Networks and Systems
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Lecture-63B
Examples and Advantages of L-Transform

Let us now consider another example, in which the Laplace transform techniques applied, to the circuit containing sinusoidal sources.

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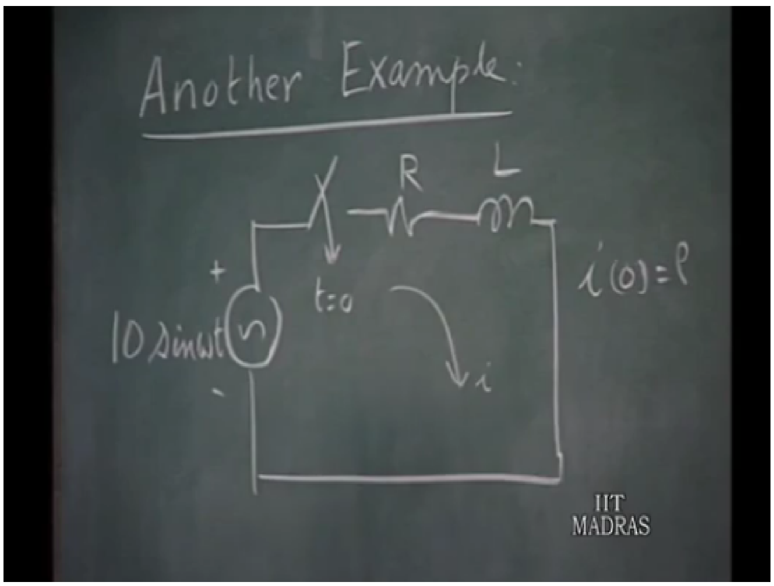

So you have a sinusoidal voltage source of 10 sin omega t, and the voltage and the switch is closed t equals 0. Let there be 2 elements in the circuit r and l, and we are interest in finding the current in the circuit for i greater than or equal to 0, and it is given that i 0 is rho, because of some previous arrangements the current in the inductor is i 0 at the time of switching. So, we make the transform diagram for this.

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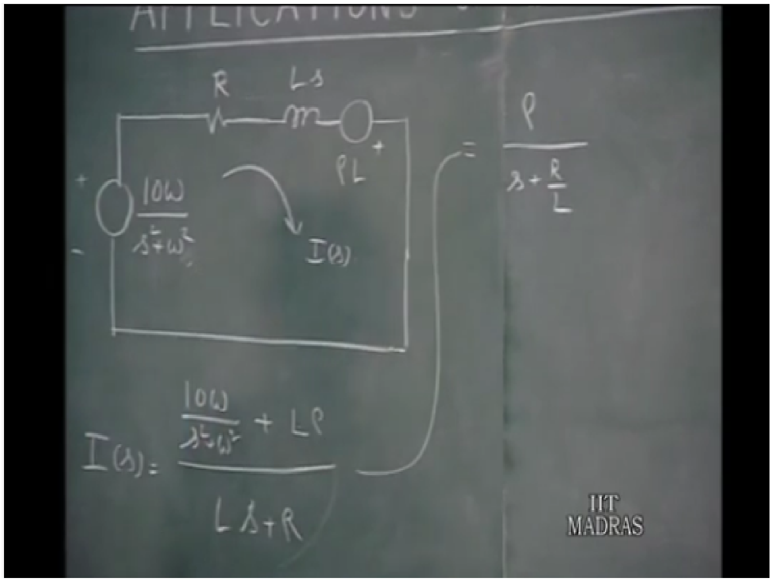

So, the voltage source has the transform 10 omega over s square plus omega square; that is the Laplace transform of 10 sign omega t, and you have the resistance r that is the generalization impedance of this. As far as inductor is concerned, you have a generalized impedance l s, and in addition you have a voltage source, representing the initial current in the inductor, which is rho times l.

