Basic Electrical Circuits
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Lecture - 97
Reciprocity in Terms of different Two Port Parameters

In this lesson, we will consider the implications of reciprocity in resistive network. We have stated reciprocity and also derived it in terms of two port parameters, but two port parameters really refer to the ratios of some electrical quantity to some other electrical quantity under certain conditions. We can talk about reciprocity even without talking about any particular two port parameter sets. So, in this lesson, we will look at what reciprocity in terms each of the two port parameters implies and also how it is useful.

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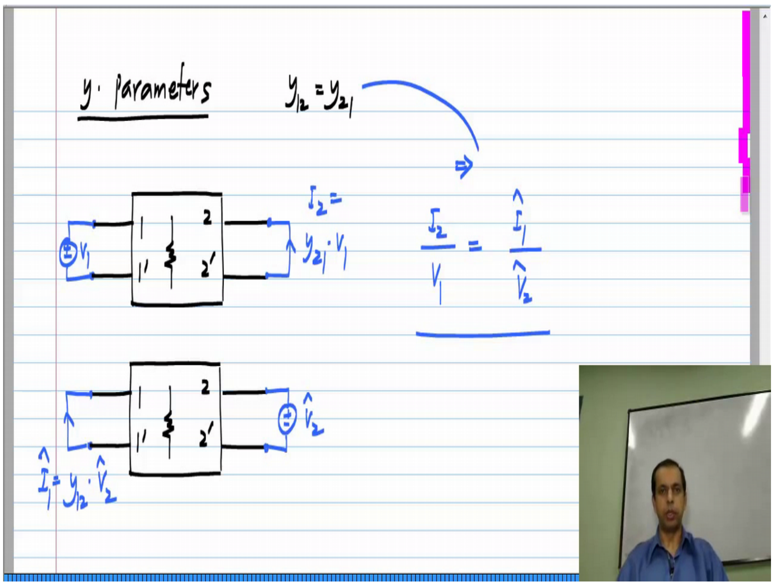

So, let us first take y parameters reciprocity says that y 1 2 equals y 2 1. So, if I have a resistive network and two ports, what is that mean let say I connect V 1 to the first side and short circuit the second side, the current here would be by definition y 2 1 times V 1. And on the other hand, if I connect V 2 had to the second port, short circuit the first port the current here would be I 1 hat which is equal to y 1 2 times V 2 hat. And by the way let me define this current as I 2. So, in this form, in terms y parameters it is says that the ratio of short circuit current in one port to the apply voltage is the same as the short circuit current in port one to the voltage applied at port two. So, this what it implies is short circuit current in port two ratio of that to voltages apply at port one, this is y 2 1 will be equal to I 1 hat divided by V 2 hat. The reason I am pointing does how it is different two port parameters have different implications in terms of exactly what you are measuring and also in different circumstances, you will have ratio of voltages or currents or voltages across open circuits and currents to short circuits and so on. So, you should be able to use reciprocity for all those situations.

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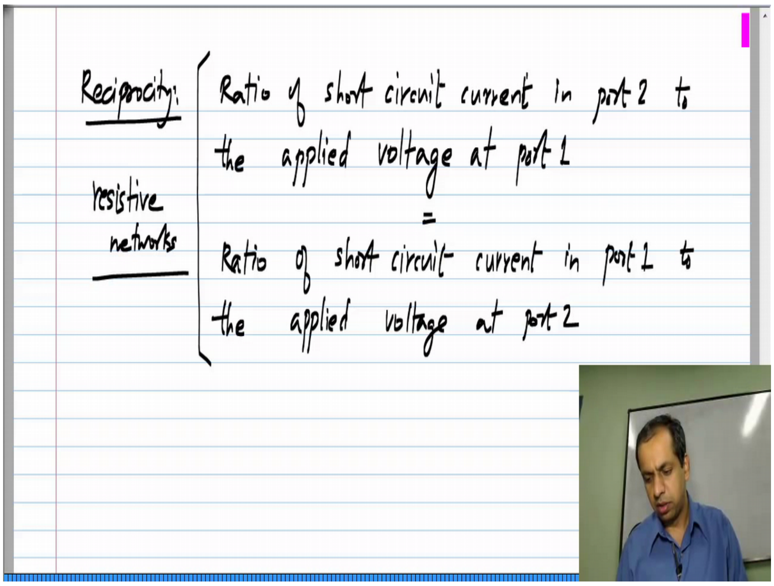

