Basic Electrical Circuits
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Lecturer - 65
Superposition Theorem

In this unit, we will look at certain circuit theorems which are useful for circuit analysis which are useful for deriving others circuit theorems, and which in some cases make some types of circuit analysis very convenient , and some of the theorems are also use for abstracting a large circuit into a simple circuit. We will see all of these things as we go along. The first theorem that will take is something that we already used it is the super position theorem. What we said earlier was that if we have independent sources and linear components then we can find the solution to the circuit by taking the independent sources one by one that is keeping one of them active and setting the values of all the other ones to zero, and find the individual solutions add them all up to find the solution when all the independent sources are acting together. Now based on the general analysis, we have done like nodal analysis or mesh analysis this can be easily proved that is what I am going to do now.

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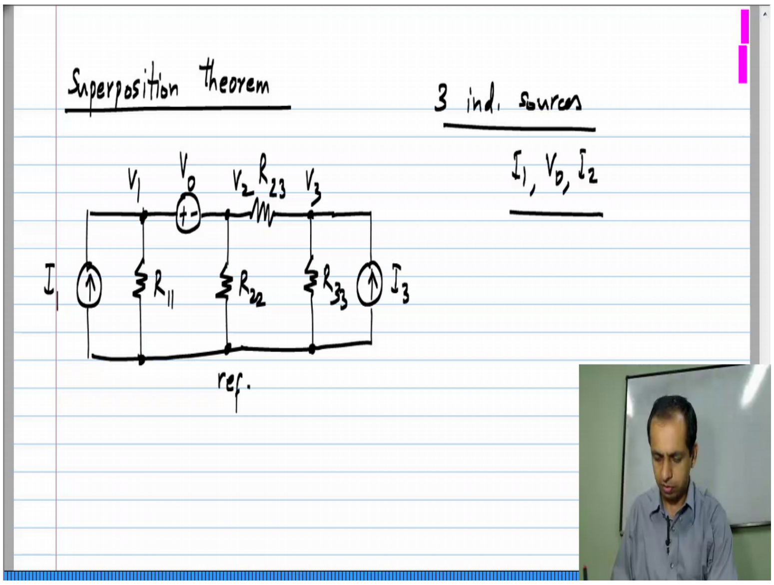

Let may take this circuit that we have considered earlier while carrying out nodal analysis; it has two current sources and the voltage source. Now if you all see have this is to be analyze you can go back to the lesson on doing nodal analysis with independent voltage sources. So, here I am just going to be use the results which we have already derived. So, now, you can see three sources here the current I 1, the voltage source V zero, and the current source I 2.

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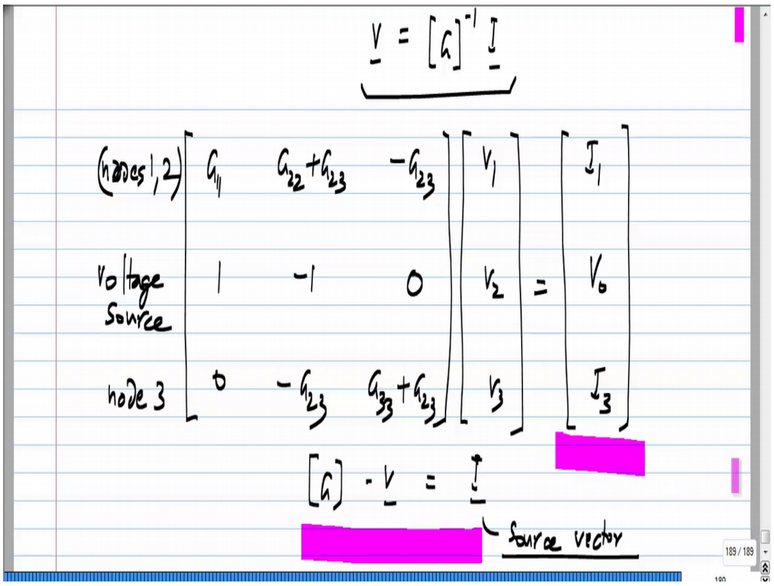

Now, if we carry out nodal analysis of this, we will arrive at these equations. This is the G matrix times the unknown vector of node voltages being equal to the vector of independent sources. As I mentioned earlier while carrying out nodal analysis, this source vector although use the letter I to denoted in this case it contains both currents and voltages. Now I have taken this circuit as an example in all cases the setup of equations will be of these form G times the unknown vector being equal to the source vector.

