Basic Electrical Circuits
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Lecture - 73
Thevenin's Theorem

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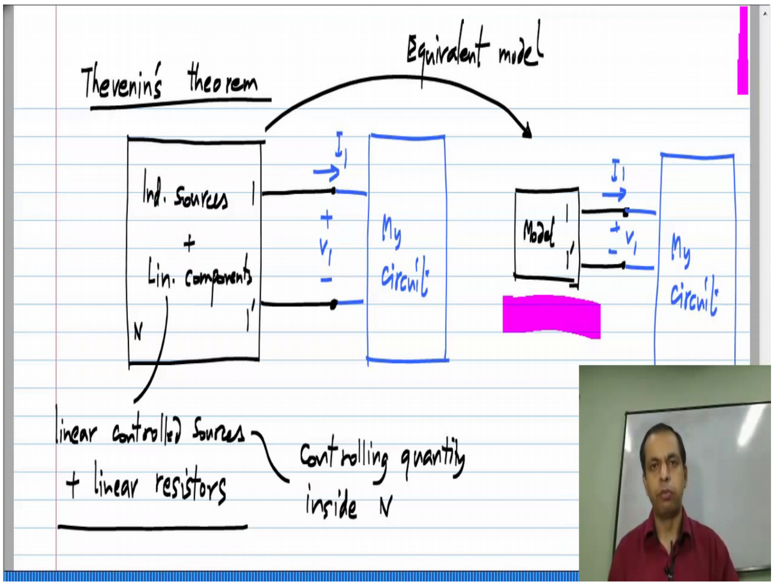

The next two theorems, I will consider will be important for representation of a complicated circuit in simple terms, and these are used very widely. It is called Thevenin's theorem named after the person who originally proposed it. Let say you have a circuit with a number of a independent sources and linear components linear components of course, mean linear controlled sources plus linear resistors basically the controlled sources and resistors we are consider so far. Now, one restriction is that the controlled sources be controlled by quantities inside this block let me call this n the controlling quantity whether it is voltage or current is inside this network N and we are two terminals available to us let me call this one and one prime.

Now, this is the common scenario I mean there are many cases where this happens it could be a power supply where inside the power supply, there is a complicated circuit, but you two terminals available to positive and negative or an amplifier or many cases where there is complicated circuit, but you can only make connection to the circuit. You cannot modify the inside of the circuit and it is also very common to have just pair of terminals tool terminals. Now, in these cases to the person who is using circuit who cannot make modification to the insight of circuit it does not matter, what the insight details are, there is no point giving them the complete schematic of the circuit what is more useful is to give a model of the circuit simplified model which I will discussed shortly. So, that they can make calculations using that what do I mean by simple model that is useful it means that let say as a user I could be connecting any other circuit to it. What I mean by a model or an equivalent of this is the following.

First of all it should be similar, otherwise I would give the whole circuit any way, but it also has to terminals. And let say I connect the same circuit this whatever my circuit is to this thing I connect my circuit to this then the model should be such that whether I connect my circuit to this complete circuit or to my model it behaves and exactly the way in particular there are some voltage V 1 across this and some current iron going through it when I connected to the actual circuit, the voltage when I connect into the model should be V 1 and the current should be I 1 then this is a useful model and they should whole true regard less of what circuit I connect then instead of specifying the complicated circuit to somebody else. Let say customer use specify only the model. So, what we going to do now is to derive such a model.

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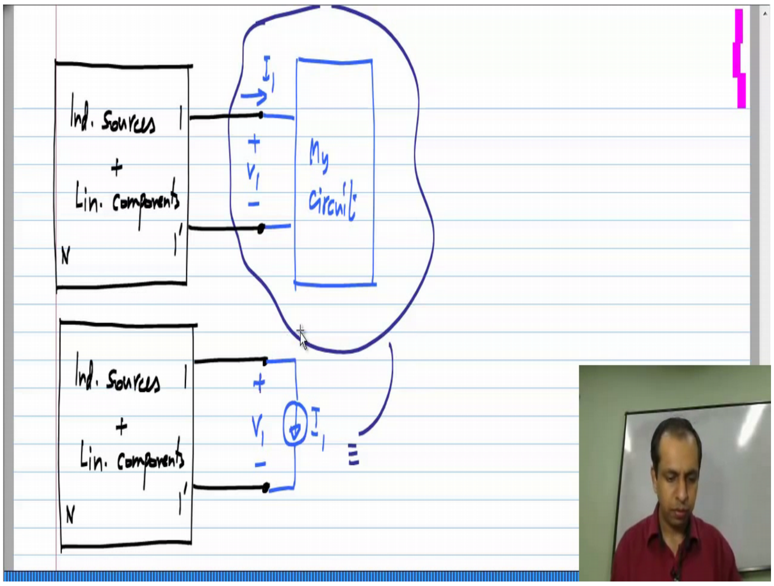