The voltage of the voltage source is rho times l l rho, so that is what we are having. So, that completes the transform diagram, and in this we are interest in finding out the current I of s. So, if you write this after all the simple loop circuit therefore, I of s can be written as 10 omega over s square plus omega square plus l rho; that is the total voltage driving in the loop divided by l s plus r; that is the total impedance of the circuit.

Now, this can be written as. After all you make the partial fraction expansion for this. So, I can write this as; first of all l rho or l s plus rho; that is straight forward; therefore, I get rho over s plus r by l, that comes from this.

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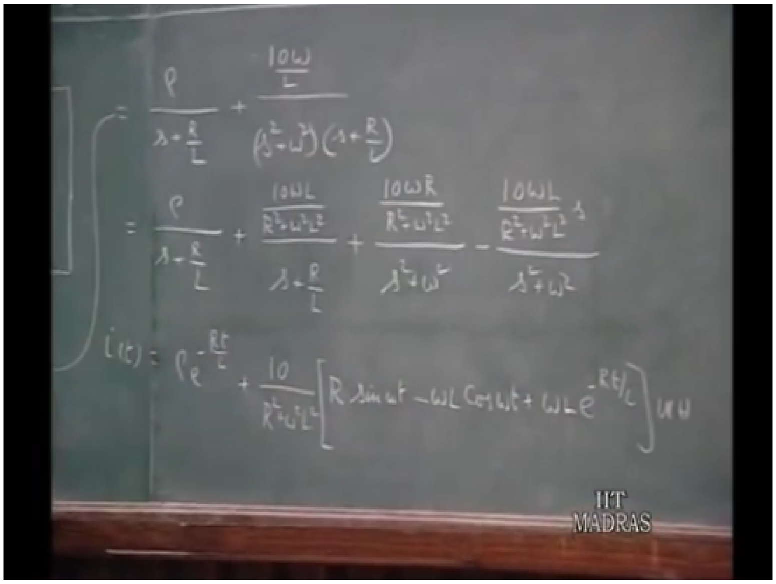

Plus in addition I have 10 omega over s square plus omega square, the ratio of the first 2 terms I will write s plus r by l, and divide this by l. So, 10 omega over l divided by s square plus omega square times s plus r by l. So, you make the partial fraction expansion of this, you will get rho over s plus r by l as before for the first term.

And for the second term you have; s plus r by l 1 term and that will turn out to be 10 omega l divided by r square plus omega square l square. You can carry this out, I do not spend time in working out the details, but one can easily arrive at this values.

And you have for the s square plus omega square term 10 omega r over r square omega square l square divided by s square plus omega square minus 10 omega l divided by r square plus omega square plus l square times s square plus omega square. Actually when you have s square plus omega square in the denominator you have a s plus b time in the numerator.

I split up in to these two portions so that this can be recognized to be the Laplace transform of the sin function. This has recognized to Laplace transform cosine function. So, i of t is, for the first time is concern rho e to the power of minus rt over l rho e to the power of minus rt over l; that is the Laplace transform of this.

As far as this is concerned you have 10 over by r square plus write let me say ten by r square plus omega square l square, that is the common factor write through. So I will keep that separately 10 have r square you have omega over s square plus omega square gives you sin omega t.

Therefore, you have r sin omega t. here again 10 over r square plus omega square l square taken out, s by s square plus omega square will yield me cos omega t; therefore, I have minus omega l cos omega t plus. Here 10 r square by r square plus omega square l square taken out the common factor; therefore, plus omega l times e to the power of minus rt by l, and of course u of t. So, this is your, all this multiplied by u of t. You notice that t equal 0.

This is 0. This cancels out with this plus cos omega t and e to the power of minus rt by l are both equal to 1 this is 0; therefore, this term is equal to 0, and this is equal to 1 and the initial value of the current it is rho which is what we know. Now, in these you observe, that this term and this term can be combined, if you wish, is decay in transient minus rt by l.

