

Basic Electrical Circuits
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Lecture – 83

We have earlier studied Y parameters or conductance parameters of two port networks, now we look at Z parameters. As I mentioned earlier, these are different descriptions of exactly the same thing and it is like specifying either the conductance or the resistance of a resistor.

(Refer Slide Time: 00:24)

z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}_{z\text{-matrix}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \quad [Y] = [Z]^{-1} \quad ; \quad [Z] = [Y]^{-1}$$

In case of z parameters, let say we have a 2 port, naturally this can contain all linear elements, but no independent sources. So, we think of currents as independent variables, you apply I 1 and I 2 and you measure V 1 and V 2 and of course, V 1 and V 2 will be linear combination of the two independent sources, which are in the circuit. V 1 will be Z 1 1 I 1 plus Z 1 2 I 2, V 2 will be Z 2 1 I 1 plus Z 2 2 I 2. As usual mind the directions of voltages and currents, they follow the passives sign convention and this is usually also represented in a matrix form, it is exactly the same thing.

So, the linear equations are placed in a matrix form, that is all and this matrix this 2 by 2 matrix is known as the Z matrix or Z parameter matrix of the 2 port. So, the definition is similar to the Y parameters, except that here currents are independent variables and voltages are the dependent variables. Now, if you compare this to y parameters, what did we have? In case of y parameters, we had I 1 I 2 to be equal to y 1 1 y 1 2 y 2 1 y 2 2

times V_1 and V_2 .

Now, clearly looking at this and that, you realize that the y matrix is the inverse of the Z matrix. So, y is the inverse of Z or Z is the inverse of y . Now, it is possible that one or the other of these is not invertible, perhaps the Z matrix is not invertible or the y matrix is not invertible, in which case you cannot define the other one. So, that is always possible and this is equivalent to having, let say a short circuit which can be specified a 0 resistance, but you cannot specify it is conductance, it is infinite. So, it is similar to that, we will see examples of this later.

(Refer Slide Time: 04:07)

Open circuit parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$I_2 = 0$ (open circuit port #2)

$$V_1 = Z_{11} I_1$$

$$V_2 = Z_{21} I_1$$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}; \quad Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

Resistance looking into port 1 with port 2 open circuited

Transresistance from port 1 to port 2 with port 2 open circuited

Now, let say you are given a network and you are asked to find the Z parameters, you need four measurements. Because, you have four parameters and by taking four possible combinations of I_1 and I_2 , you will get four equations, you take four possible combinations of I_1 and I_2 measure V_1 and V_2 for each case, you will get four equations and that is it, from that you can solve for the four parameters.

Now, the convenient combinations to take a, first of all you set I_2 equal 0 that is you open circuit port number 2. So, under those conditions what happens, we will get V_1 to be $Z_{11} I_1$ and V_2 to be $Z_{21} I_1$. In other words, you leave the second port open and you apply current I_1 to the first port and you measure both V_1 and V_2 . So, from that we easily see that Z_{11} is V_1 by I_1 with I_2 set to 0 and similarly, Z_{21} is V_2 by I_1 with I_2 set to 0. Now, because you make these measurements with the second port open circuited, the Z parameters are also known as open circuit parameters.

So, you set I_2 equal to 0, so only one of the independent sources comes into play, that is all that it would. Like I said if you take different combinations of I_1 and I_2 , but this is the most convenient. So, you have Z_{11} and Z_{21} , Z_{11} you can say is nothing but, the resistance looking into port 1 with port 2 open circuited and Z_{21} is the trans resistance from port 1 to port 2 with port 2 open circuited. Trans resistance means that it is the ratio of voltage to current. So, it has dimensions of resistance, but the voltage and current are not measured at the same place, current is applied to port 1 and voltage is measured at port 2.

So, and the principles are exactly as same as with y parameters, so I will go quickly through these remaining parameters. In case of y parameters, we short circuited one port and measure two parameters, then short circuited the first port and measured the other two parameters, here instead of short circuiting we are open circuiting. Because, here the independent variables are currents, we set current to 0 that means a port becomes an open circuit, port is open circuited.

(Refer Slide Time: 08:33)

Open circuit parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$I_2 = 0$ (open circuit port #2)

$$V_1 = Z_{11}I_1$$

$$V_2 = Z_{21}I_1$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} ; Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

Resistance looking into port 1 with port 2 open circuited

Transresistance from port 1 to port 2 with port 2 open circuited

So, measuring the other two is quite simple for measuring Z_{12} and Z_{22} . Open circuit port number 1 that is, you do not connect anything to port 1, you connect I_2 to port 2 and measure both V_2 and V_1 . So, the first equation reduces to V_1 equals $Z_{12}I_2$, because $Z_{11}I_1$ is 0, because I_1 is 0 and V_2 will be $Z_{22}I_2$. So, from these we get Z_{12} to be V_1 by I_2 with I_1 set to 0, that is port 1 open circuited and Z_{22} is V_2 by I_2 with I_1 set to 0, that is Z_{22} is the resistance looking into port 2 with port 1 open circuited and Z_{12} is the trans resistance from port 2 to port 1 with port 1 open circuited.

So, these are pretty simple definitions similar to y parameters, it should be pretty clear.