limit is taken as t plus. And alternative version, after all i associated with s with x of s. I could have taken associate s with as instead of x of s I could x s times a of s.

So, in alternative version would be y t will be a 0 plus times x of t plus 0 plus to t plus a prime tau x of t minus tau d tau. So, that is an alternative way of representing this, because after all x a s the symmetrical in a s and x s. So, whatever role x t plus here, a t will also play here. So, these are alternate.

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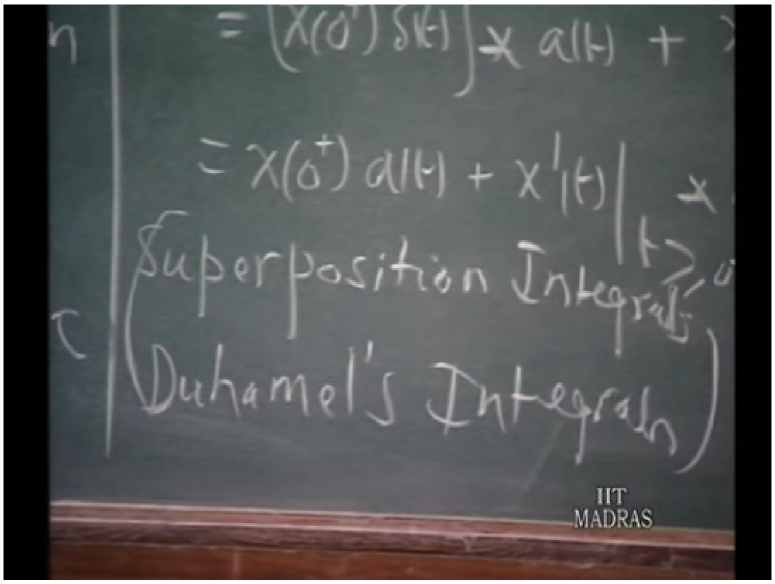

Now these 2 integrals what we have here, are also referred to super position integral literature. They also refer to Duhamel's integrals. So, this are alternative names for that, even the convolution integral terms of impulse response is also called super position integral, that a literature this is called the super position integral. These are also super position integral or Duhamel's integrals Duhamel's integrals is of course particularly with a step function.

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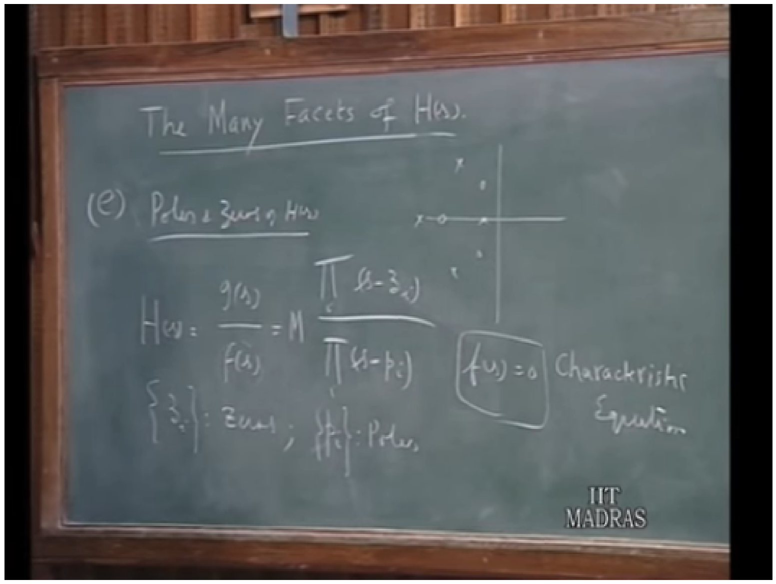

Another property of the system that you like to make a note of. This is the significance and poles of poles and zeros of h of s. So, h of is the ration function as mentioned as the ratio as 2 polynomials. Therefore, it is ratio of g s over f of s. So, it is in general in constant multiplying factor, and product of several such factors sum down i and in the denominator we have products of factors sum down i.

And the set z i are called zeros of this h of s, and the set of p i values are called the poles of this something which we already mentioned. So, h of s is specified by the locations of zeros and poles, as well as the constant multiplying factor m. So, the poles in zeros are locate are general impulse in the complex plane, by the z location z i location are indicate by zeros, pole locations by crosses.

And it has to be kept in mind that; since g of s and f of s are polynomials with real coefficients of s. whenever a complex pole occurs its conjugate also must be present. Similarly a complex 0 is always accumulate conjugate; that means, complex zeros and poles occurring conjugate pairs not singly.

As far as the real poles are concerned, there is no problem, it can appear with a unit multiply be seated. Now, an important factor that one would notice in solving a system is.

Suppose we have different types of output that you looking for, and naturally you have different system functions, relating the different outputs for corresponding input.

