Basic Electrical Circuits
Dr Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture – 99

In this lesson we look at some example problems, in which you could exploit reciprocity of resistive networks. In general, if you are told that some network consists only of resistors, reciprocity is one of the things you should think about. It is not that every problem involving resistive circuits exploits reciprocity, but that is one of the things that is possible. There are many other things that are implied by having a network, which has only resistors.

For instance, if a network has only resistors it means that it will only dissipate power and so on. So, there are many things that are implied by having a purely resistive network, one such thing is reciprocity.

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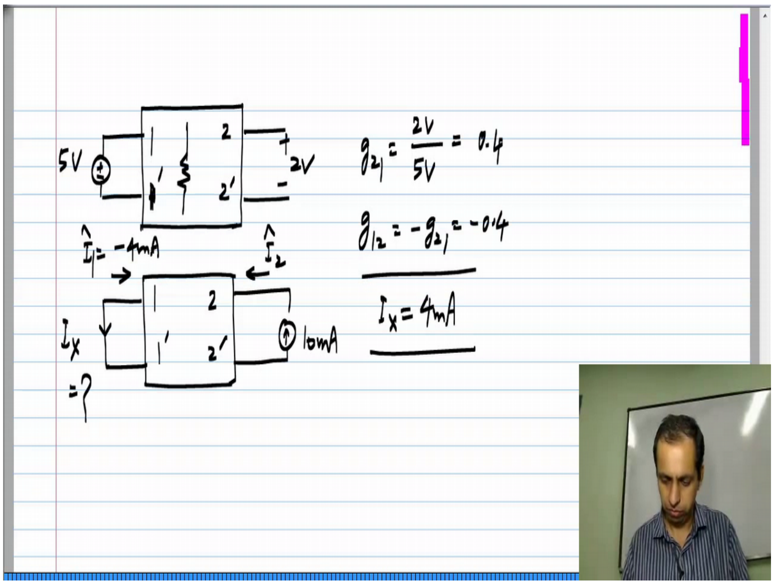

Firstly, you could be given a straight forward two port style question. So, let us say you would be asked I apply 5 volts here and I see 2 volts over there, then if I apply 10 milli amps over here to port 2. What current will I see that way? So, this is a straight forward reciprocity problem and you can think of the g parameter equivalents for this, because from this part, from the first circuit you see that g 2 1 is 2 volts by 5 volts, which is basically 0.4 and the second part is implicitly asking for g 1 2. You apply current to port 2, measure the current in port 1.

You know that for a reciprocal network g 1 2 is minus g 2 1 is minus 0.4. So, I 1 in this direction which is the direction we take for currents in a two port. So, if this is 10 milli amperes; that is, if I use my old notation I 2 hat is 10 milli amperes, I 1 hat will be g 1 2 times 10 milli amperes which is minus 4 milli ampere. Now, this I x is opposite to I 1 hat, so the answer is that I x is 4 milli ampere, so this is one such thing. This is a very simple problem, where you can exploit reciprocity and things could get slightly more complicated

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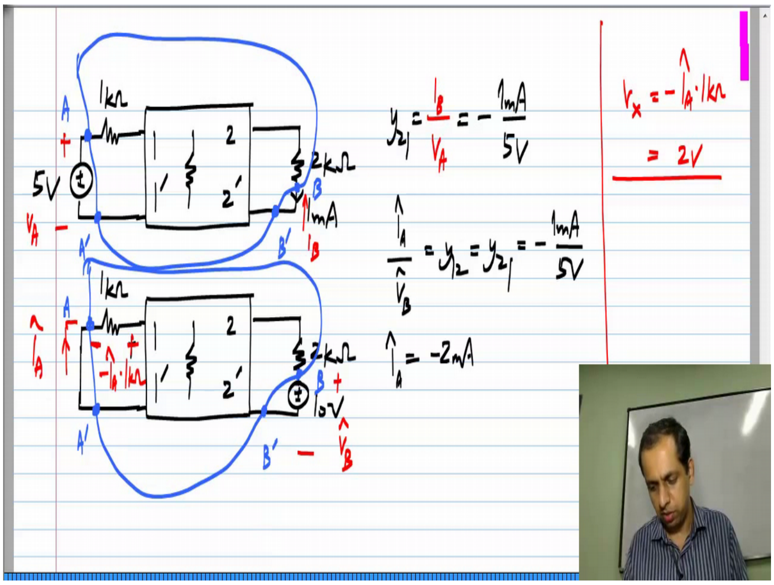

