Basic Electrical Circuits
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Lecture - 94

In this lesson we look at an important property of resistive networks, which is known as reciprocity.

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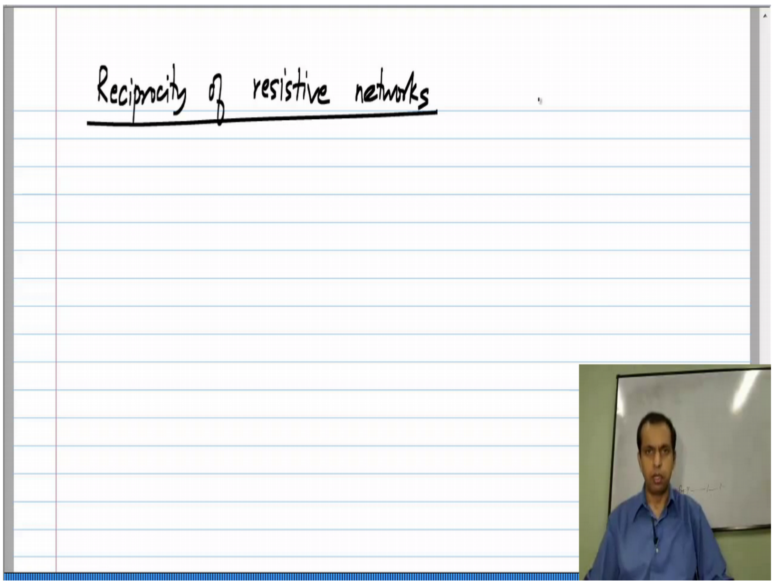

Now, let us go back to the examples for which we evaluated all for two port parameters.

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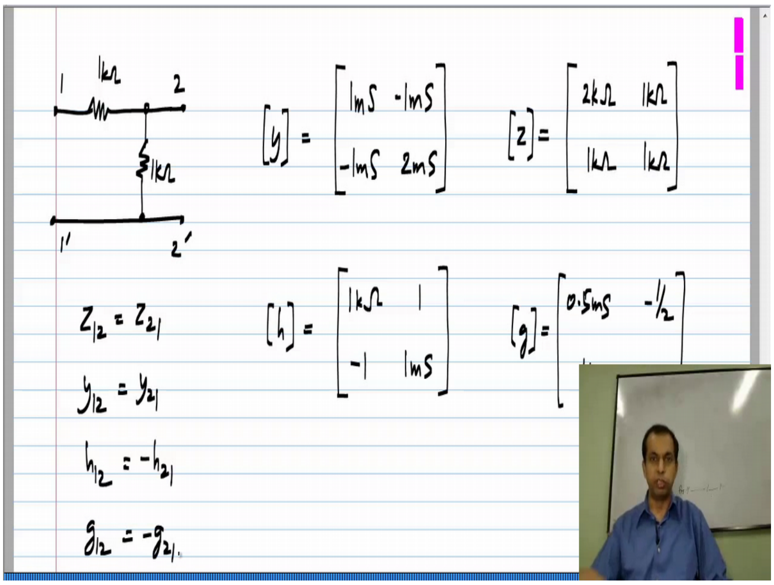

So, one of the circuits we evaluated for, this is very simple circuit, just have two resistors of 1 kilo ohm each and these were the parameter sets, this is the y matrix, z matrix, h matrix and g matrix. Now, the interesting thing here which you may have noticed is that, the half diagonal elements that is y 1 2 and y 2 1 are equal to each other. Similarly, z 1 2 and z 2 1 are equal to each other and if you look at the other two parameters, h 1 2 is not equal to h 2 1, but h 2 1 is exactly minus h 2 1. Similarly, g 1 2 is exactly minus g 2 1, z 1 2 is z 2 1, y 1 2 is y 2 1, h 1 2 is minus h 2 1 and g 1 2 is minus g 2 1.

So, there appears to be some relationship between the parameter 1 2 and parameter 2 1; that is, between the transmission from port 1 to port 2 and from port 2 to port 1. So, now, the question is, is this just a coincidence. Of course, there is no such relationship for z 1 1 and z 2 2, they can be arbitrary; here also they are quite different from each other. Now, this relationship between parameter 1 2 1 2 1, is it a coincidence or is it some special property of networks or to be sure, we have also seen networks for which the property which I just now mentioned does not hold true.

