Networks and Systems
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Lecture-61
Laplace Transform Method for Mutual Inductance

In the last lecture, we acquainted ourselves with concept of transform diagrams, and how they help us in writing down the equation of performance of a network, the Laplace transform domain.

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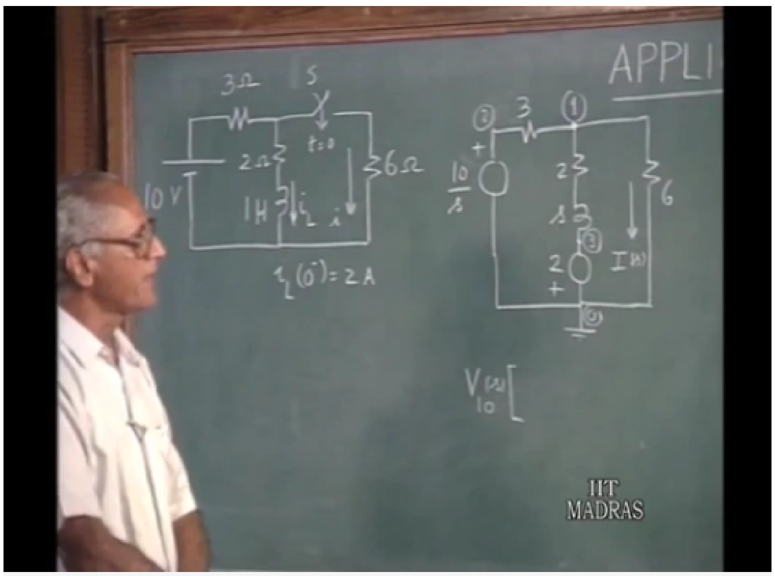

We took up this particular example, where the switch s is closed t equals 0, and you are ask to find out this current i. And then taking note of the current of the initial the 2 amperes. We replace that initial condition in the current by equivalent voltage source in the transform diagram, and we establish this transform diagram.

As I mentioned in the last class, once we have the transform diagram that can be analyzed in almost the same lines, as what you would observe in the case of dc circuits. Writing down the various equations, we took up in the last class; the loop current method for the solution of this network. We may as well the node voltage method. We can use super position technique.

We can use thevenin's theorem, whatever, all the, the whole gamete of techniques that is available, for solution of dc circuits or ac circuits can be applied to the transforms diagrams as well. Just for the sake of illustration, let me solve this same circuit by the node voltage method.

Suppose I take this at the datum node, and if I solve for this node voltage, then you can find out the current in the 6 ohm resister, by dividing that voltage 6. Therefore, I would like to use it node to datum voltage method. This is the node voltage to be solved for. Suppose this is node 2; the value of the node voltage with reference to the datum is 10 by s this is already known, and suppose I call this node 3, the voltage of node 3 the datum is already known.

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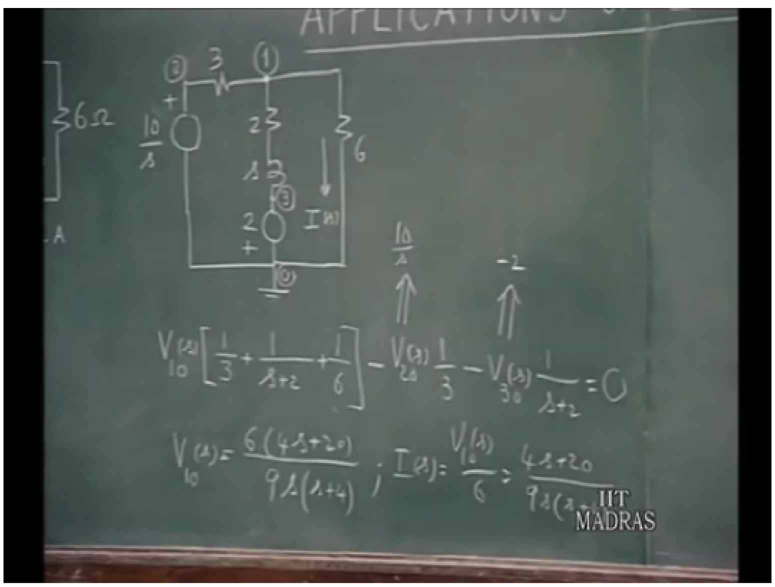