To do that to derive a model of this circuit that I have circuit n first I will use the substitution theorem this is why I discuss that earlier what I will do is I will substitute this entire thing with a current source. So, let me copy over the network N which I am interest to model. And I know we already discussed it earlier that this circuit whatever I connect to it. If it happens to draw a current I 1 or have a voltage drop across it which V

1 I can replace it by that current. Keep in mind that this is just a step in the derivation of the model we do not need to know the value of this I 1 to make this model. So, what I am trying to do is to not deal with some arbitrary circuit, but single current source I 1 another I will show that whatever the value of I 1 is the model I derive will equal into the original circuit. So, hopefully there is now doubt about the equivalence this because we have discuss the substitution theorem extensively we have proved it earlier. Now, if I substitute it with I 1 I expect that there is a voltage V 1 over there. Now, let us see the relationship between V 1 and I 1 and what comes out.

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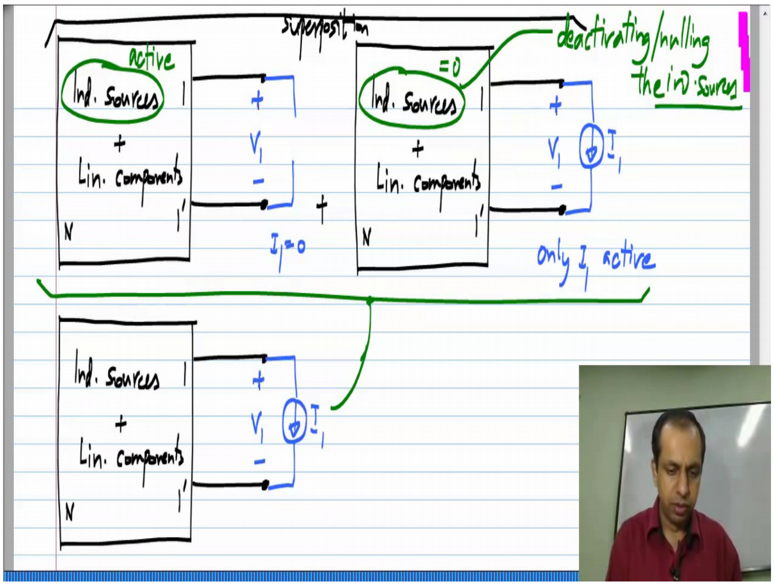

Let us say I have given this circuit for analysis and I have to find value of V 1. What are the possibilities one possibility is to use super position. So, I will think of this as a super position of two cases let me copy this over twice this is my circuit. I think of this as superposition of these two cases, I will mention exactly what the cases at the moment, in the first case, I will set I 1 to zero that is the I open circuit, this one this corresponds to I 1 being zero and the independent sources inside the circuit, these are all active.

Then the other cases, I consider only I 1 to be active, the independent sources in the circuit are all set to zero. So, this is known as deactivating or nulling the independent sources; it is most important to keep in mind that I am nulling the sources only the independent sources not the control sources; the controlled sources controlling quantity is remain exactly further, and they are only the independent sources are exactly where they are, and how they are only the independent sources are nulled. So, this case is clearly a super position of these two cases because in this case we have I 1. So, the independent sources inside the circuit and I think of it a superposition of when I 1 is zero here and when all the independent sources in the circuit are zero, this is the reason I discussed the exception to the superposition theorem earlier. Now, let us look at these two cases one by one.