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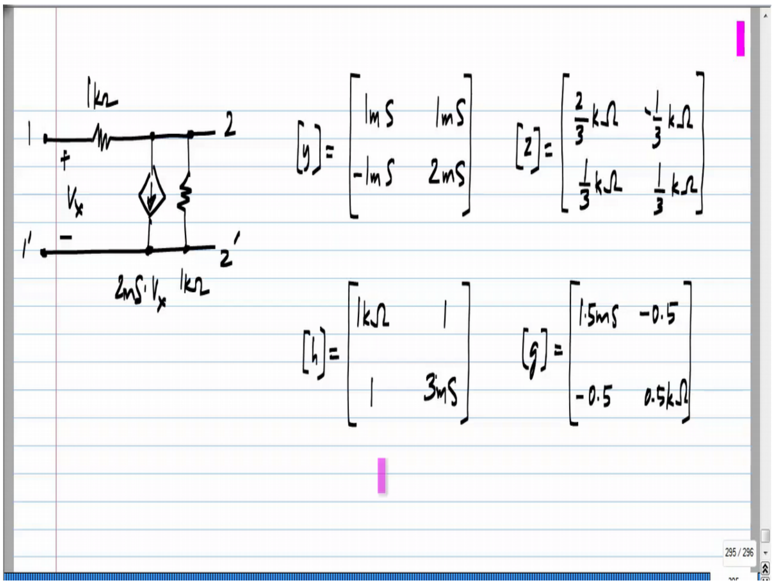

For instance, this is another circuit and in this case certainly y 1 1 is not equal to y 1 2 and z 1 1 is not equal to z 1 2 and so on. So, they have some numbers which are not necessarily equal to each other. So, there does not appear to be a special relationship between the forward and the reverse parameters in this case. Now, ((Refer Time: 02:56)) as part of activities or assignments you would have solved for the two port parameters of other networks, which consists of only resistors.

The difference between this network and the other one which we looked at is that, this circuit includes controlled sources, whereas this one does not. So, now, the question is, is this coincidence or is this some special property of resistive networks. It turns out that all resistive networks have this property, you would have noticed this not only from this one, but also from other problems which we have solved and it turns out, this is known as the reciprocity; that is, if you have a purely resistive two port network, then the forward transmission and reverse transmission are closely related.

If you describe them in terms of y or z parameters, they are equal to each other and then g or h parameters; they are the negative of each other. So, later in this unit we will go on to prove the reciprocity theorem. So, now, I am just stating it.

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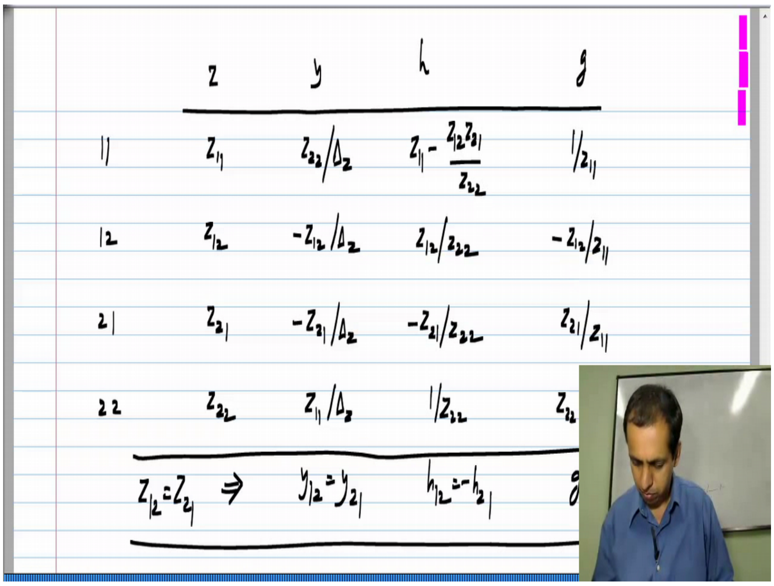

Now, here I have tabulated the relationship between different parameter sets. So, I have listed parameter 1 1, 1 2, 2 1 and 2 2 and I have taken z parameters as my primary means of representation and I have represented all other two port parameters in terms of z parameters. For instance if you look at y 1 1, this you can do by inverting the 2 by 2 matrix, y 1 1 turns out to be z 2 2 by, this delta z means that determinant of the z parameter matrix.

Similarly, y 1 2 turns out to be minus z 1 2 by delta z, y 2 1 is minus z 2 1 by delta z and y 2 2 is z 1 1 by delta z. Now, the point is you clearly see that if z 1 2 equals z 2 1, then y 1 2 will also be equal to y 2 1. Now, it is not that a circuit is reciprocal in z parameters, but not in y parameters or other parameters. Similarly, these are the expressions for h parameters, now I will look at only h 1 2 and h 2 1, h 2 is z 1 2 by z 2 2 and h 2 1 is minus z 2 1 by z 2 2.

