We have proved the reciprocity of resistive two port networks, but only for three terminal two ports. Now, I will say how it can be very simply extended to four terminal resistive two ports.

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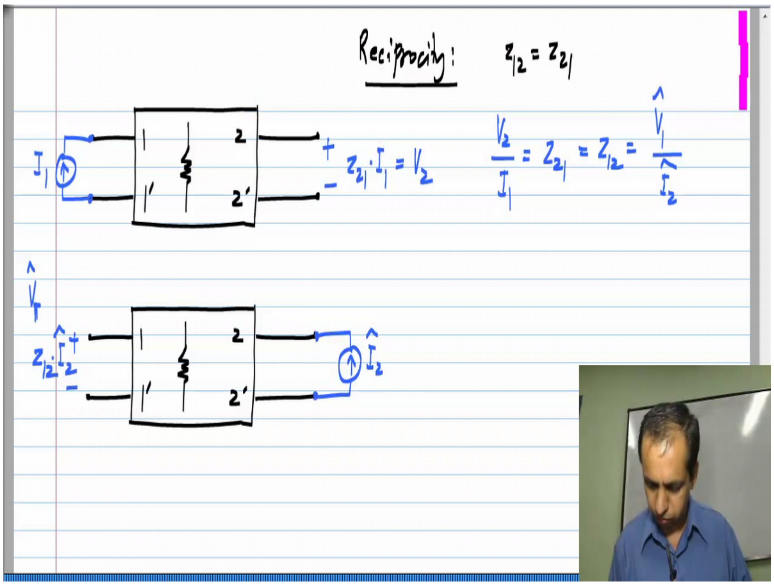

So, let say we have a four terminal two port like this and it consists only of resistors. Now, what is it that we want to prove? We can prove this in terms of any of the four parameters, let say we think of proving Z 1 2 equals Z 2 1 for this network. Then, what is it really mean? So, let say I connect I 1 to port 1 and nothing to port 2, I leave port 2 open circuited. Clearly, what comes out at the port 2 between these two points is Z 2 1 times I 1, this is from the definition of Z parameters.

And similarly, let say I connect I 2 to port 2 and leave port 1 open circuited, the voltage that I get here would be Z 1 2 times I 2. Now, what I have to prove is, let me call this V 2 and just to distinguish between these two let me call this I 2 hat, then the voltage here would be Z 1 2 times I 2 hat and I will denote that voltage by V 1 hat. So, the hat refers to the second case, without the hat it is the first case. So, essentially V 2 by I 1 would be Z 2 1 and I have to prove that, that is the same as Z 1 2 which is V 1 hat by I 2 hat.

So, essentially the open circuit transmission parameters from one side to the other side;

that is, what I have to prove to be equal for both sides from port 1 to port 2 and port 2 to port 1. So, this is how I will go about proving the reciprocity for four terminal two ports and I will use the results I already know from three terminal two ports.

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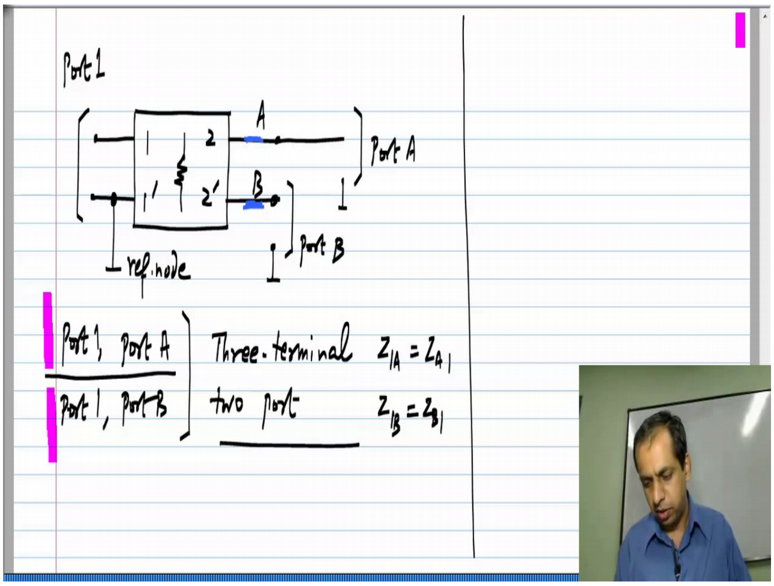

What I will do is, since I already have the proof for three terminal two ports; that is two ports in which there is a terminal common to the two ports. I will also try to define this in the same way I will define this 1 prime as the reference node. So, then this is port 1 as we already know, my actual second port is between 2 and 2 prime, but of course, nothing prevents me from defining another port, which is between, let me call this as A, between node A and the reference node.