Reciprocity means ratio of short circuit current in port two to the applied voltage at port one equals ratio of short circuit current in port one to the applied voltage at port two now this is the of course, true for purely resistive networks. So, this is one particular statement of reciprocity in terms of short circuit currents.

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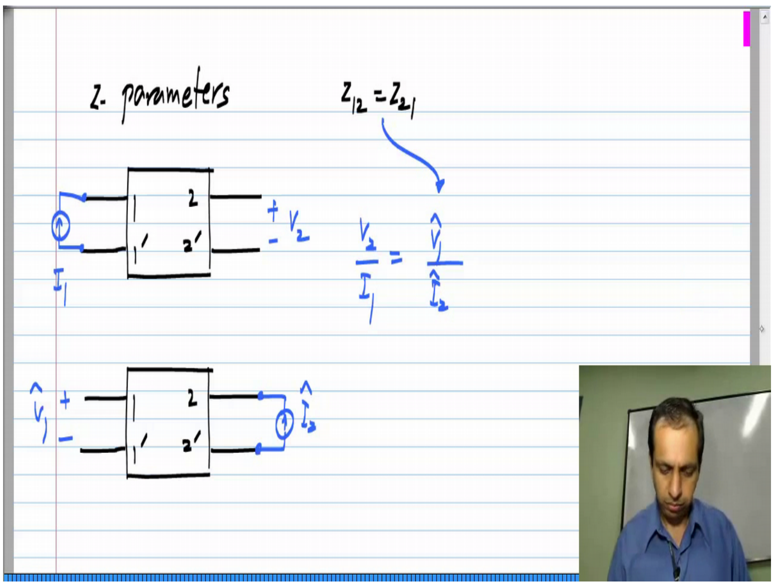

Now, similarly let say what it means in terms of z parameters which is z 1 2 equals z 2 1. So, when you are measuring z 2 1 you will apply a current port one and measure the open circuit voltage at port two and when you are measuring z 1 2 you apply current two port two and measure the open circuit voltage at port one and this statement our reciprocity means that V 2 by I 1 equals V 1 hat by I 2 hat. So, here the ratio open circuit voltage to the applied the current is the same whether you applied the current at port one and measure the voltage at port two or you applied the current at port two and measure the current at port one.

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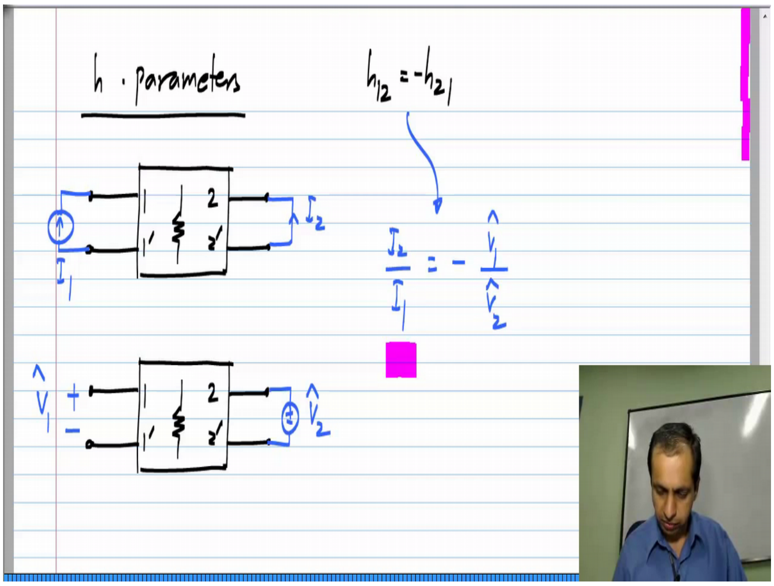

