Basic Electrical Circuits
Dr Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture – 77

In this lesson, we will discuss the maximum power transfer theorem, it is a theorem of great practical importance. If you have a circuit with a number of independent sources and other linear components, such as resistors and control sources and you have access do of it is terminals, the question is how do you extract the maximum power by connecting a resistor, a load between these two terminals. So, what we will work out is the condition for maximizing the power that can be drawn from these terminals.

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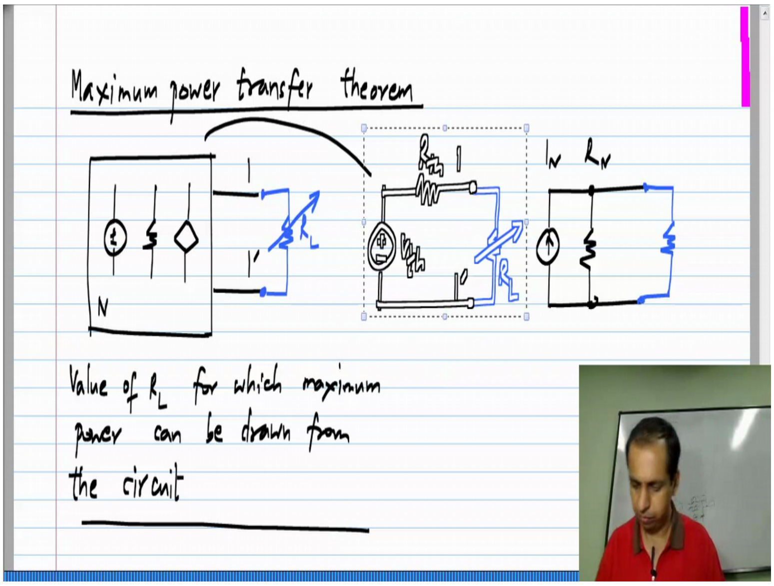

It is known as the maximum power transfer theorem; I have a network N with independent sources, linear resistors and linear control sources, self content. So, all the controlling quantities are within the same circuit N and we have access to two of it is terminals 1 1 prime. We can connect a resistance R L between 1 and 1 prime and there all out vary R L and we are required to find out at what value of R L we can draw the maximum power from the circuit; that is, at what value of R L well maximum power be dissipated in R L and how much that power is.

Now, we know that every network, every circuit for which two terminals are accessible can be modeled by either the Thevenin equivalent, a voltage source V t h in series with resistance R t h or the Norton equivalent, a current source I N in parallel with a resistance R N. So, we will use these models, we do not want to deal with the specific of the circuit, but we know that all these circuits can be reduced to this model or that model. So, we will consider one of these models to find the condition for maximum power transfer, we can use either one, the answer will be exactly the same with either case.

So, now if you have a voltage source V t h in series with R t h, this part is fixed anyway, because what we are allowed to vary is the load R L. So, as R L is varied, when does it draw the maximum power; that is what we want to find.

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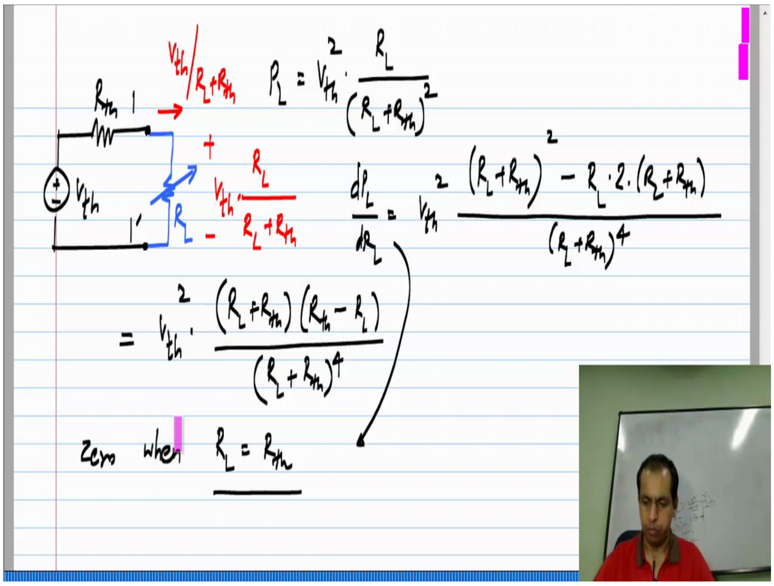