For instance, you could connect some things to the network. So, let us I have 5 volts, 1 kilo ohm and here also I have some resistor 2 kilo ohm and let us say here I have 1 milli amp flowing. And then, let say you are asked this question, I apply 10 volts over there, what is the voltage drop across this 1 kilo ohm resistor. Now, this looks slightly more complicated. Now, one thing you can of course, do is because this is a resistive network, you can describe it with a two port parameter set, whichever it is and apply the reciprocity constraint; that is, y 1 2 equals y 2 1 or z 1 2 equals z 2 1 and so on.

Now, the better thing to do is to realize that this is exactly the same problem as before and slight disguise. I will define my new two port to be something like that, this is A and A prime that is one of the ports and this is B and B prime; that is the second port and that is terminated in a short circuit. And again I will take the exact same two port here, I have to take the same thing that I took earlier, where originally we had a short circuit, we have now a voltage source and this makes sense, basically we get this by reducing this voltage source to 0.

You are not allowed to change the network, but you can change the values of the independent sources, in all these problems when you set a voltage source is 0 it becomes a short circuit. Now, you cannot apply reciprocity after changing the network, you should keep the network exactly the same. So, now, you can think of whatever is inclusive within blue stuff as, the inside part does not change at all, it is only the sources that are applied outside that change. Here on port A we applied 5 volts, port B terminates with the short circuit, here port B we have applied 10 volts and port A is terminated by a short circuit.

Now, clearly this scenario where this side we apply a voltage that side a short circuit corresponds to y parameter definitions. So, now, what is y 2 1 of this circuit? It is remember for two port definitions current I B, let me show it here, I B is defined that way. B is over there and this is V A applied to port A and y 2 1 is nothing but, I B by V A which is minus 1 milli amp by 5 volts, because this 1 milli amp is an opposite direction to that definition of I B.

Now, what do have in the other case? I have V B hat which is 10 volts and what I was asked to find was the voltage drop in this resistor; that we know is related to the current through the short circuit. So, I will find I A hat, so the again that is the direction with which we define the two port currents. So, you know that I A hat by V B hat is basically y 1 2 of this network which of course, is the same as y 2 1 and that is given by minus 1 milli amp by 5 volts.

So, I A hat itself will be this number times V B hat which is 10 volts. So; that means, that I get I hat to be minus 2 milli amperes. Now, the voltage that I want, this voltage is nothing but, minus I A hat times this resistor which is 5 kilo ohms. If I call that V x, my final solution is V x is minus I A hat times 1 kilo ohm, which is given by 2 volts. So, that is plus 2 volts, this voltage drop here is plus 2 volts.

So, here there is something that looks like a two port and you have some stuff out sided it, but you define your own new two port and you can answer questions. Now, exactly in the same spirit would be instead of connecting a voltage source here I could connect a current source across this 2 kilo ohm resistor and so on. So, that is another possibility. Now, of course, you could be given problems that is, do not say anything about a two port network.

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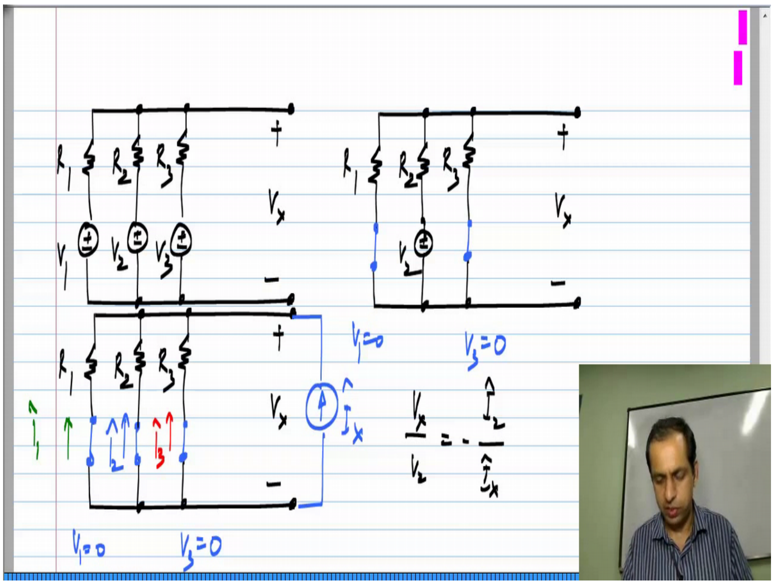