It we do it with h parameters you will have h 1 2 equal h 2 1 and this is my resistive two port when you are measuring h 2 1 you will apply like a current two port one terminate port two with short circuit and measure the current in port two and when you measuring the h 1 2 will apply a voltage to port two and have an open circuit across port one and measure the voltage across that open circuit this of course, implies that I 2 by I 1 equals minus V 1 hat by V 2 hat. So, basically the current again from port one to port two which is port given by this is the negative of the voltage gained from port two to port one, so you get this negative sign. If you choose the direction of currents, I show on here I you had chosen the current to be in the opposite direction you want get that, but we always choose the current direction consistent with the two port parameter definitions. So, we get this I 2 by I 1 2 be minus V 1 hat by V 2 hat.

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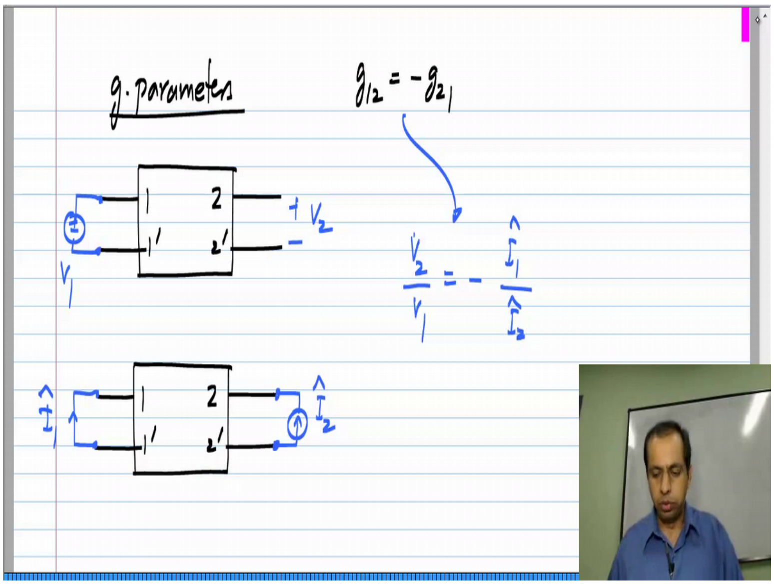

g parameters, we have exactly the same thing g 1 2 is minus g 2 1 and I have my two port. When I measuring g 2 1 I will apply a voltage to port one and measure the open circuit voltage across port two, and when I measuring g 1 2 I will apply a current to port two and measure the shot circuit currents through port one. And this implies that the voltage gain from port one to port two, and this voltage gain is measured with an open circuit termination equals the negative of the current gain port two to port one which of course, is measured with short circuit termination. These are all the implication reciprocity in terms of two port parameters we have defined.

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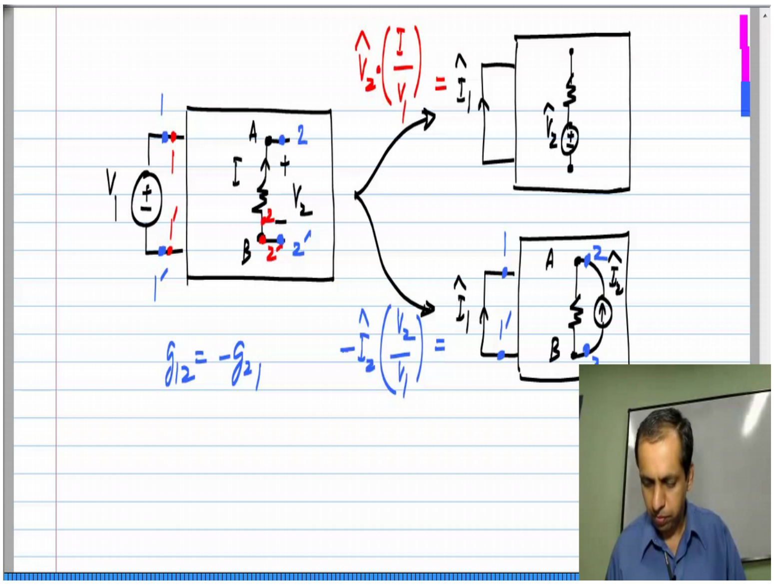