Now, what is the power dissipated in R L? P L will be equal to the voltage across R L times the current through R L. The voltage cross R L from voltage divider expression is V t h times R L by R L plus R t h, the current through R L we have V t h across R t h plus R L, so it will be just V t h by R L plus R t h. The power P L which is the product of the two will be V t h square R L by R L plus R t h square. Now of course, to maximize the function we know that we can differentiate the function and set it to 0; at least that will find the extremes of the function, whether it is maximum or minimum and by finding the second derivative, you can figure out whether it is maximum or minimum.

Now, by differentiating this with respect to R L which is our variable here, which is what we are allowed to vary, we have V t h square which is the constant as far as we are concerned and the denominator will be square. We have denominator times the derivative of the numerator, which turns out to be R L plus R t h square times the derivative of R L which is 1 minus the numerator R L times the derivative of the denominator, which is 2 times R L plus R t h.

So, this whole things turns out to be V t h square times R L plus R t h, R t h minus R L and here we have R L plus R t h to the power of 4. Now, it is very easy to see that, this goes to zero when R L equals R t h and R L equals R t h the derivative goes to zero. So, that is fine, the extreme has occurred and it turns out that is the maximum.

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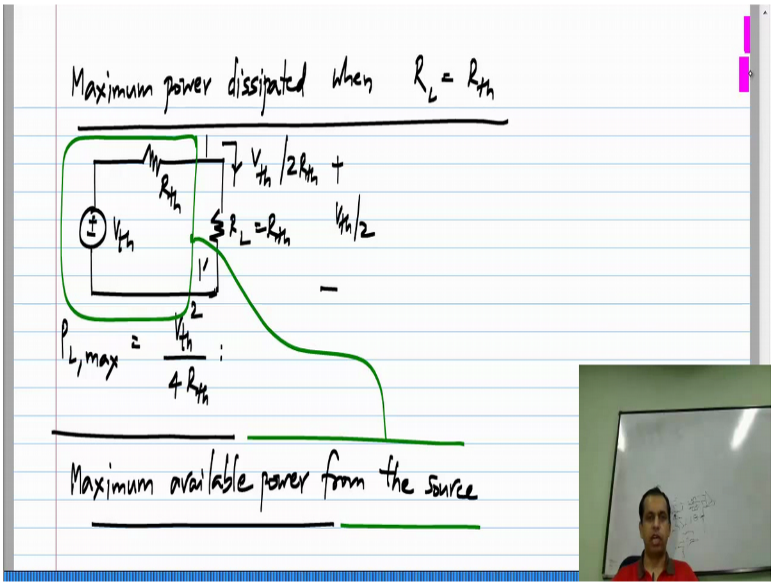

What does it say, if the load resistance equals the Thevenin's resistance seen across the two terminals 1 1 prime, then it draws the maximum amount of power, this condition is known as matching of the load to the source. The source has some internal resistance which is R t h; that is the equivalent resistance, terminal resistance of the source and if you have a load which is the exactly the same value, then you draw the maximum power. Now, how much is that the power?

So, if this is the case, the current here will be V t h by 2 R t h and the voltage across this will be just V t h by 2. So, the maximum power that can ever be dissipated in R L as you vary R L for a given source is the product of this voltage and current which is V t h square by 4 R t h. Note is that, this is the property of just the circuit, the load when it is arranged to draw maximum power, it is resistance will be equal to the source resistance.