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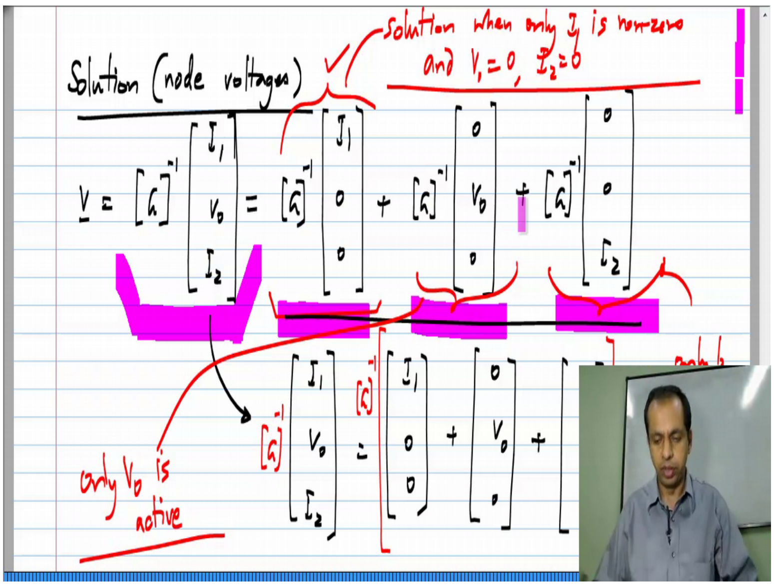

So, clearly we can solve for the node voltages the unknown vector V by inverting G. So, let me write this down, for this particular case, taking this source vector. So, the solution that is the unknown node voltages can be obtained by this expressions and I will write out the source vector explicitly like this. Now clearly this vector over here can be written as I 1 0 0 plus 0 V 0 0 plus 0 0 I 2. So, now, this whole thing equals G inverse times this entire thing the summation on the right hand side. So, essentially, we have formed this summation and multiply the whole thing by G inverse on the left side that will be equal to G inverse times the source vector.

Now, I expand out each of the terms to get what it says as that the complete solution to the node voltage is V is the sum of this, this and that one. Now if you look at each of these terms let us take this for one instance what is it, it is G inverse times I 1 0 0 basically this is this whole thing here is the solution when only I 1 is non zero, and V 0 and I 2 are both 0; in another words this is the solution when only the current source I 1 is the acting and the other sources are deactivated. Similarly this one is the solution when only the voltage source is acting and the other source deactivated, because you can take this general solution the completely solution over here said I 1 equal to 0 and I 2 equal to 0, you will get this one. And similarly you get the third one, when you said I 1 equal 0, and V 0 equal 0. So, this is the solution when only I 2 is active. So, in this case only, I 2 is active and in this case only V 0 is active.

So, it says that the complete solution when all three sources I 1, V 0 and I 2 acting together equals the solution when only I 1 is the acting and the other deactivated; the solution when V 0 is acting and the other two are deactivated; and the solution when I 2 is acting and other two are deactivated, and this exactly the statement of super position theorem.