Therefore, this term and this term together are the transient terms that decay with time actually both of them can be combined. Whereas these two terms are the steady state term. You have driving this particular circuit with sinusoidal forcing function; therefore, this is steady state solution, and this is what we are having.
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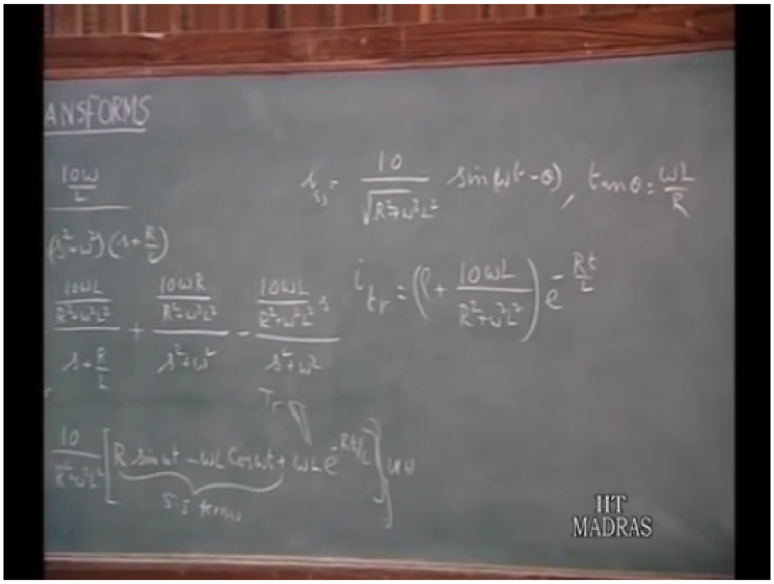

You can show this steady state term to be 10 by root of r square plus omega square l square; that is the impedance circuit; sin omega t minus theta, where tan theta equals omega l over r. So, this is the steady state solution. You have here in ac circuit in which having a driving force 10 sin omega t, and you have a r and l in the circuit.

Therefore, the impedance is square root of r square plus omega square l square, and the power factor angle equals 10 inverse of omega l over r, so that is the steady state solution. You combine this two and you can show very easily, that the sum of these two indeed this. the transient power is the solution, equals rho plus 10 omega l over r square plus omega square l square e to the power of rt by l; that is the transient power of solution.

Rho e to the power of minus rt by l plus 10 by r square plus omega square l square time omega l e to the power of minus rt by l, this is the transients solution. So, in this we can see that once we have a solution, we can identify the terms which corresponds to this steady state, behavior of the current, and what are the terms which correspond to the transient portion of the response.

So, both this are simultaneously brought out in the final solution. You can combine these terms and show it to be so. And of course, once you have the total solution that is valid only for t greater than are equal to 0; therefore, u of t will always there, to specify that i of t that expression whatever we have, is valid only for t greater than are equal to 0.

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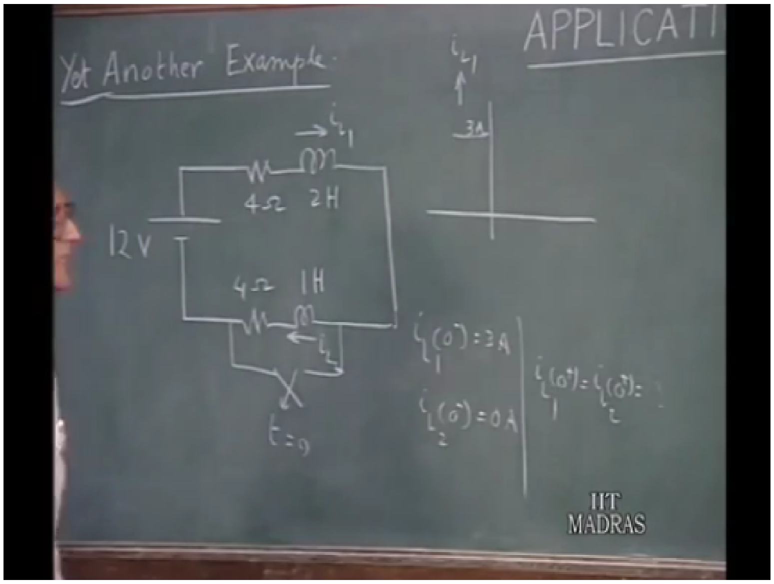

Let us work out one more examples. Let us now consider this example, which we had earlier discussed in the context of the classical, whether differential equation approach to the solution of transients. We have set up here in which the two inductors. The two inductors are originally carrying different currents, but once the switch is opened both of them are perforce, required to carrying the same current.