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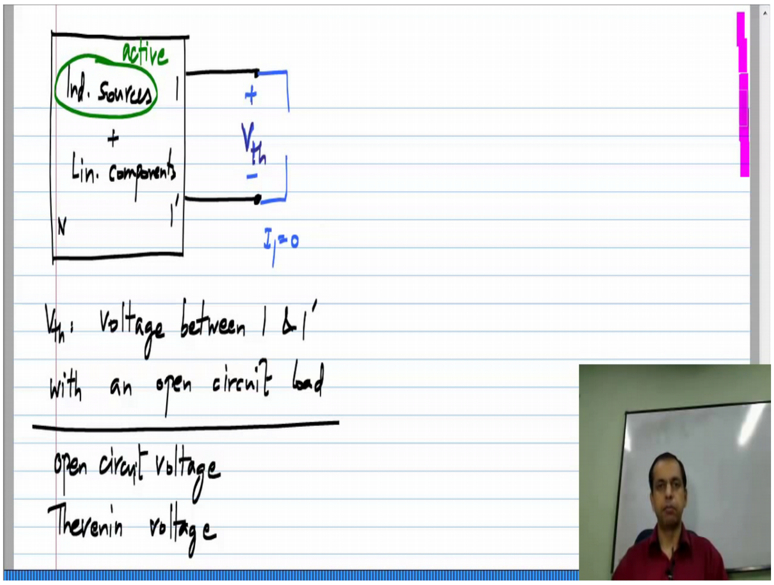

This is the case with I 1 equal to 0; remember in this case, I am analyzing only this circuit right I am not analyzing what is connected to it this is what makes the derivation independent of circuit I connected to although in the intermediate step s I assume some circuits and then it draws out some current I 1 and so on. What I am modeling is only this circuit and it is independent of what circuit it is connected to. So, what am measuring here is I have this circuit N with two terminals exposed one and one prime, and I 1 equal to 0. And let me call this let me call this voltage V th, I will explain the terminology later, basically what is V th its the voltage between one and one prime with the specified polarity of course and you specify a voltage and also specify the polarity.

You have an open circuit between one and one prime that is you can think of it does open circuit load; it is not loaded at all and then measure the voltage across this with the independent sources in the circuit being active, this V th is also known as the open circuit voltage for obvious reasons it is measured with an open circuit at the output which is an open circuit or the Thevenin’s voltage in relation to this theorem. So, you will get some value. All you do this you take your circuit and you do not connect anything between the terminals one and one prime where you are trying to find the equivalent and find the open circuit voltage across those terminals.

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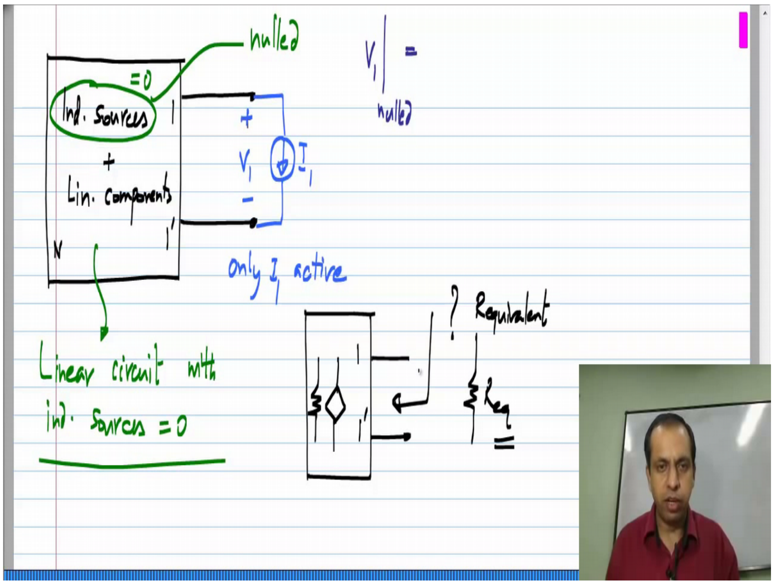

Now, in the second case only I 1 is active and the independent sources in the circuit are nulled. And I have this current I 1 and what is V 1, we have earlier discussed this remember know with independent sources nulled this is a completely linear circuit. With the independent sources set to 0, now we know that this voltage V 1 here, in this case, V 1 with independent sources nulled; it is simply we proposes to I 1, because in earlier lessons and assignment you have solved problems of this type, where your asked there given a circuits and two terminals exposed and you are asked to find out what the circuit look like. And if the circuit is linear that is a consist of only resistors and control sources then looking in here what it look like is a resistor of some value some equivalent resistance I will call it Req.