So, the maximum power that is dissipated in the load will be equal to V t h square by 4 R t h and this is known as the maximum available power from the source. By source I mean, this part the combination of V t h and R t h which could be by itself like that, that is you could have a voltage source and series with the resistance or it could be a representation of a very complicated circuit. We know that any complicated circuit can be represented in this form, a voltage source and series with the resistance.

So, whatever it is the maximum available power from the source is this much; that is, if you do all you can to maximize the amount of power drawn from the source, this is what you will be able to draw. So, in summery if you have a source which is represented by a voltage source V t h in series with the resistance R t h, then to draw maximum power from that you should connect a load resistance which is exactly equal to R t h; that is, you should match the load to the source and the amount of power you can draw which will be the maximum available power from the source is V t h square by 4 R t h.

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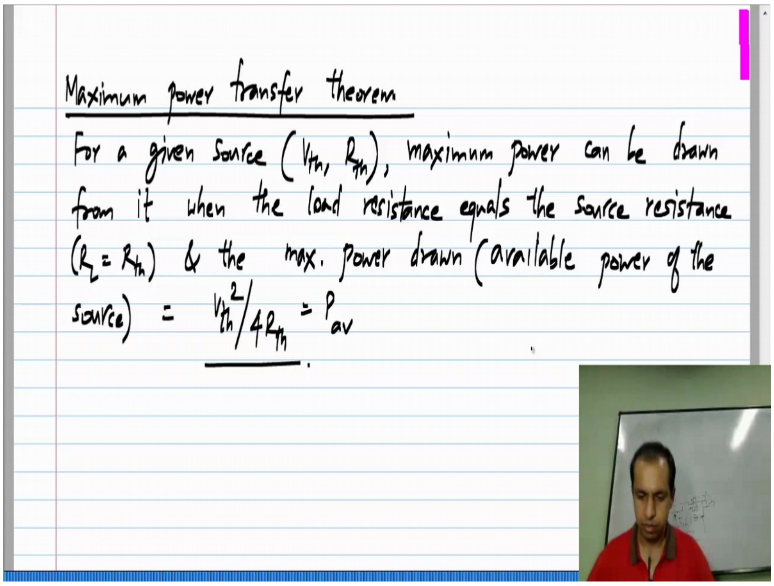

So, here is the statement of the maximum power transfer theorem. For a given source, when I say given source the Thevenin voltage and the Thevenin resistance are given, maximum power can be drawn from it when the load resistance equals the Thevenin resistance of the source. I will call it the source resistance R L equals R t h and the maximum power drawn, which is defined to be the available power of the source equals V t h square by 4 R t h, this is the available power.

Now, I made some implicit assumptions here, I assume that R t h was positive and R L is also positive; that is this is the case with many sources such as antenna. An antenna can be represented as a voltage source and series with some resistance, which constitutes the impedance of the antenna and then, the load it is always dissipative, so it also is a positive resistor. So, under these conditions when you make R L equals R t h, you draw maximum power.

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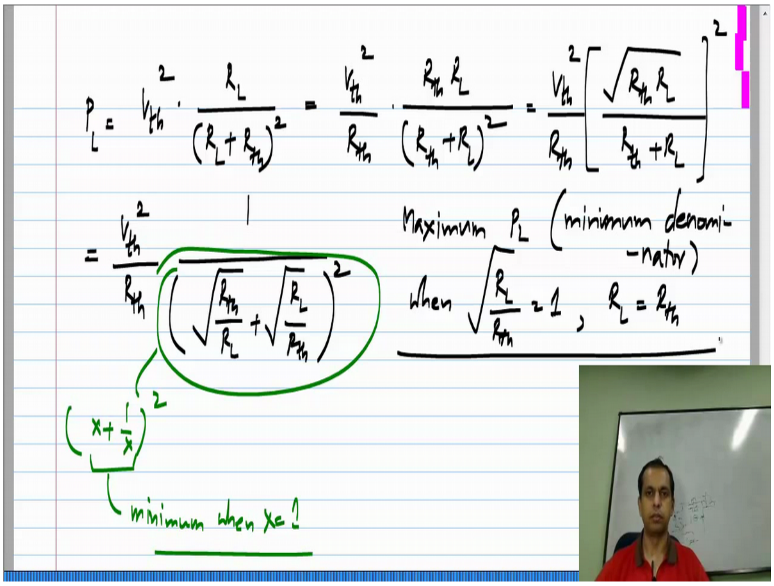