So, our aim is, to write the node equations to solve for v 1. So, there is only 1 unknown node voltage; therefore, let us say v 1 0 of s times the sum of the impedances connecting that node. One admittance this is generalized admittance I am talking about. The impedance is 3 therefore; the admittance is 1 by 3. The admittance of this combination for entire branch; the impedance has s plus 2; therefore, the admittance is 1 over s plus 2.

The impedance of 6 generalize impedance, the impedance is 1 by 6; that is the self node admittance that node 1. Therefore, the mutual terms you have to take v 2 0 of s, and admittance connecting node 1 in 2 is 3. Therefore, 1 by 3 therefore, that is the coupling term that we are having.

Further you also have coupling term with reference to 3 0 of s. So, the admittance joining node is 1 and 3 is, 1 over s plus 2. This must be the sum of the currents entering node 1, to the various currents of sources. In this particular example there is no such current source; therefore, is 0.

Now, in this equation the only unknown is v 1 0 of s, because v 2 0 of is known; that is equal to 10 by s. v 3 0 of s is also known, since the polarity is plus with, the 0 as positive polarity reference to 3, v 3 0 can be written as minus 2. On substitution of this values per v 2 0 and v 3 0, we can get v 1 0 of s. v 1 0 of s can be calculated and shown to be, 6 times 4 s plus 20 divided by 9 s times s plus 4.

But we are interested not v 1 0 of s, but the current in this 6 ohm resistance/ therefore, i of s obtained by dividing the v 1 0 of s by 6. And when you do that this becomes 4 s plus 20 divided by 9 s time s plus 4. This exactly the same expression that we got for the current i of s, if the loop current methods, and once we got a this point further work is the same, as what we have done in the case of loop current method.

So I will not do that, so from this you can get i of t. The point is that once we have the transform diagram, you can analyze the transform diagram, by any one of the known techniques that is available to you, in the whatever you do in the case of dc circuits can be applied to this transform diagram as well, except in terms of resistances.

We have generalize impedances, except in terms of tight dc voltage sources, we have transform voltage sources instead of dc current sources you transformed current sources. So, everything is in terms of s, but otherwise the principle, the technique is the same.
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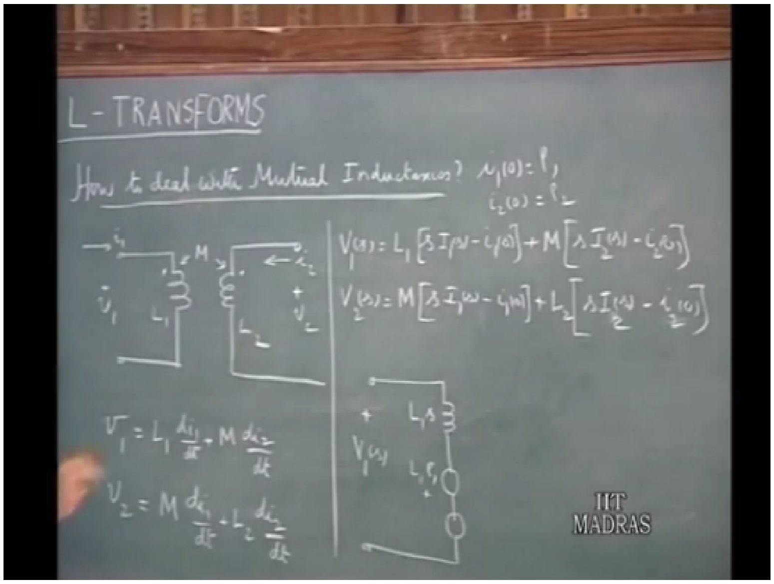

Now, next question that you like to ask ourselves is how to deal with mutual inductance. We have so far addressed ourselves, to finding out the generalized impedances r and l and c elements. What we do when we encounter mutual inductances. So, suppose I have two couple coils, having self inductance l 1 and l 2.