Therefore, there is the case of discontinuous in inductor currents, in the transient from t equals 0 minus to 0 plus. In the case of differential equation approach, we have calculated the t equals 0 plus conditions separately. Now, let us use the Laplace transform other than show, that once you circuit t equals 0 minus conditions 0 plus conditions, almost immediately follow in the normal routine.

You do not have to make separate calculations. So, let us use 0 minus conditions, and solve for this circuit. You recall that the current was originally 3 amperes, when the switch is closed, the current in the i l 1 0 minus is 3 amperes, and i l 2 0 minus is 0 amperes.

But once the switch is opened i l 1 0 plus and i l 2 0 plus are going to be the same, and you would like to know what the value is, and we use the constant fluxing case theorem, or some similar effect to find out the new current in the inductors. Let us use Laplace transform techniques here. So, let us draw the transform diagram for this.

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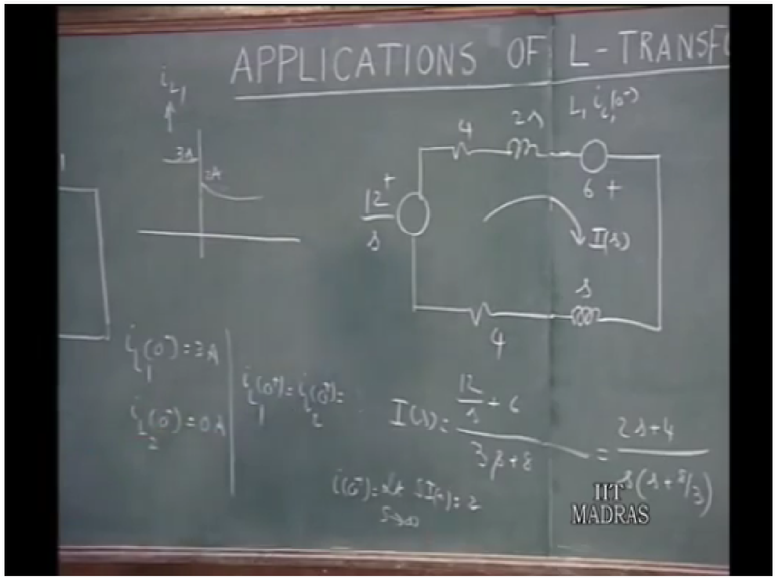

You have 12 by s is the transform of the voltage source. you have 4 2 s, and as for as the inductor is concerned we use the 0 minus values, 0 minus value is 3 amperes, so 3 times 2 6 l 1 i l 1 0 minus; that is 6; and down here you have inductance of value s impedance s. and the initial current is 0 t equals 0 minus.

Therefore, you may as well do not have that, and you are having 4. So, what is the current here? This is very simple circuit, so i of s will be 12 by s plus 6 divided by 3 s plus 8; that is what we are having. So, this will be 2 s plus 4. Simplify this you get 2 s plus 4 times s plus 8 by 3. Now, we can make the partial fraction expansion find out the i of t, but before that find the initial value the current; i 0 plus equals limit as s tends to infinite of s times i of s.

So, when you multiply this by s, and take the limit as tends to infinite that becomes 2, 2 s square by square; therefore, this is 2; that means, this current from 3 amperes, we have jump to 2 amperes at t equals 0 plus and something else equals. So, automatically you find out the i 0 plus.