This can also be delivered in a slightly different way, the expression for the power was V t h square by R L plus R t h square times R L which can be rewritten as V t h square by R t h R t h R L by R t h plus R L square and I will further rewrite this as V t h square by R t h times square root of R t h R L by R t h plus R L this whole thing square. And finally, taking the square root term in to the denominator we will get V t h square by R t h 1 by square root R t h by R L was square root R L by R t h square.

Now, if you observe the denominator that is this one it is of the form x plus 1 by x square, now this form x plus 1 by x may be familiar to you. Now, as x increases the first one increases and the second one decreases and if you try to minimize the value of this, you will find that the minimum will occur when x exactly equals 1. So, this has a minimum when x equals 1, when x tents to 0 this one goes towards to infinity, when x tents to infinity this one goes towards infinity in the middle some were it will reach minimum and it turns out that it will be 1 x equals 1, you can work this out for yourself.

Now, if you see the denominator of this expression it is of this form x plus 1 by x squared. Now, to maximize this function the power in the load we have to minimize the denominator remember R L is only in the denominator, now were else in the expression. So, we have to minimize the denominator and that will be minimum when x equals 1 that is each of these terms equals 1, you can think of one of them as x and other one is 1 by x.

In other words, in the maximum of the L which occurs when the denominator is at, it is minimum when square root R L by R t h as 1 or R L equals R t h. So, it is another way of deriving the same result.

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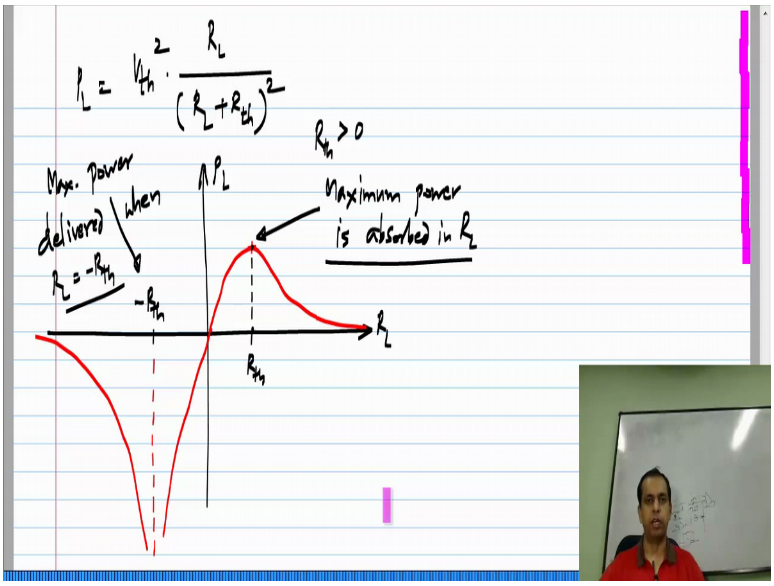

Now, let us explores this a little bit further this expression P L being V t h square times R L by R L plus R t h squared you plot P L of the function R L. This is what you will see, if R t h is positive and R L is redistricted to be positive, then this function will be maximum when R L equals R t h and than for both R L equal to 0 and R L equal to infinity the powers of all to 0 for R L equal to 0 the load is a short circuit there can be no voltage across it and it does not draw any power, it does not dissipate any power.