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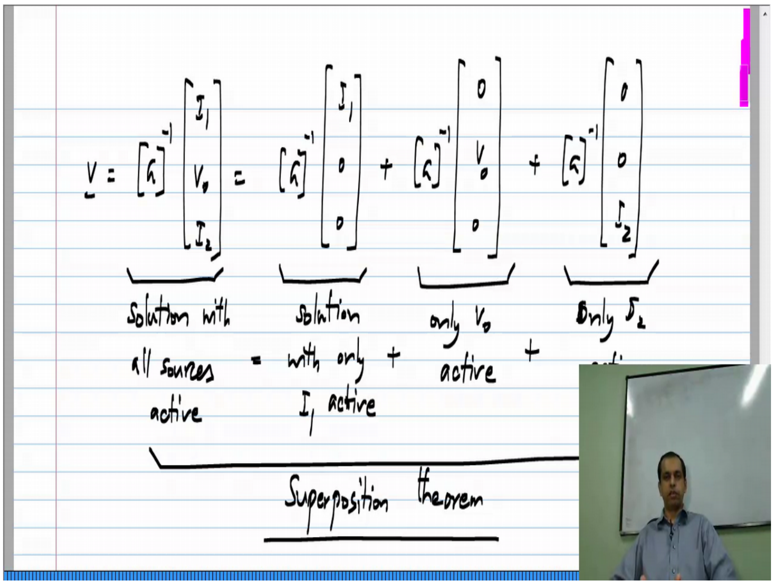

This is the solution with all sources active; this is nothing but the solution with only I 1 active and this is the solution with only V 0 active, and finally, this is the solution with only I 2 active. So, this solution with all sources active being the sum of solution with only I 1 active plus the solution with only V 0 active plus the solution with only I 2 active this is the super position theorem is basically result of linearity of equations. In this case, we have expanded out the matrix multiplying the sum of three vectors as the matrix multiplying individual vectors and summing them together. So, the consequence of the equations governing the circuit being the linear is super position, now why is that useful it is useful because sometimes it simplify analysis instead of analyzing the circuit with all three sources acting together you can analyzed with one source acting at a time and add up the solutions.

Now, I illustrated with the particular circuit in which had three sources now that is just for example, you can easily see you can have an hard very source vector and you can always a represented as summation of number of source vectors where in each source vector only one of the sources is non zero, and you will get exactly the same result. So, if you want you can just go and complete the proof for some general cares and not for the particular circuit I will sure, but the reasoning I am shown applies to all the circuits. Now of course, you can also show the same proof using mesh analysis, it is not the analysis method that not important here is the linearity of the result thing equations which governed the circuit. So, if we have independent sources and linear components in the circuit, we can always represent the equations as a set of terms which are linear in the variables being equal to some terms which are a combination of the independent sources and because these equations are linear, you have this property of super position.

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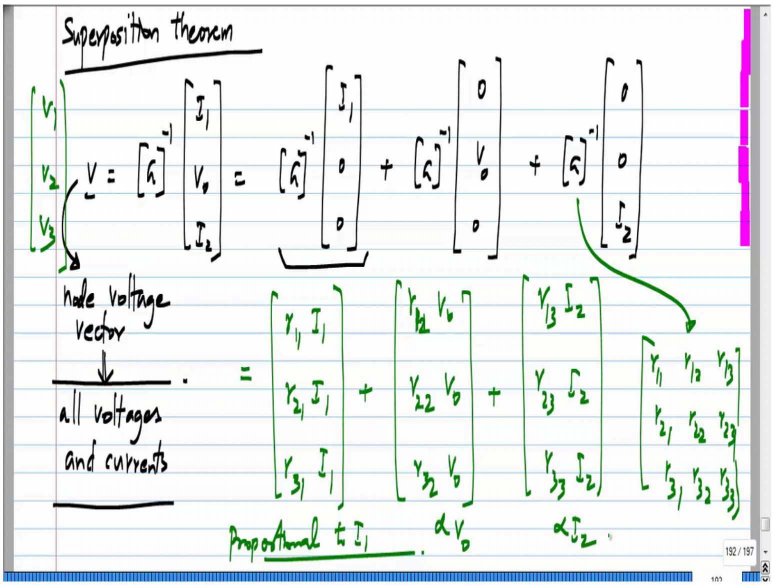

We have seen that the node voltage vector in presents of three independent sources can be written as the solution with only I 1 active and the other two set to 0; only V 0 active and the other two set to 0 plus only I 2 active and the other two set to zero, this is the super position theorem. As I mentioned, we do not need this particular circuit or only three sources, we can have any number of sources and you can superpose the results from the when you looking at node voltages and consequently any voltage or current in the circuit all voltages and current is the circuit can be obtained from super position.