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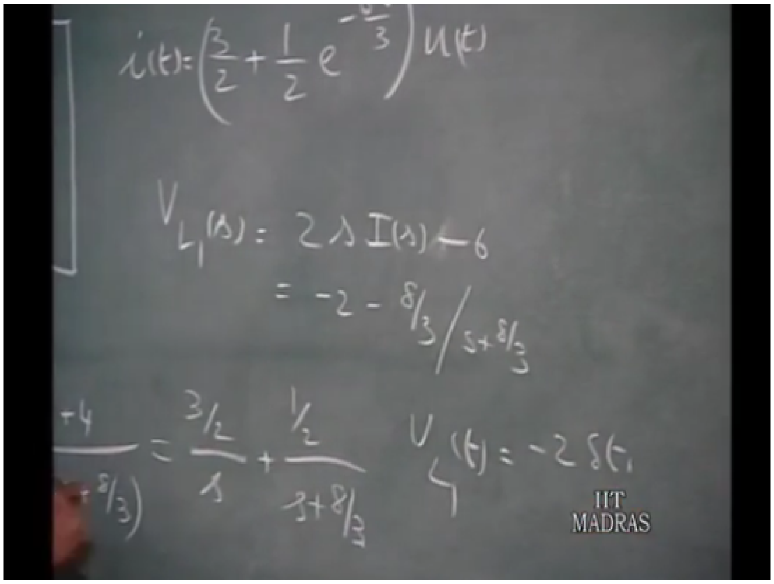

So, if you want to find i t separately, you make the partial fraction expansion, this becomes 3 by 2 s plus half of s plus 8 by 3. So, you get i of t in the loop as 3 by 2 plus half of e to the power of minus 8 t by 3 u of t. So, that current was originally 3 amperes in i l 1, jump to 2 amperes, and then finally, indicates to 3 by 2 amperes.

Now, what about the inductor voltage; suppose i want to find out the inductor voltage vl 1 of s. When you want to find vl 1 of s, now that we have got i of s 2 s times i of s plus; that is the drop here minus 6. So, when you want to find the inductor voltage, you must take the voltage between these 2 terminals, not across the inductor alone this portion only.

You must take the initial current also in account, because these are the known terminal of this inductor in the circuit. This inductor has been replaced by not only an inducting impedance, but also initial current source. So, when you want to find out vl 1 of s, if this is vl 1 of t the corresponding Laplace transform, is the voltage in the transform diagram existing between these two terminals.

You should not make the state of computing mainly this portion. So, 2 s times i of s minus 6; that is v l 1 of s, and you can show that this will be minus 2 minus 8 by 3 divided by s plus 8 by 3, which means vl 1 of t is minus 2 delta. I will write this separately here.

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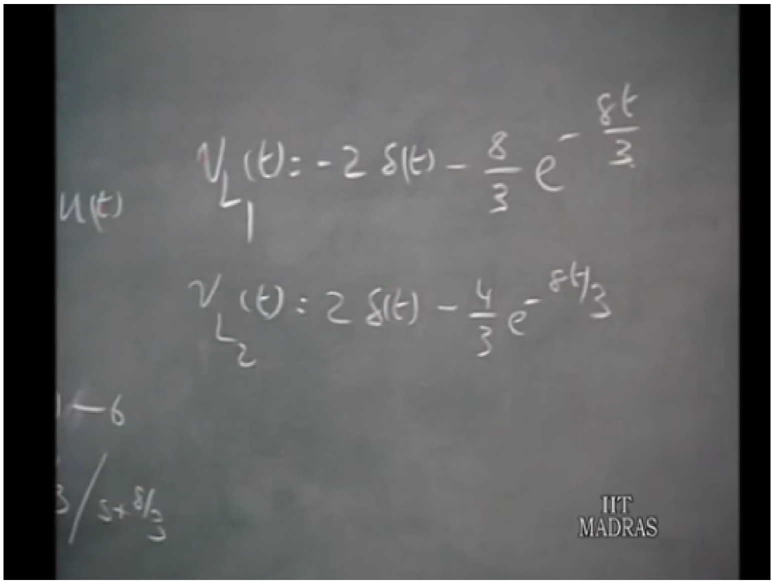

V l 1 of t is minus 2 delta, delta t minus 8 by 3 e to the power of minus 8 t by 3, and likewise we can calculate v l 2 t; that is the voltage across this, between this 2 points v l 2 t likewise we calculate, you will get this result 2 delta t minus 4 upon 3 e to the power of 8 t by 3. So, this both the currents are jump from t equals 0 minus 2 0 plus values in no doubt, there will be impulse functions.