Similarly, when R L equals infinity it is an open circuit that can be no current through it and again there is no power through R L. Now, if you allowed R L to be negative what happens is that when R L equals minus R t h you have a short circuit across the voltage source, the series combination gives you 0 resistance. So, that gives you the maximum possible current that is infinite current through R L and infect R L will be delivering power instead of observing power.

The power actually goes to infinity when R L equals minus R t h when it becomes more negative than minus R t h it again comes back to 0. So, here I assume that R t h s more than 0 and then this will be even maximum power is observed in R L. If you say maximum power observed or delivered by R L, the maximum power of the delivered when R L equals minus R t h, but of course, this not a very use full result because this means that R L itself is delivering power and to R t h for that R L cannot be physical resistor, but equivalent of some complicated circuits with sources inside and so on.

So, this is not a very useful result, but I just wanted to highlight that it is possible to have infinite power also, but then it will be infinite power being delivered by R L, when R L exactly equals minus R t h, the current in the loop goes to infinity and you have infinite power as well of the result of real significance is this when R L equals R t h and you have the maximum power observed and example of this happens when you have an antenna, antenna can be represented by as a voltage source and series with resistance many standers antennas have a 50 ohm internal resistance.

So, you have a voltage source in series with the 50 ohms resistance and to draw maximum power form that your circuit must appear like a 50 ohm resistance, again even this load resistance is a representation of the input of your circuit it may or not be a physical resistance, it is a physical resistance power will be dissipated unit, if it is the equivalent of something power will go elsewhere, but to draw the maximum power from antenna which has a 50 ohm internal resistance your circuits should also present 50 ohm resistance and those of you will go on to study R F circuit later you find that a lot of effort is expended on by in trying to make the input of the circuit look like 50 ohms.

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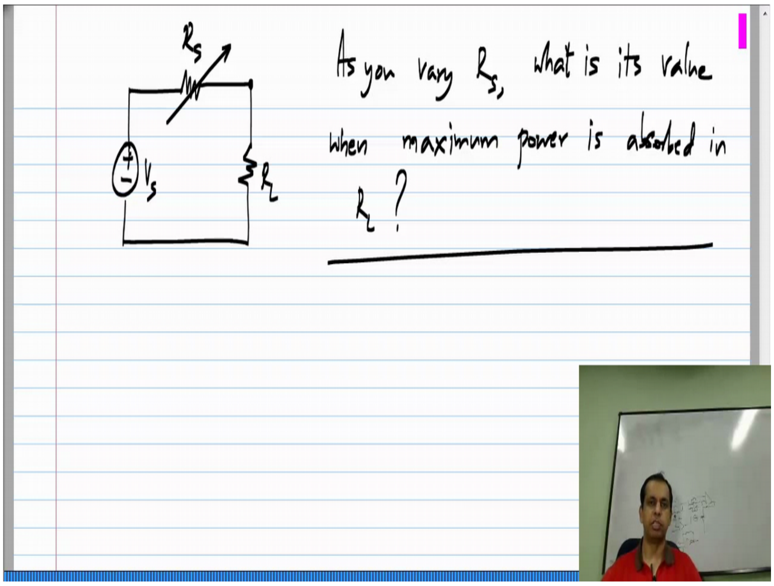

As the very last think I just want to touch up on something I won't discuss is in detail this scenario is sometimes confuse with the scenario that we all ready discussed that is let say we have a source V s and it is source resistance R s and you connect a load to it, exactly the same situation is before I simply change the symbols around that is all. Now, let us say that we vary R s not R L as you vary R s, what is it is value, when maximum power is observed in R L I won't work this out you can do it yourself and you can do it by differentiation or why thinking about it in any other why. Of course, you can for now assume that all resistances are positive they are non negative, think about it answer is very simple, but do not confuse this situation with the other one where wearing R L and trying to fine when the maximum power is observed in R L.