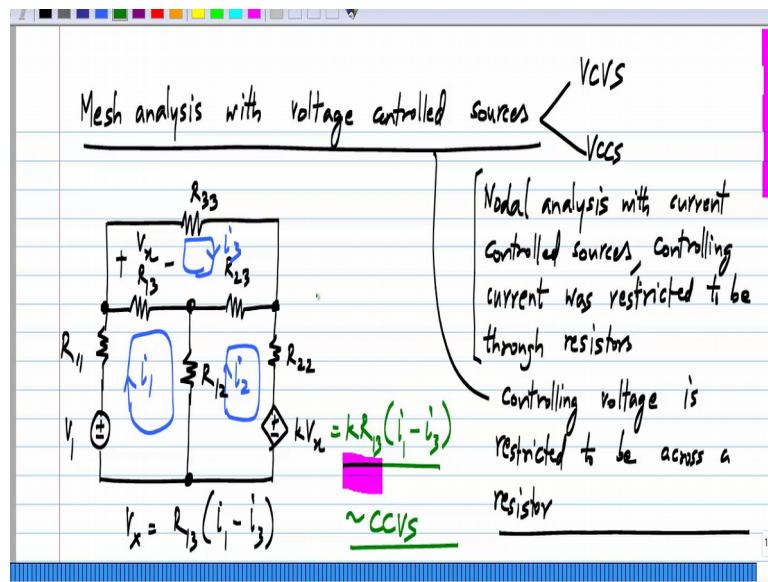


Basic Electrical Circuits
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Lecture – 63

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Now, we look at how to carry out mesh analysis, when we have voltage controlled sources; that is a voltage controlled voltage source or a voltage control current source. Now, if you recall when we had nodal analysis with current controlled sources, the controlling current was restricted to be through resistors. Now, similarly in this case when we have mesh analysis with voltage controlled sources, we restrict the controlling voltage is restricted to be across a resistor.

Now, I will emphasis again, it is not that when the controlling voltage is elsewhere, let say across a current source you cannot analyze the circuit, you definitely can. What we want to do is to carry out mesh analysis without introducing auxiliary variables. If you have the controlling voltage across a current source, you have to introduce an auxiliary variable for that voltage drop. Now, we do not want to get into that, so we have made these restrictions just to tackle only the simple cases.

That is the same reason why we restricted the controlling current in nodal analysis to be only across a resistor. If the controlling voltages across a resistor, clearly it can be return in terms of the current through the resistor, so it can be treated analogously to current control sources;

that is why we introduce this simplification. So, let me take this circuit and I have defined this voltage V_x across R_{13} and in the second mesh, instead of fixed voltage source in our original circuit I have a voltage control voltage source $k V_x$.

Now, we know that V_x is R_{13} times the current through R_{13} , which is i_1 minus i_3 . So, this voltage source can instead be written as $k R_{13}$ times i_1 minus i_3 . Now, clearly instead of thinking of this voltage source as k times the voltage V_x , we can always think of it as $k R_{13}$ times some controlling current i_1 minus i_3 . So, with this rewriting of the expression for the voltage source, this becomes analogous to a current controlled voltage source, which we have already studied.

So, what I suggest you to do is, please write down the mesh analysis equations for this circuit, like I said this is the analogous to having a current controlled voltage source. Go back to that case and see how we did that and write down the three mesh equations, because we do not have any current sources, we can write the 3 mesh equations without any problem. Now, I am going to put down the answer, the machine analysis equations, but please carried out yourself and compare it to the answer that I gave, so that is why we have the better practice of doing mesh analysis.

