Basic Electrical Circuits
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Lecture - 68

In this lesson, we will consider another elementary circuit theorem known as The Substitution Theorem.

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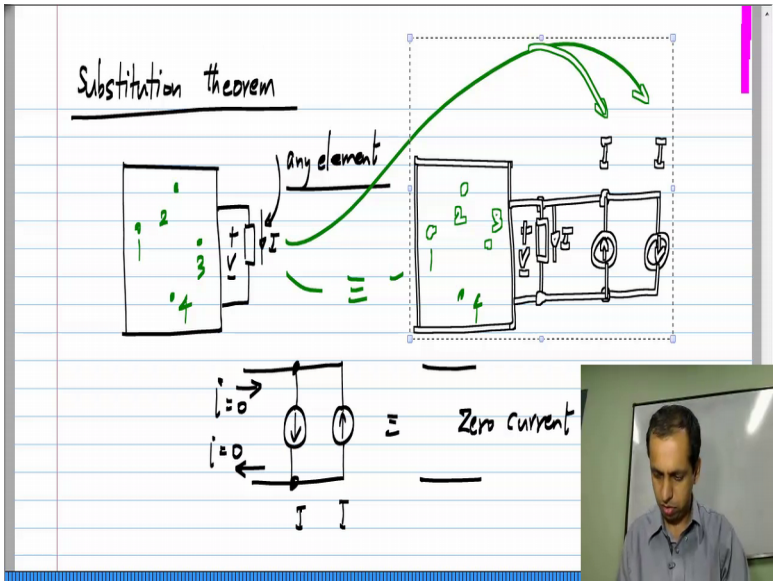

Now, let us consider a circuit with some element, so again this element could be non-linear as well and let say it has some voltage V across it and some current I through it. Now, consider the following modification to the circuit. What I will do is I will take the same circuit and across this element I will connect a current source like this and a current source like that and I will choose the values of this current source to be exactly I, where I is whatever current is flowing through the element V.

Now, if I have two identical current sources connected in anti parallel fashion like this, it is nothing, it is an open circuit, this is 0 value current source. If I have this, both being I and I, then this is equivalent to an open circuit, because whatever current comes in here goes that and goes that way, so the current here is 0 as is the current over there, so it is really an open circuit. So, all I am saying is, I will connect the open circuit across the element and; obviously, that will not change anything in the circuit.

Because, I mean I have this element and to say that I will connect an open circuit across it; means that, I will really not change the circuit anyway, this is just a fancy convoluted

way of showing an open circuit, but there is a point to it as we will see. Now, keep in mind that the circuit is operating in some condition, it has some values of independent sources and so on and the voltage across this happens to be V and the current through that happens to be I and I choose the exact same current that is flowing through this, make two copies of it, connect them in anti parallel across the element.

So, clearly these two cases are identical to each other; that means that, the voltage across this does not change and not only that, you can have a number of nodes in the circuit. The voltage at any of them does not change in fact; there will be no change at all through any branch voltage or any branch current in the circuit. Because, I have really not made any changes, I just connected zero current source across the element and I have got a fancy way of getting this zero current source.

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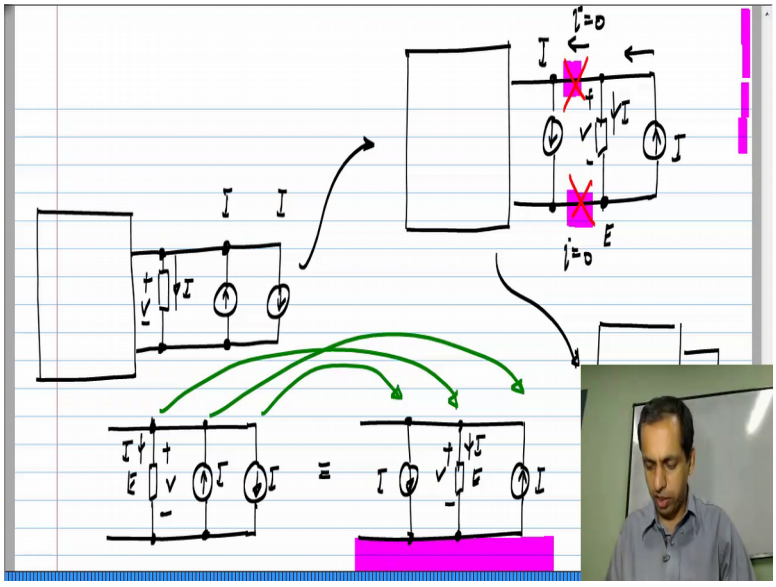