So, let say I call this port A and this 2 prime I will label as B and I will call this port B, the actual second port I am interested in is between 2 and 2 prime. So, that is what I will get to prove, but now essentially I have defined three ports with a common terminal. This 1 prime is defined be the common terminal and this between this terminal 1 and the reference node; that is one of the port's, terminal 2 and the reference node that is the another port. Just to distinguish it from the other two ports that we already have, I will call that port A and between terminal 2 prime or B and the reference node I have the third port which is port B.

So, now, if I take any two of these, so let say port 1 and port A or port 1 and port B, so each of these is a three terminal two port. Remember the network is resistive, I can define the ports anywhere I want, the ports given to me are between 2 and 2 prime and I will eventually prove reciprocity for that port. But, I am now defining two other ports with the same common terminal as port 1, so I can take any two of these ports and use my earlier result for three terminal two ports.

So, what does my earlier result for three terminal two ports say? If I take this case; that is take port 1 and port A as the two port network, I know that Z 1 A is Z A 1. And similarly, if I take port 1 and port B as my two ports I will have Z 1 B equals Z B 1, fine.

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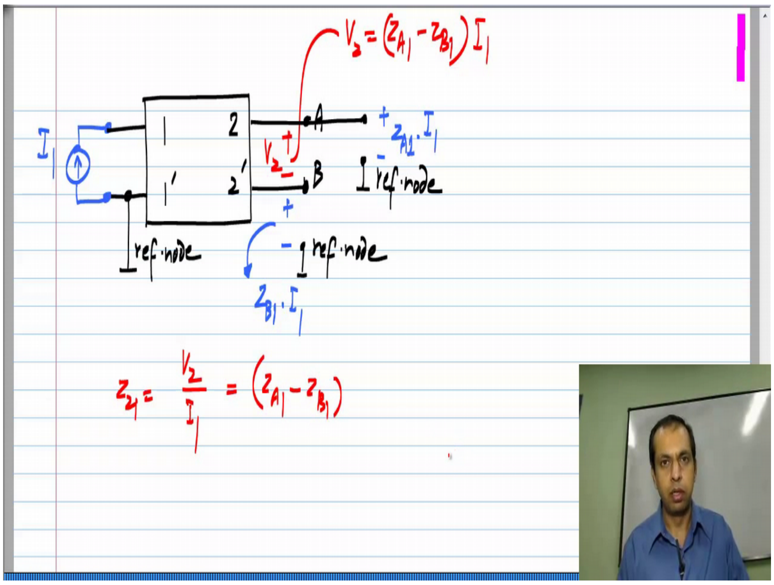

So, now, let me try out the two cases. In the first case, I will apply I 1 to port 1 and leave port 2 open circuited and let me define this as the reference node. If I leave port 2 open circuited, then port A is also open circuited, because I do not have anything between this and the reference node. Remember, this is terminal A and my reference node is over there and this is terminal B and the reference node. Since I open circuit 2 to prime, I also have an open circuit at port A as well as at port B.

So, what is the voltage that appears at port A? Clearly, I can think of port 1 of as one of the ports, port A as the other port, so if I inject I 1 into port 1, what comes out here is Z A 1 times I 1. Z A 1 is the parameter from port 1 to port A and similarly, here at port B I will have Z B 1 times I 1. So, I calculate these two voltages by first considering 1 and A as one of the two ports and then, 1 and B as one of the two ports.

Since I do not connect anything to these two ports; that is okay, if you really want to I could draw two separate pictures for this and that, but when I apply I 1 I will get some voltage here and some other voltage there, which I can express in terms of the two port parameters between port 1 and port A and port 1 and port B. Now, what is my actual second port voltage V 2? This is V 2 and clearly you see that, by using KVL around this loop will have V 2 to be Z A 1 minus Z B 1 times I 1. So, I have applied a current here and found out the voltage on port 2.