So, if you take it linear circuit and look at it between two terminals one is says look at it the applied by V characteristics what you get is the iv characteristics of a resistor know because you can have a controlled sources inside this resistance need not always be positive, but it always be a resistor all that is saying is that this V 1 will be proportional to I 1. Now, exactly what is it if you at the directional I 1 it is being nulled from this where V 1 measured with this terminal being positive, so now, when you want to measure the equivalent resistance of a particular circuit, what you do you connect test voltage, let say with this polarity and measure the current flowing in and the equivalent resistance is given by V test by I test.

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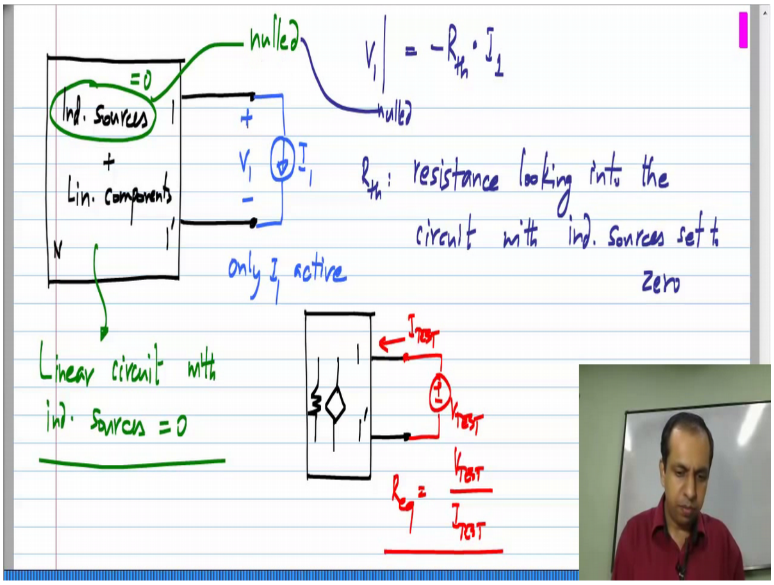

Now, if you absorb this V is in the same polarity as this, but I 1 is the opposite polarity is being pulled out. So, R equivalent will appear with a negative sign V 1 with the independent source nulled this is the nulled source the I am referring to will be minus some resistance time I 1, where R th is the resistance looking back into circuit; Rth is the resistance looking into the circuit with independent sources set to zero that is by nulling the independent sources. Again just to ensure that you understand the reason for this negative sign.

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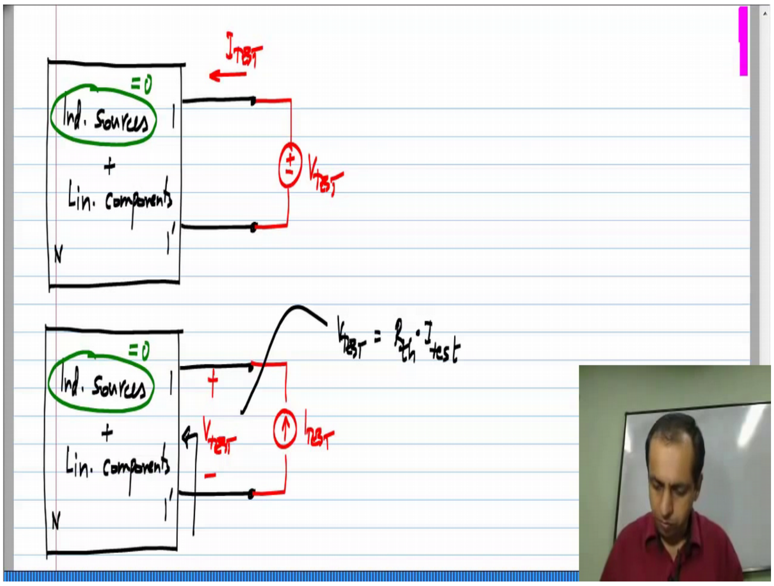