But the main use of reciprocity is that let say you already know some result, with the particular source. So, let says the network and it consist of resistors and let say I have the single independent source some where I will consider that to voltage source. Let say you already solve for this case that is I know that if I apply V 1 here between these two nodes I get particular V 2 what reciprocity theorem enables to do is let say you have alternative scenario where for instance let me call node a node b and between node a there is resistor. So, let say we have alternative scenario where the voltage source V 1 replays by for circuit and between nodes a node b. We have the resistance in series with that we have voltage source let me call that V 2 hat now I already know the result one V 1 is apply certain V 2 appears across a and b and; obviously, you also know this current which flows through the resistor.

So, now, let say you have the scenario or an alternative scenario where a still have a short circuit were the voltage source was and between a and b, I have a current source in a parallel, let me call the I 2 hat. And I want know how much current this I 1 hat here or I 1 hat in this case I do not need to solve for the circuit again I can use reciprocity from this one to get these result because in the first case essentially what I am doing is I am defining let say this is port one and this link here is port two then you can think of the original case as the case with port to be short circuit at and this current.

I have marked this really the current port two I marked it top of the resistor, but now the resistor part of the two port it is really the port current. So, now, clearly this V 2 hat applies to the second port over there and the first port short circuit at. So, I 1 hat I can find out without doing any further calculations is the same as V 2 hat times the ratio of the hat here which is I by V 1 I by V 1 would be nothing but y 2 1 with this as the first port and that as the second port.

Similarly, in this case I can relate this case to the original case by defining something else as two ports let me call this port one which is same as before and now I will call this between node a and b define the second port two. So, now, if you do that we can see that basically these V 2 by V 1 is nothing, but g 2 1 this port two is open circuit at here nothing else connected there and now this port one which is short circuited and this is port two to which this current I 2 hat is applied. So, again using reciprocity in terms of g parameters I can find out I 1 hat and I 1 hat would be equal to negative of I 2 hat times V 2 by V 1 this negative sign comes because g 1 2 is minus g 2 1 and this V 2 by V 1 here is nothing, but g 2 1 between these two ports shown in blue and here what I want is g 1 2 that is nothing, but the negative of g 2 1.

So, this is the point of the reciprocity theorem, we do not have to be presented with something that is defined at the two port network. You could be ask question like this that you apply voltage here and you gets something ales there and in other case the voltage apply on this side and what response appear from that side you can use reciprocity by defining your on two port.

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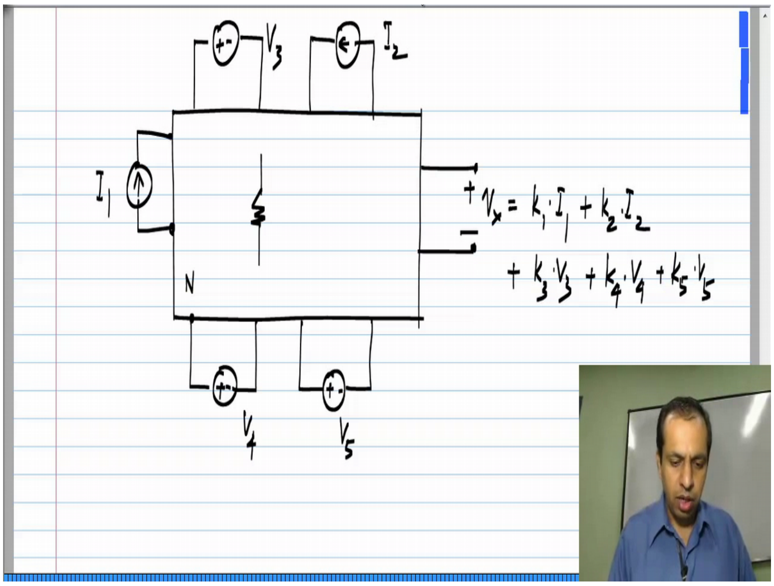