These are the dot points, and I have a mutual inductance m between them, and let me say the voltages and the currents, are the 2 coils are indicated as follows, as given here. Then in time domain the equations are performance of this mutually coupled pair of coils is v 1 equals l 1 di 1 dt plus m di 2 dt, because the two currents, both the currents enter the dot points.

Therefore, beside of the mutually induced m is the same as the self induce m; therefore, this is plus that is also plus v 2 likewise, is m times di 1 dt plus l 2 times di 2 dt. These are straight forward standard equations, governing the behavior of the pair of couple coils which we know. When we transform this equations, in term by term we transform. So, v 1 will have Laplace transform v 1 of s.

When you transform this, this will be l 1 times s times i 1 of s minus i 1 0; that is the Laplace transform of l 1 di 1 dt. Similarly, the Laplace transform of m di by dt be m times s times i 2 of s minus i 2 0. Second equation will be v 2 of s m times s times i 1 of s minus i 1 0 plus l 2 times s times i 2 of s minus i 2 0.

So, this is straight forward application of the transforming this equations. Now we have to draw a transform diagram, which repeats these equations. So, what you have now is, v 1 of s is l 1 s times i 1 s plus ms of i 2 of s plus minus l 1 i 1 0 minus mi 2 0. Therefore, if I have in the transform diagram; for the first coil v 1 of s is the transformed voltage, terminal voltage the coil, and the coil has the self inductance l 1 and it has got a generalized impedance l 1 of s; this is self impedance of the first coil.

And you have 2 sources voltage sources represents the initial currents in the inductors therefore, this is l 1 i 1 0. Suppose i 1 0 i write; as rho 1 as i 2 0 as rho 2 to simplify my notation.

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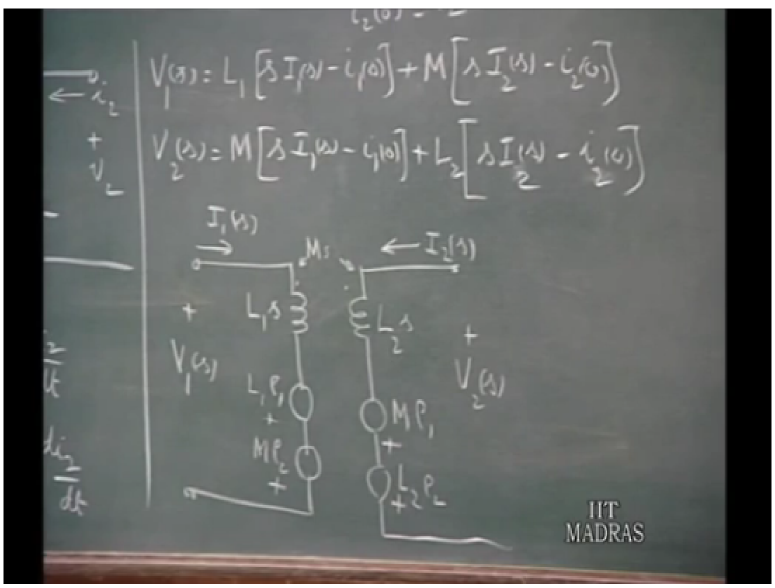

So, l 1 rho 1 and minus mi 2 0 therefore, you have another term m rho 2. So, you have 2 voltage sources, representing the initial currents in the inductor, and the second coil likewise we have a representation like this. Where v 2 of s is the terminal voltage and the current is i 2 of s, and the Laplace transform of the current here is i 1 of s, l 2 of s and then here you have once again minus mi 1 0, l 1 of s minus i 1 0 that is m times rho 1 and here you have l 2 times rho 2.

In addition we have a coupling impedance ms. so; that means, once you have write down this equation, the transform diagram in this fashion. We say the voltage terminal voltage v 1 of s are this coil is i 1 of s passing through l 1 of s we create a l 1 of s times i 1 of s. In addition i 2 flowing to this coil will have a mutually induced voltage here, whose Laplace transform is ms times i 2 of s.