Let me take this circuit and have nulled the independent sources how would I measure the resistance looking at the circuit between one and one prime I would apply V test like this and measure I test or I would apply I test and measure V test. If you compare this picture to what we had earlier the voltage is in the same polarity, but current is in the opposite polarity so that is why you get this minus Rth times I 1. Because in this case, you know that V test is Rth times I test, where Rth is the equivalent resistance to the circuit, looking back that way. Another thing I want to point out here is that again all though I started with some specific circuit connected to it what I measuring here only belongs to the circuit N. So, finally I am modeling only the circuit N. So, we have this case where the independent sources in the circuit are nulled and we have this case where I 1 is zero meaning, we are measuring the voltage with in open circuit.

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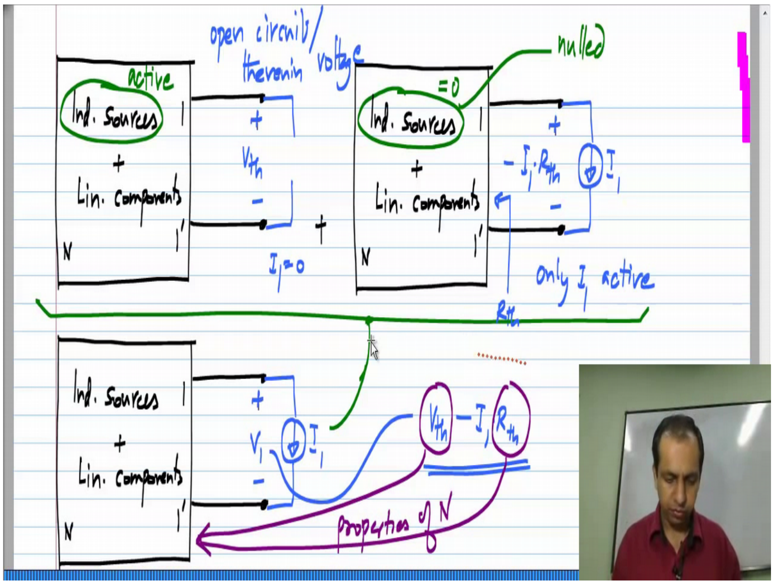

So, combining these two what do we get, so this by original case is superposition of these two cases and we saw that the voltage here in this case is nothing but V th the open circuit or Thevenin’s voltage and the voltage here is nothing, but minus I 1 times Rth where Rth is the resistance looking into the circuit are those two terminals. So, clearly this V 1 here is superposition these two which says that its V th minus I 1 times Rth. Now, this is true regardless of value of t one because V th and ith are simply properties of n. So, emphasize that let me show that V th and Rth both are properties of n now what uses all this the point the following.

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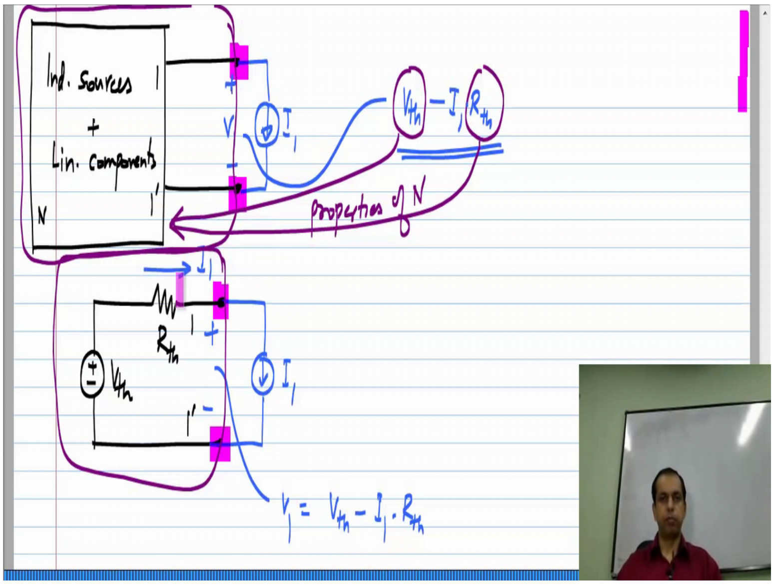