Now let me take this and manipulate it further. What I will do is, I have these three elements in parallel. I have this element E let me call it, it has a voltage V and a current I through it and I have this current source this way and a current source that way. Obviously, this is the parallel connection and it does not matter the order in which, I show the parallel elements. Because, this is exactly equivalent to I, the element E, the voltage across it and a current I through it and the current of value I. All I have done is, I have moved this here and I have moved this over there and I have moved this over there, I simply redrawn it differently; that is all what I did, so these are exactly identical.

Now, I can redraw the complete circuit with this instead of that, so if you do that this will

change to this circuit, where I draw the current source of value I first, then the element E with the voltage V across it and a current I through it and the other current source I pointing upwards. It is clear that there will be absolutely no change when you go from here to there; that means that, all voltages will remain exactly the same and all currents through all the elements will also remain exactly the same.

Again, I have not really changed anything I have just changed the way I have drawn the three parallel elements that is all. Now, the interesting thing is that, if you examine the current in this wire, what it is. The current in this wire is equal to the current flowing here minus the current flowing downwards to E. So, clearly the current in this link is exactly equal to 0, similarly the current in this wire is also exactly equal to 0. So, that comes from the fact that this element has a current I flowing downwards and this has the current high flowing upwards and this connection simply circulates here with 0 current through these wires.

Now, the moment I know that I have 0 current through a wire I can just cut it off; that is, I can turn it into open circuit without altering the circuit at all. Because, after all it was carrying 0 current anyway, so if I turn it into an open circuit it will still carry 0 current and no circuit equation has changed as the result of this, so the circuit conditions will remain exactly the same as before. So, what I am going to do is cut of these wires and I will be left with this I.

Now, if you recall, this is the same as the original circuit and this is the same as this circuit and this is the same as that circuit. When I say same, what I mean is, all branch voltages and currents in the circuit are the same in this case and the original case.

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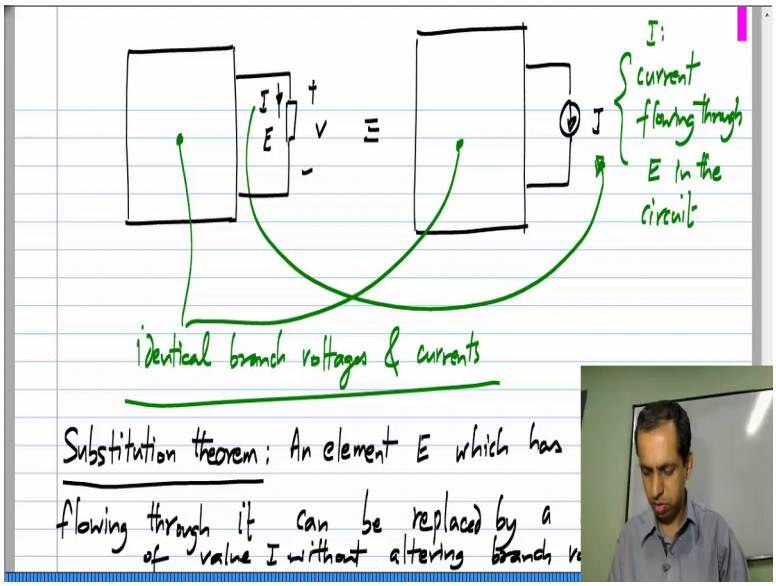

So, clearly it follows that if I have a circuit with an element E, which has a certain voltage V across it and a certain current I through it, then the element E can be substituted by a current source of value I, where I is the current that is actually flowing through E in the circuit. I have to use that value of current otherwise, the whole thing does not hold. All my reasoning is dependent on the current source value here being exactly same as the current through the element E.