So, you have not only l 1 of s l 1 s times i 1 of s plus also ms times i 2 of s. L 1 of s i 1 of s plus ms times i 2 of s. In addition representing the initial current in inductors we have 2 voltage sources. Likewise the same situation is there in the second coil also. This is very similar to what you do in the case of study state circuit analysis ac circuit analysis, where instead of self inductance is represents as impedance j omega l 1.

The mutual impedance is j omega m. So, instead of j omega we have s; otherwise it is exactly the same, that the situation that we have in the case of ac circuits. Now, you may as well think of v plus in the initial currents, through current sources. So, if you do that, you can write this equation in this form.

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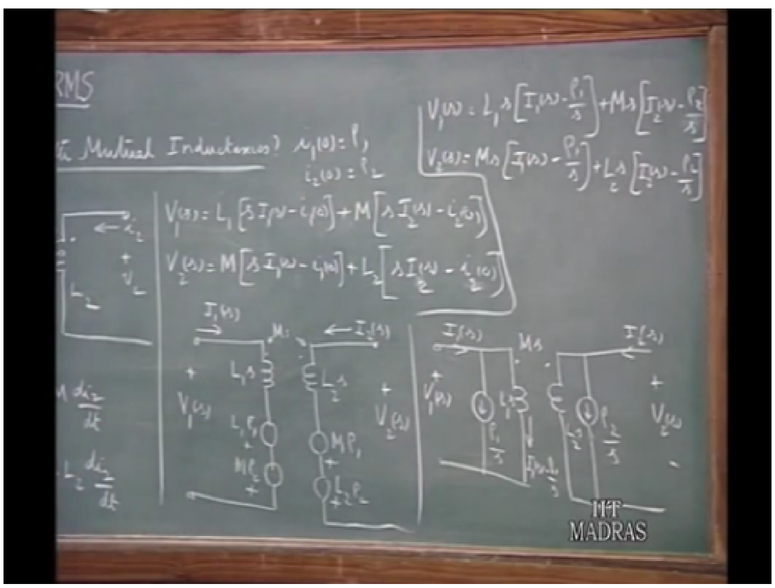

You can write v 1 of s, you can write as l 1 s times i 1 of s minus rho by s. So, instead of writing minus l 1 rho 1, I can write l 1 of s times minus rho 1 by s. So, this term here, is put in this form plus ms times i 2 of s minus rho 2 by s. and you can write v 2 of s exactly going to the same method; ms times i 1 of s minus rho 1 by s plus l 1 of s time i 2 of s minus rho 2 by s; that is what we handle.

So, is you look at these two equation, you can represent them in this fashion. You can have a current source here. So, let the current source be of strength rho 1 by s. So, the current in the actual inductor here, is i 1 of s minus rho 1 by s, and that current passing through l 1 of s will set up a voltage drop l 1 of s times i 1 of s minus rho by s. In addition you will have a mutually induced voltage ms times, the current in this coil, and what is the current in that coil, in a symmetrical way you have i 2 of s here this is v 2 of s, and you have another current source here ,which is rho 2 by s.

So, the current in this coil is i 1 of s minus rho 1 by s, and the current here is i 2 of s minus rho 2 by s. So, now you see that this voltage v 1 of s is obtained as the drop in this inductor. What is the drop in this inductor to self induced v m f i 1 of s minus rho 1 of s times l 1 of s. and the mutually induced voltage is ms times the current here, which is i 2 s minus rho 2 by s.

So, this exactly the term that you have having here. Similarly the voltage induced in this which is equal to v 2 of s, is the self induced voltage i 2 of s minus rho 2 by s by l 2 of s, and the mutual induced voltage this is ms times i 1 of s minus rho 1 by s. So, this are 2 alternative representation of the initial conditions the mutual inductor, if you low with the voltage sources use this diagram.

If you like to use the current sources you can use this diagram. The advantage of the current source representation is, that one side you have only rho 1only figures here, when you put the voltage source both rho 1 and rho 2 come on both sides; that is the minor advantage. So, this is the way in which we can analyze the transient problem, involved in mutual inductances. Let us work out an example to clarify these ideas.