The question is can we come up with a simple circuit which gives a same voltage when I 1 is connected to it and its look like it is very easy. If I have a voltage source V th and a resistance of value Rth. And let say I will connect this current source I 1 which by the way represents whatever circuit I connect to my original circuit clearly the current flowing here is I 1 and the voltage here is V 1 equals V th minus I 1 times Rth. So, a series combination of Vth and Rth will model, this circuit the only condition is this circuit should have only independent sources and linear components why linear components because we use super position in the derivation this which is valid only for linear circuits. So, as long as the circuit has only independent sources and linear components it can be modeled exactly by these two quantities V th and R th, where V th is the open circuit voltage of this at the terminals one and one prime. Let me also mark the terminals this one and this one prime and the equivalents hold. So, only at these two terminals for nothing else it holds for the voltage across these two terminals and the current flowing out through these terminals.

So, this simple model here is equivalent to this circuit; however, complicated it is and this is known as Thevenin's theorem. So, very useful because you got have how are complicated circuits you want with hundreds of resistors and controlled sources are even thousands and millions it does not matter as long as your concern only about the behavior of the circuit at these two terminals one and one prime. We can model it with single voltage source and a single resistance I also showed how to determine the voltage source and the resistance values the voltage source value is nothing but the voltage that you see across the terminals one and one prime with an open circuit between them. And the resistance is nothing but the resistance you see between the terminals one and one prime after nulling all the independent sources in the circuit.

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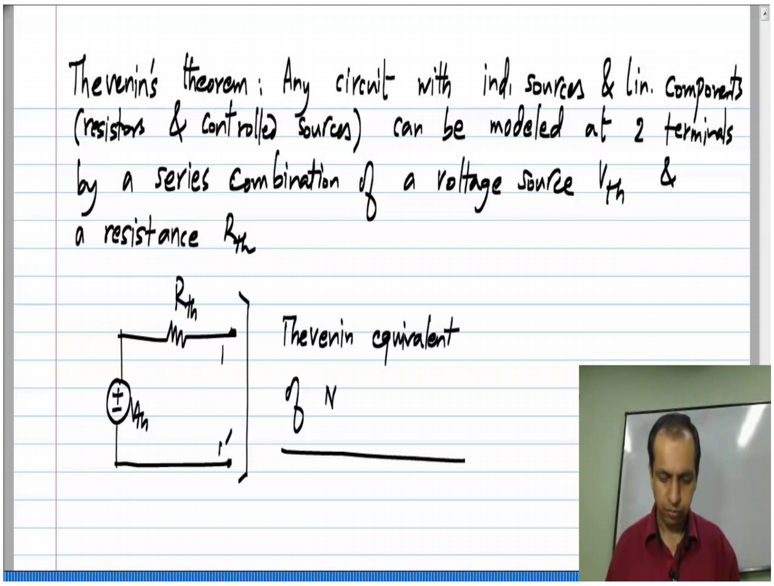

So, Thevenin’s theorem say that any circuit with independent sources which means independent voltage sources or current sources and linear components that is resistors and controlled sources can be model at two terminals. The two terminals will be given to you by a series combination of a voltage source of value Vth and a resistance Rth and the series combination Vth and Rth between one and one prime, this is known as Thevenin’s equivalent of N so that is Thevenin’s theorem. And you have the Thevenin’s equivalent or Thevenin’s equivalent of your circuit n and this is how you determine the value of V th you place an open circuit between one and one prime measure the voltage that develops here always minus.

The polarities here I am measuring with one being positive and one minus being negative that is why I am measuring the voltage and that how we have to place it in the equivalent as well, and this is how you measure the resistance. You deactivate all the independent sources you will left it only linear component that is a resistors and controlled sources then looking back into the circuit between terminals one and one prime you determine the resistance Rth. So, these two together will give you the equivalent of n. So, clearly you can see that this is the much simpler model then the original circuit which would have hundreds or thousands or any number of components so that is the reason it is used so widely.