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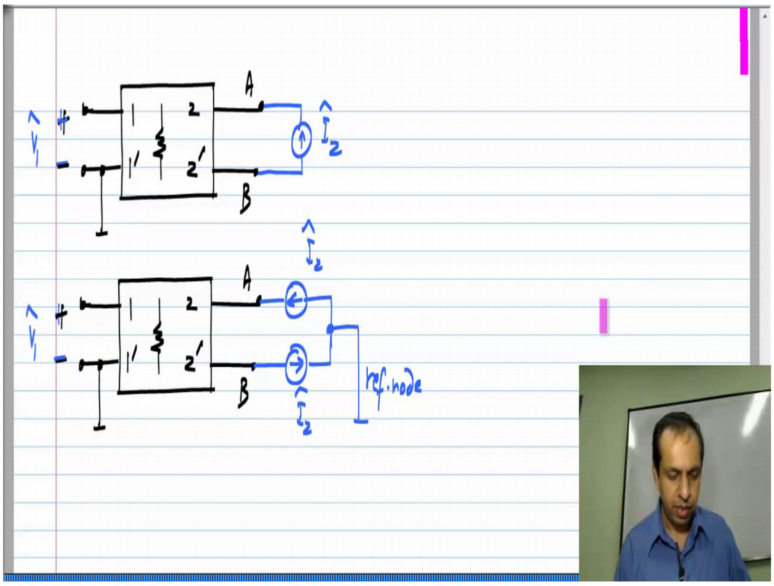

Now, let me take the other case, where I open circuit port 1 and I want to measure V 1 hat when I 2 hat is applied over there. So, again it is the same network of course, the same resistive network that have been taking all node, now this is also an opportunity for me to use one of the earlier theorems that we discussed, what I will do is the following. My calculations work with this point as the reference node and I had defined two other ports, port A and port B with this node being the reference node.

This was port A, it was between this terminal and reference node and this was port B, which is between this terminal and the reference node, so what I will do is the following. I have copied this over, then I will split the current source I 2 hat has two identical current sources in series. So, I can do this, because the series combination of two identical current sources is exactly the same, we have seen this in the earlier theorem, that we discussed as splitting of the current source and we also saw that we could connect this node to any point in the circuit and I will choose to connect it to the reference node.

Now, why do I do this? By doing this, I see that this upper current source is basically exiting port A and this lower current source is exiting port B. I already know the results between port A and port 1 and port B and port 1, I know that reciprocity holds. So, that is why I do it like this.

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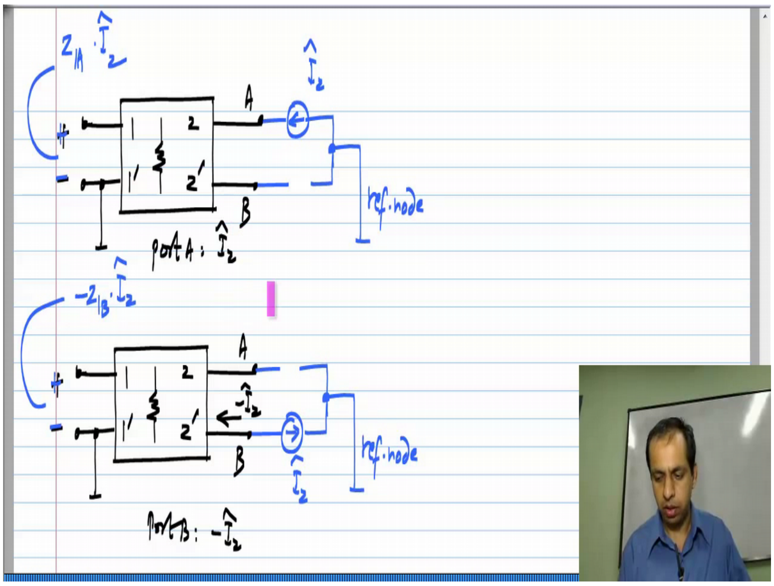

So, now, since I have two sources here I leaves super position, so I will split this into two cases, where I have only the upper current source active over there and only the lower current source active over there. Now, what is the voltage I get at port 1, so now, port A is fed with I 2 hat, so at port 1 I will get Z 1 A times I 2 hat with only this current source being active, if you look at the lower picture port B is exited with minus I 2 hat, because of the direction of the current source.

So, here port A is exited with I 2 hat and here port B is exited with minus I 2 hat, because the current flowing into port B is minus I 2 hat. So, the voltage here is minus Z 1 B times I 2 hat.