So, this is the node voltage vector by implication we can also super post or voltages and currents. So, that is the proof one of the possible proof because you can prove that super position holds good for a set of linear equations in many ways. Now another point I want to highlight here is that if you look at this individuals solutions, this will consists of a vector of three numbers vector of length three, let me just take a general form of G inverse and I will call the entries r 1 1 r 1 2 r 1 3 r 2 1 r 2 2 r 2 3 r 3 1 r 3 2 r 3 3.

So, now, if you compute G inverse times I 1 0 0 it will pick out the numbers from first the columns of this, we will have this will be r 1 1 I 1 r 2 1 I 1 r 3 1 I 1. So, this will be the vector this is the response due to I 1 acting alone the important point I want to mentioned here is that this is simply proportion to I 1. So, if you have a single independent source in the circuit then the solution everywhere in the circuit whether it is voltage or current will be proportional to that dependent source. You can see that this is now a vector of three voltages V 1, V 2, V 3. So, V one is proportional to I 1, V 2 is proportional to I 1, and V 3 is proportional to I 1 when I 1 is acting by itself. So, as a said with the single independent source always the case that every response in the circuit whether it is voltage and current is proportional to the excitation.

Similarly when V zero is acting by itself, we will have the solution being proportional to V 0. And finally, when I 2 is acting by itself all solutions will be proportional to I 2. So, this is proportional to V 0 and this is proportional to I 2.

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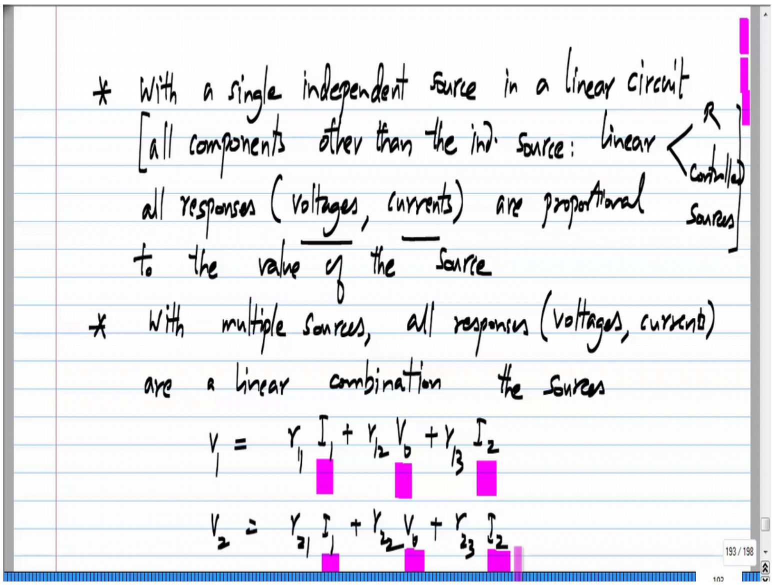

So, to summarize, we have a single independent source in a linear circuit what this means is that all components other than the independent source are linear; these are linear; that means, they can consists of R resistors and controlled sources capacitors and instructors are also linear, but there solution comes out slightly differently. So, we want deal with them now. So, the single independent source and linear circuit all responses and by responses I mean voltages and currents. So, these are the responses that we considered are proportional to the value of the source.

If you have multiple sources all responses that is whether they be voltages or currents are a linear combination of the sources should take a particular variable node voltage V 1 it will be r 1 1 I 1 plus r 1 2 V 0 plus r 1 3 I 2 the point is that it is a linear combination of I 1, V 0 and I 2 the proportionality constants will be different for different quantities. If we take some other quantity, let say the second node voltage V 2, it will be r 2 1 I 1 plus r 2

2 V zero plus r 2 3 I 2. So, it is also linear combination of I 1 V 0 and I 2. So, this is something again you have to keep in mind that if you have multiple sources every quantity in the circuit and by quantities I mean voltages and currents not power specifically, each one will be a linear combination of all the independent source values. So, you can use that fact also to find the solution in certain circuits in certain situations.