But taking the l 1 and l 2 put together, the impulse functions can cancel each other out; that means, even though the inductors have impulse voltages in them. In the local loop the impulses get canceled; therefore, all the other voltages are finite, and therefore, the total voltage law is still satisfied, because the impulse voltage generated here, is cancelled by the impulse voltage are generated here.

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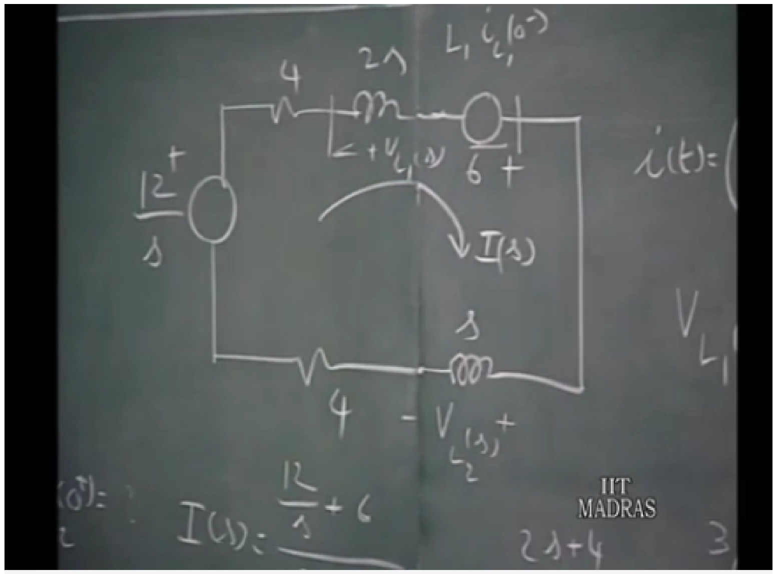

Because this will be, now v l 2 of s this v l 1 of s define theses 2 points, and the sum of this transient be 0. So, we have worked out, a number of examples illustrate in the Laplace transform techniques. As I mentioned once we have the transform diagram, we can use the whole range of network analysis technique; that is available to us, in the context of d c circuits or a c circuits.

We can use the node analysis, the loop current method analysis. So, various network theorem we can apply all this, follow in a straight forward duty in fashion. Later on we return to a discuss on network theorem and some point, and then we can pick up this thread at the time, but at this time, from what we have seen.

Let us now see what are the advantages of the Laplace transform domain in the transient analysis of the electrical networks. We find there are number of advantages compared with the classical differentiae equation approach. First, when you have functions of time which are discontinuous, or which are discontinuous derivatives; like your step function, or ram function.

Then the Laplace transforms of these time functions can be simple expressions, where as if you look at the classical differential equation. Suppose you have the function is described by different electrical expression in different regions of time. So, you have to find the solution for one interval of time, find the final solution, and put this for the initial condition for the next interval and so on and so forth, that becomes quite complicating.

Whereas in the transform domain, for the entire range of time you have one single simple expression, for the Laplace transform domain. Secondly, we observed that we do not have to do involve ourselves with calculus operations; no integration, no differentiation. All the equations describe in the behavior of the system, are put in purely algebraic form, algebraic equations are involved.

So, calculus is converted in to a kind of algebra, because differentiation corresponds to multiplication by s, integration corresponds to division by s. So, calculus is converted to algebra. A third feature is, that you get the total solution; the transient part the solution, and the steady state part of the solution both together in 1 unit.