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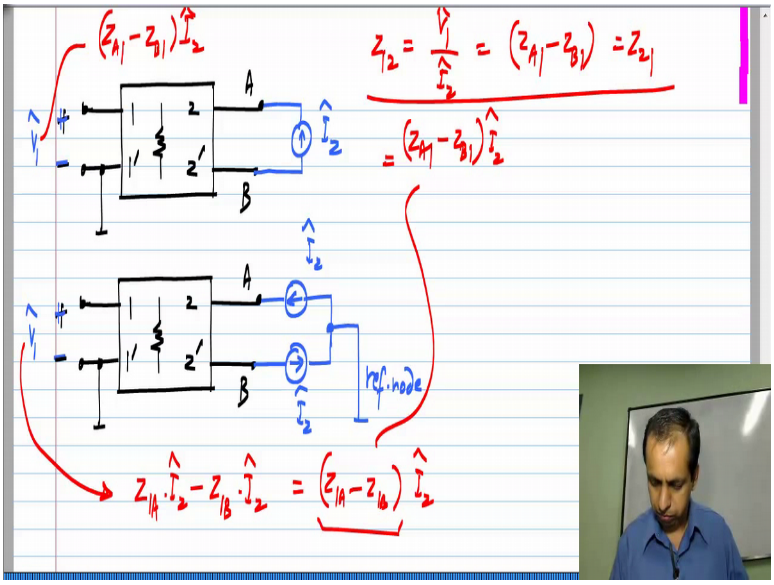

Now, in the original case I have both of these sources being active, so the actual voltage V 1 hat would be the sum of this voltage and that voltage. So, the voltage here would be Z 1 A times I 2 hat minus Z 1 B times I 2 hat or Z 1 A minus Z 1 B times I 2 hat. So, pretty simple I split the current source took 1 current source at a time got the outputs and I have this as my output at port 1 when port 2 is exited.

I also know that port 1 and port A for my reciprocal to port, port 1 and port B for my reciprocal to port. So, this is nothing but, the same as Z A 1 minus Z B 1 times I 2 hat. So, the voltage that is developed here V 1 hat is the same as Z A 1 minus Z B 1 times I 2 hat I think you are a already able to see the reciprocity here, but just for completeness the parameter Z 2 1 will be V 2 buy I 1 with port 2 open circuited that is V 2 I by I 1 in the circuit which transferred to be Z A 1 minus Z A 1 and the parameter Z 1 2 would be V 1 hat by I 2 hat in the circuit, which is also Z A 1 minus Z B 1, so that is equal to Z 2 1.

So, by a simple extension we can prove reciprocity for a general four terminal resistive 2 ports.

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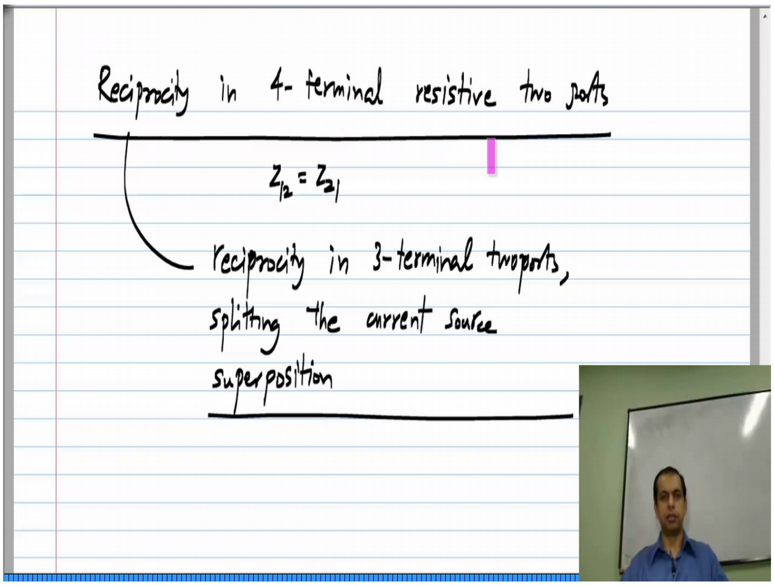

So, this is proved basically we have proved that Z 1 to equal Z 2 1 and in doing that we used reciprocity in three terminal resistive two ports. And we also use the earlier theorem on splitting the current source and super position, because we are using reciprocity in three terminal 2 ports we had to define some other ports with a single common node as the reference node and we were able to do that quite easily hope that is clear.