I is the current flowing through E in the circuit and this circuit and that circuit will have identical branch voltages and currents. On this idea that an element E can be replaced by a current source, whose value equals the current flowing through the element in the original circuit is known as substitution theorem. An element E, which has a current I flowing through it can be replaced by a current source of value I and the of course, the point is without altering branch voltages and currents, basically without changing the circuit conditions. So, that is the substitution theorem, I will illustrate it with an example.

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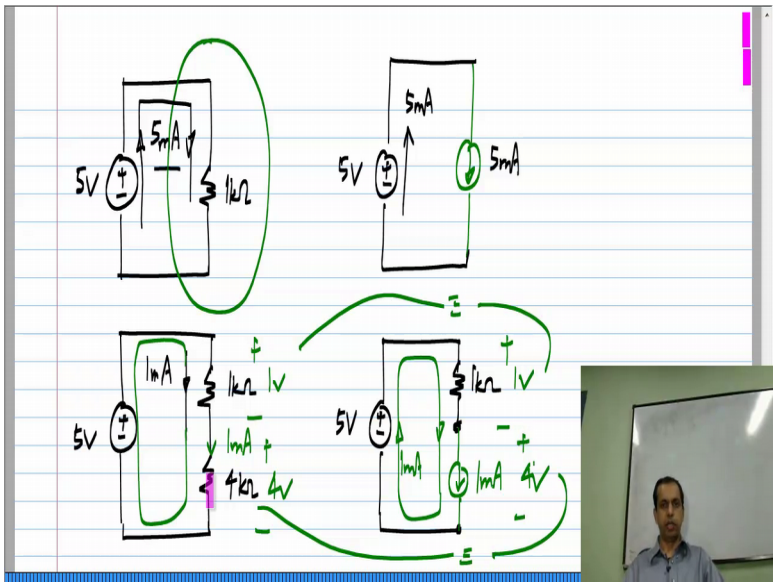

First lets me take an extremely simple example, let us say I have 5 volts and 1 kilo ohm, then clearly we know the that 5 milli amp flows through this resistors. Now, let me substitute this resistor what should I substitute it with according to the theorem I just stated I have to substitute it with the 5 milliohm current sources. Now, what was the current originally flowing through the 5 volt source it is 5 milliamps upwards and in this case also it is 5 milliamps upwards in the voltage source.

So, the circuit conditions have remained exactly the same of course, is the very trivial example, let me make it slightly more complicated this is 5 volts and say this is 1 kilo ohm this is 4 kilo ohm and in this case will have 1 milliohm flowing through the whole circuit. And let me choose the substitute the this 4 kilo resistors; that means, that I will retain the 5 volt source and 1 kilo ohm resistor, but I will substitute the 4 kilo ohm resistor current flowing through it I know the current flowing through that 1 milliohm appear, so the current here is 1 milli amp.

So, now, let us calculate some quantities in the original circuit this 1 milli amp circulates like this. So, there is a current in 5 volts source of 1 milli amp pointing upwards and the voltage across this is 1 volt and voltage across this is 4 volt, now let us go back to the this circuit we have 1 milli amp which means that current 1 milli amp is circulating this way. So, the current through the 5 volts source is still 1 milli amp pointing upwards. So, same as before and this 1 milli amp is flowing through 1 kilo ohm source.

So, across this will have voltage of 1 volt also the same as before and finally, the

resistance had a voltage of 4 volts across it the current source, which substituted the resistance also as 4 volts across it because 5 volts minus 1 volt is 4 volts. So, again nothing has changed, now I already proved it an general case these two cases are just for illustration its looks almost like a trivial theorem and it is some ways it is very easy to understand, which mean you value is in proving other theorems.

Also sometimes it is useful for circuit enthusiast it turns out that we can also back substitute in this case in place of a resistors I substituted a current source. We can also go back words we can substitute current source with the resistor subject to some conditions and that helps in deriving new circuits, because sometimes it is more compulsion to realize current sources, but you could in many cases realize the same functionality replacing the current source with the resistors.