So, clearly if z 1 2 happens to be equal to z 2 1, then h 1 2 will be minus h 2 1, so that is the reciprocity condition in terms of h parameters. Similarly, if you look at g parameters, g 1 2 is minus z 1 2 by z 1 1 and g 2 1 is z 2 1 by z 1 1. If z 1 2 equals z 2 1, then clearly g 1 2 will be equal to minus g 2 1. So, what I am trying to point out here is that, reciprocity is a property of the network; it is not related to what parameters you choose to describe the network.

So, reciprocity refers to some special relationship between a forward parameter and reverse parameter of a two port network and that holds good; that is, reciprocity holds good regardless of which parameter set you choose to represent the two port in. So, clearly from these relationships z 1 2 equal to z 2 1 implies y 1 2 equaling y 2 1 or h 1 2 being equal to minus h 2 1 or g 1 2 being equal to minus g 2 1. Is this fine?

These turn out to be very useful relationships as well, because many times they give you tremendous short cut in solving for certain types of circuits for certain types of questions. Now, one more thing I want to point out is that, again let say z 1 2 happened to be 0; that is, there is no reverse transmission in z parameters. Then; that means, that the reverse transmission parameter regardless of which one you choose y 1 2, h 1 2 and g 1 2 will all be zero and such a network, where the reverse transmission parameter is zero is known as a unilateral network.

Similarly, if the forward parameter is zero in z parameters; that is, if z 2 1 is zero, the forward parameter y 2 1, h 2 1 or g 2 1 will also be zero. So, these properties being unilateral or being reciprocal are just a property of the network and that will hold good regardless of which parameter set you choose to describe the network in.

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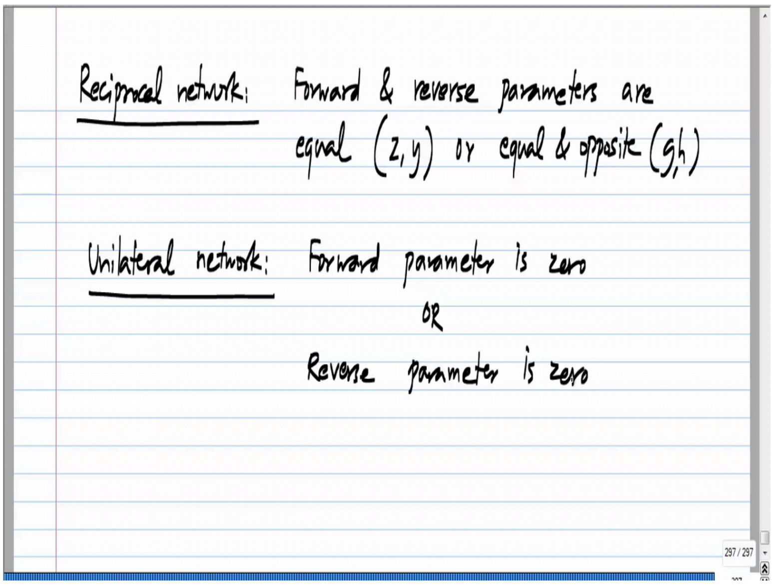

A reciprocal network, reciprocal two port network means that the forward and reverse parameters are equal and if you choose z and y parameters or equivalent and opposite for g and h parameters. Now, a unilateral network this is when, let say either the forward parameter is zero, this means that there is a control from port 2 to port 1, but not from port 1 to port 2 or the reverse parameter is zero; which means, that there is control from port 1 to port 2 and not from port 2 to port 1.

Of course, if both these are zero if both the forward and reverse parameters are zero; that means, that there is no communication between port 1 and port 2. You may as well have two separate circuits, two separate resistances, one connected to port 1 and one connected to port 2. Of course, this is not a very interesting two port network, because if there is no relationship between the two ports, you can simply treat the whole thing as two separate one port networks.

So, the main thing I want to emphasize here is that, there is some property called reciprocity and another one called being unilateral and these are unrelated to which parameter set you choose, whichever you choose they should come out to be the same and we notice that, resistive networks have some special relationship between the forward and reverse parameters. They seem to be reciprocal; that is, the amount of forward transmission equals the amount of reverse transmission.

Like I said, it is not just a coincidence due to the particular example we chose, it is a general property of resistive network and we are going to prove that in the next lesson.