So, let us try one such thing, now all of these, there are many ways to solve the problem, there are many ways to do circuit analysis, you should be familiar with large number of ways, so that you can attack the same problem in multiple ways and that is how you also build intuition. Now, as you go to more and more complicated circuits, some techniques are not to be easier than others. So, you should really have horizon of all possible techniques.

Now, this problem some of you may already be familiar with, let us I have V 1 R 1, V 2 R 2, V 3 R 3 and I am required to find the voltage here, let me call this V x. How do you find it? Of course, you can apply regular circuit analysis and find it. Now, just for the hike of it I will use reciprocity to do this. So, what do I do for that? Essentially I should imagine that, I have only one of these sources I can find V x due to that one and then I repeat it for other sources and apply super position.

Now, when I have only one of these sources and I am trying to find this for instance, now I have the same circuit with V 2 set to 0 and V 3 set to 0. So, I can apply super position and do this, I can also calculate V x in this case and it is not very difficult, but like I said we are discussing reciprocity, so let us do it using reciprocity relationships. So, now, I know that, if I apply V 1 here I get some V x. The ratio of V x to V 1 in this circuit is exactly the same as, let us say I short circuit this and I apply a current source over here I exact and I find I 1 hat.

I know that V x by V 1 in this circuit will be exactly the same as minus I 1 hat by I x hat.

Now, why is this approach advantages at all it looks like a I am going around in loops I mean making the problem only more complicated, the point is let me now take the case where V 2 acting by itself I have said V 1 and V 3 to 0 and I can calculate V x. Now, I also know that the reciprocal case for this is again instead of going from V 2 do V x I can go from I x hat to I 2 hat over there.

The point is this network is exactly the same as before I am simply calculate I 2 hat previously I calculated I 1 hat over here. And now you can easily imagine that if I had only V 3 acting here and the other two were set to 0, the reciprocal case of that would be when I have I x hat here and I simply calculate I 3 hat. So, this network the reciprocal network remains exactly the same for all these three cases.

Remember, I have three sources here V 1, V 2, V 3 I can do super position by taking one of them at a time, each one of them will give a different network by different network I mean in one case V 2 and V 3 I replace by short circuits, in the other case V 1 and the V 3 are replaced by the short circuits and in the third one V 1 and V 2 are replaced by short circuits.

Now, if I take the reciprocal cases for each of these three, the network is exactly the same I apply the current from this side and I simply have to calculate these three currents. So, and that is very easy. So, for this reciprocal case I know that V x by V 2 exactly equals minus I 2 had by I x hat.

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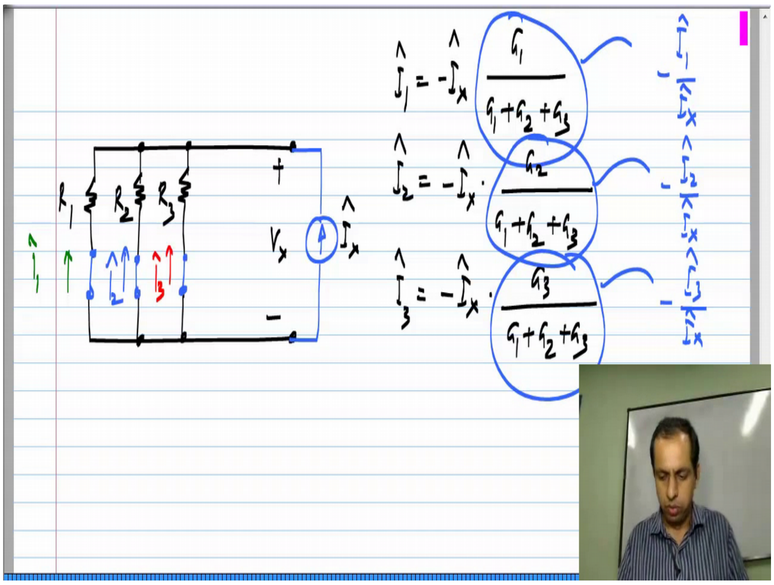