It turns out suppose h 1 of s h 2 of s h 3 of s depending up on the y's that you are talking about; y 1 y 2 y 3. All of them have got the same f of s in the denominator. So, f of s equals 0 is what is called the characteristic equation of the system. This is the characteristics of the system.

So, no matter what response quantity you are looking for, the denominator here f of s is the same, for all transfer functions, for all system functions, for a given system this called the characteristic function equation.

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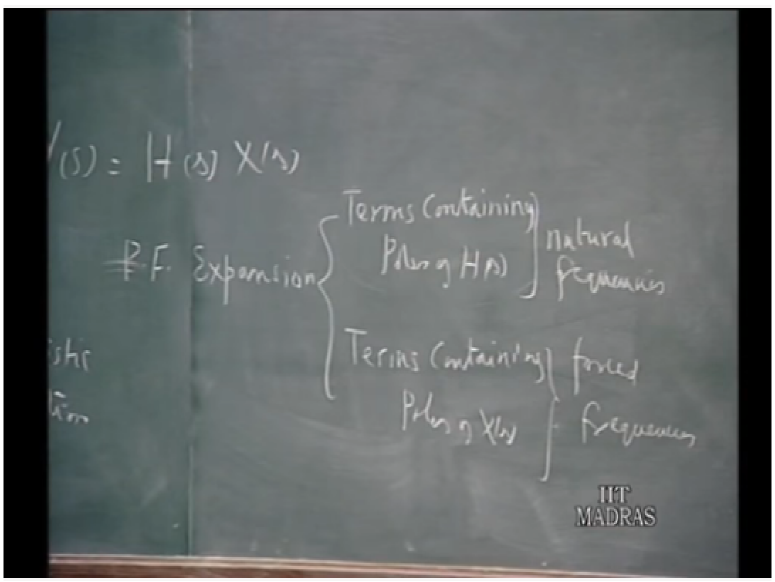

And when you talk about y of s here your h of s times x of s, and you make the partial fraction expansion of this, in y of s. Then the partial fraction expansion of y of s, will involve terms containing poles of h of s, terms involving poles of x of s. In the partial fraction expansion what we have is, terms containing poles of h of s are representing the natural frequency system. They are the PICL.

There are the roots of the characteristics equation. So, the natural frequency of the system, are the roots of the characteristics equation. when I talk natural frequency I talk about frequency complex domain their roots of the characteristics equation, they are the poles p i. On the other hand you are h the terms containing, poles of h of s x of s are the forced frequencies.

They are the frequencies that come as the result of the particular input. So, when you have got y of s, when you make the partial fraction expansion, you get some terms corresponding to the poles of h of s, sometimes corresponding to the poles of x of s. The terms contain the poles of x of s, if you add the time domain response of all of them, this is the force response.

On the other hand if you take into account only the poles of h of s, they represent natural frequencies, the summation of all the corresponding time domain expressions, represents the natural response the system, and the total response is what you obtain take the inverse Laplace transform of this.

So, whenever you want aggregate the total response is natural response and force response. We can always look at this, as emanating respectively from the poles of h of s by the poles of x of s respectively. As far as zeros are concerned, they do not have any particular connotation like this, the only vary the amplitude of this various components.

Suppose I have one particular response quantity e to the power of s t a. It is the amplitude of the multiplying factor, of the complex frequency term; that is dictated by the zeros. The poles however represent the nodes, the time expressions, the complex frequencies, dictated by the poles, where as the zeros will vary the amplitude of the various terms.

So, in this lecture what we have done so far is, acquainted ourselves the definition of the system function in terms of the Laplace transform. You recall that the system function is defined as the Laplace transform of the response quantity, to the Laplace transform of the excitation quantity, with 0 initial conditions.

Prior to the application input and the system function is general the ratio of 2 polynomials, for the type of system that we are taking about number of parameter systems, which is f of s g of s over f of s. And we looked at various property of the system function. The system function is the Laplace transform of the impulse response. We also saw the system function is also related to the Laplace transform of the step response.

We saw the various types of convolution integrals, how to get the output for the general input in terms of the impulse response h of t, or the step response a of t. We also finally, looked at the nature of the ratio of two polynomials, which comprises, which specify the locations of zeros and poles of complex plane, and we mention that the poles of the h of s indicate the natural frequencies of the system.

They are invariant for a particular system, and whatever type of output that is looking for in the system. They do not depend up on the input quantity. On the other hand x of s we have some poles which are characteristic of the particular excitation function that you have, dealing with at the point of time.

So, y of s which is the product of h of s have some x of s, will have poles partly coming from h of s, partly coming from x of s. The poles coming from h of s presents the natural frequency, and the associated response as the natural response. The poles coming from x of s are the forced frequencies, and the associated times response, is the forced response. More about the system function will be taken up in the next lecture.