In the case and which can be particularly useful. So, let say we have these two points and measuring the voltage and then we have only resistor in this circuits let say we have number of sources connected both current sources and voltage sources let say this is the circuit we have multiple sources and we have the output now you are required to find out the contribution all of these things to the output. So, let me call this V 2, V 3, V 4, V 5. So, now, we have to find out this V x from all of these sources, now sometimes it could be easier turn this problem around and realize that the transfer from I 1 to this even all these us other sources are will be the same as this point to that and so on. So, now, we this V x clearly be linear combination of all of these sources. So, let me call that k 1 times I 1 plus k 2 times I 2 plus k 3 times V 3 plus k 4 time V 4 plus k 5 times V 5 naturally because this is the voltage and these two are currents k 1 and k 2 will have dimension of resistance, and k 3, k 4, k5 will be dimension less constants.

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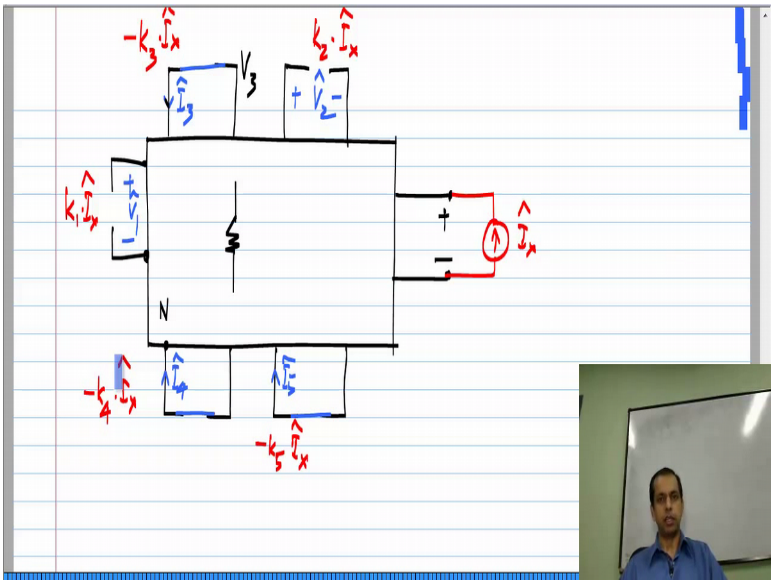

Now how do we use reciprocity in a situation like this. What we do is we deactivate all these sources that is short circuits all the voltage sources and we open circuit all of the current sources let me define this is I 3 hat I 4 hat I 5 hat and this V 1 and this is V 2 these directions are consistent with that two port convention that I have already use the current sources and voltage sources that on a implies. Now because in the original circuit, this is terminated in open circuit, I will apply a current source ix hat and do the analysis. Now the advantage of this is that we have single source in the analysis and this is many times easier to carry out now is given hat would be k 1 times I x hat this V 2 hat would be k 2 times I x hat and this I 3 hat would be minus k 3 times I x hat, this I 4 hat would be minus k 4 I x hat and I 5 hat would be minus k 5 times I x hat.

What I did was the first consider this and that as the two port, and use the z parameters version then this and the that as the two port use to the z parameters version, then this and the that as the two port and use the g parameter version and so on. So, I get these ratios. The point is by analyzing the circuit with single source, I get these coefficients of the linear combination k 1, k 2, k 3, k 4, k 5. So, this is the advantage of using reciprocity in certain situations. Now it turns out that in circuit analysis in particular we have something known as noise analysis every resistor in the circuit in general any component in the circuit trades noise and you need to calculate the transfer function from every noise source to the output and multiply that by the actual noise would be created to find output noise.

Now in this case in terms out the particular simple to turn the whole picture around and apply an excitation at the output and find what appears in place of each of the resistors that should be the transfer function which is also the same as the transfer function from the resistor to the output. And by using this reciprocity the simulator is able to calculate the noise as functions very easily so that is the application of it.