Now, what do I do with this? This is the reciprocal case and I have to find I 1 hat I 2 hat and I 3 hat over here and I have this current I x hat injected from this side and this is just current division into three resistors or three conductance's and that is very easy I know that I 1 hat is simply minus I x hat, I know that I x hat is injected in this direction. So, the actual current in R 1 will be in the opposite direction in terms of I x hat. So, I will have minus I x hat times the ratio of conductance's, this is the current divider case.

So, G 1 which is 1 by R 1 divided by G 1 plus G 2 plus G 3, similarly I 2 hat will be minus I x hat times G 2 by G 1 plus G 2 plus G 3 and finally, I 3 hat would be minus I x hat times G 3 by G 1 plus G 2 plus G 3. And if I look at the coefficients minus I 1 hat by I x hat this is what I want, this is V x by V 1 minus I 2 hat by I x hat which is V x by V 2 and minus I 3 hat by I x hat. So, these ratios would be simply what is inside this circle, this one and that one.

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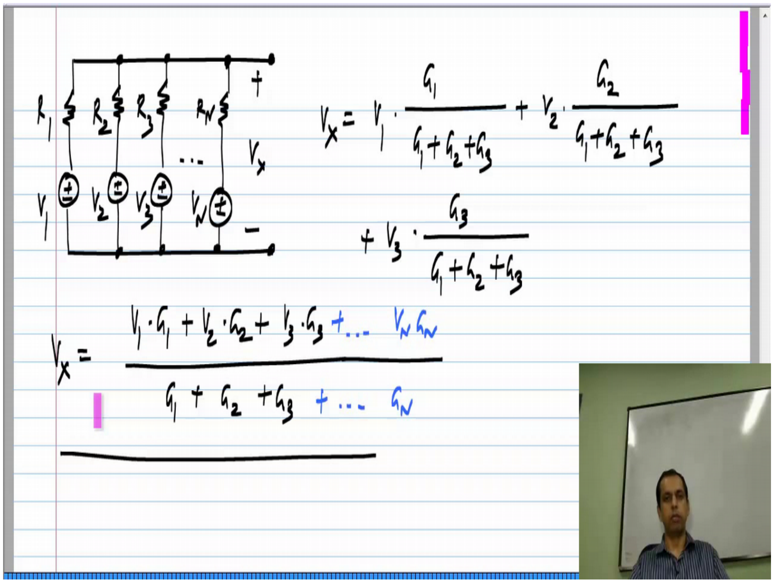

So, in my original circuit now I got to do super position and that is quite easy V x is simply contribution from V 1 which is V 1 times the ratio which I wanted and that ratio is nothing but, G 1 by G 1 plus G 2 plus G 3, because this minus I 1 hat by I x hat is nothing but, V x by V 1 when V 1 is acting by itself. Similarly, minus I 2 hat by I x hat is when V 2 is acting alone and finally, minus I 3 hat by I x hat is when V 3 is acting alone.

So, V x would be in this case V 1 times G 1 by G 1 plus G 2 plus G 3 plus V 2 times G 2 by G 1 plus G 2 plus G 3 plus V 3 times G 3 by G 1 plus G 2 plus G 3. In other words V x in this case is V 1 times G 1 plus V 2 times G 2 plus V 3 times G 3 divided by G 1 plus G 2 plus G 3. So, I quite a simple formula and you can generalize this, if you have a number of more sources. So, let us say I connect in of these together V N and R N I will simply have to continue this formula V N G N and in the denominator also all the way to G n. So, this is what we will have?

Now, like I said you can derive this many ways in particular I would encourage you to I try and derive this by using terminal and Norton equivalents suitably. But, this is one way in which you could use reciprocity, my original circuit and multiple sources and I wanted to calculate this particular output instead of that I can term this around and use reciprocity I connect the single source here and after that it was very easy in this cases especially, because it was just a current divider and to find to all these branch currents and each one has some ratio to the original current and I multiply all these voltages by corresponding ratios to get the final voltage. So, that is how you can use reciprocity in this case and also in many other cases.