Whereas in the class steady the differential equation you have the particular integral solution, the complementary function. So, there are found separately and combine them, but here you do not have to do that. You find out the total solution in 1 step. But once you got the total solution, you can identify the steady state part and the transient part in the final solution, using common reasoning which is, and these terms will become evident.

If you see what are the terms which come to forcing function, and what are the terms which come from the system itself. This will be clearer when we discuss the system function and its ramifications later on. But the main advantage of the Laplace transform technique is, that the initial conditions pertaining to the system are plugged in into the solution in the equations at the very beginning itself.

Therefore, you do not have to evaluate separately the arbitrary constants, that you have to do, in the case of differential equations. And this is very important, because if the order of the system, the number of reactive elements becomes large. Suppose there are 4 or 5 reactive element, and you have the fourth order differential equation.

Then you need to know 4 initial conditions, not only the initial value, but the values of the first 3 derivatives. At this becomes quite complicated, because they are not evident, they are not given the problem, and you have to calculate this, I have to find out the initial derivatives second derivative third derivative, to separate manipulations.

All that unnecessary as far as the Laplace transforms technique is concerned. All the initial conditions pertain to the reactive elements, are use in the transform diagram, are equivalently in the differential equation, we substitute them and you get the solution. So, you separate evolution separately arbitrary constant is not necessary at all, in the transform approach.

And in this context it is seen that, you do not have to find out the 0 plus conditions separately from 0 minus conditions. If you have 0 minus conditions, you straight away plug them in u 0 minus conditions, and the solution that you get is 1 that starts from 0 minus, automatically it will proceed.

And if you in fact interest in finding out 0 plus conditions, you can use the initial value theorem, refine the expression and find out what 0 plus conditions are. So, all these are the advantages of the transform approach, to the solution of transients in networks. To summarize once again, because this is the very important aspect, let me recapitulate the very less advantages that we have mentioned.

First of all, functions which are discontinuous and which are discontinuous derivatives are easily handled in the transform domain, because you get once a single expressions valid for all time. Secondly, we find that calculus is converted into algebra. So, the manipulations are that much easier.

Thirdly we find the transient and steady state solution come together in one package, but you can separate them at the final solution into the 2 separate components if you so wish. And most important of all, is the fact that the initial conditions are used in the problem, to start with the data regarding initial conditions in fact in the problem the very beginning itself. So, separate evolution of arbitrary constant is avoided.

This troubles of feature as for the classical differential equations solution is concerned. After having said all this, must also be fair to the fundamental approach of the differential equations. What is the differential equation can do, which Laplace transform perhaps cannot do. See the differential equations very fundamental solutions, fundamental approach.

Suppose you have given some data, not the initial condition t equal 0, but you want to specify some conditions on the response, initial value equals so much at t equals 0. The derivative is equal to so much at t equals one second, the second derivative so much at t equals two seconds.

So, suppose the data which regarding the arbitrary constants are specified in a different way. Not all a t equal 0, then the Laplace transform domain, is not very convenient to handle such type of data, if all the initial conditions t equals 0 or given then it is nice. But if it is not so, then the differential equation is much more unable to giving as the proper solution, and it is certainly is a more fundamental approach.

So, one will also have keep in mind, one should also know how to handle transient problems, using the classical differential equation approach, even though Laplace transform has lot of advantages, and we for the most part is at Laplace transforms, one should not completely ignore or forget, the differential equation approach to the solution of the transient problems.

Particularly the simple cases you can write down the differential equation solution by the mere inspection. You do not have to transform those equations every time. So, is a powerful technique no doubt, but in very simple cases. The straight forward application of differential equations will fetch the result quickly, and also with little bit of inside in to the working at the problem.

So, we have covered in this last 2 lectures; the application of Laplace transforms, to the transients in networks. Now in the next lecture we will take up the question of how to use the Laplace transform technique, to general analysis of system. We go back to the system function that we already talked about in the preliminary set of lectures, and see how you connect it up the Laplace transform technique. This will be the topic in the next lecture.