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The image shows a handwritten matrix equation for mesh analysis, labeled "Asymmetric". The equation is $[R] \cdot \underline{i} = \underline{V}$. The matrix $[R]$ is a 3x3 matrix with the following elements:

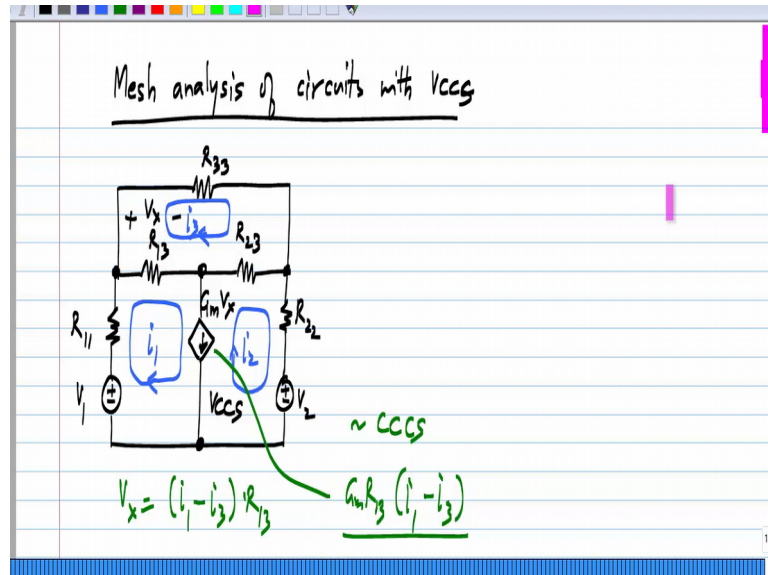
- Row 1 (Mesh #1): $R_1 + R_2 + R_3$, $-R_2$, $-R_3$
- Row 2 (Mesh #2): $-R_2 + kR_3$, $R_2 + R_3 + R_3$, $-R_3 - kR_3$
- Row 3 (Mesh #3): $-R_3$, $-R_3$, $R_3 + R_3 + R_3$

The current vector \underline{i} is $\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$ and the voltage vector \underline{V} is $\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$. The terms kR_3 in the second row are highlighted with pink boxes.

These are the mesh equations for the circuit I showed, now the control source appears here and there in the equations for the second mesh as you expect, because the controlling source is part of only the second mesh. I have written it as R times i , the unknown vector being equal

to the independent voltage source vector. The important thing is that to this R is asymmetric now.

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Now, let me consider a case with the voltage controlled current source. The voltage controlled current source is here and its current is defined as G_m times V_x and V_x is across this resistor R_{13} . So, V_x can be written as current through R_{13} , which is $i_1 - i_3$ times R_{13} . So, this controlled source can be rewritten as $G_m R_{13} (i_1 - i_3)$. With this restatement, you see that there is no difference between having a voltage controlled current source or a current controlled current source. So, this is analogous to having a current controlled current source.

This is because the controlling voltage is across the resistor, so it is proportional to the current through the resistor. So, whether you think of this as a voltage controlled current source or a current controlled current source, you will have exactly the same set of equations. So, again because you already treated the current controlled current source case, please write the mesh equations for this by yourself and compare it to what I put down.

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[R]: hybrid

$$\begin{array}{l}
 \text{mesh \#1, \#2} \\
 \text{Supermesh} \\
 \text{mesh \#3} \\
 \text{VCCS} \\
 \text{CCCS with } k \text{ replaced by } \underline{G_m R_{13}}
 \end{array}
 \begin{bmatrix}
 R_{11} + R_{13} & R_{22} + R_{23} & -R_{13} - R_{23} \\
 -R_{13} & -R_{23} & R_{13} + R_{23} + R_{33} \\
 -G_m R_{13} & -1 & G_m R_{13}
 \end{bmatrix}
 \begin{bmatrix}
 i_1 \\
 i_2 \\
 i_3
 \end{bmatrix}
 =
 \begin{bmatrix}
 V_1 - V_2 \\
 0 \\
 0
 \end{bmatrix}$$

So, these are the mesh equations, we have to form a super mesh, because we have a current source and we do know the voltage drop across it. So, if you form a super mesh by combining meshes 1 and 2 and mesh 3, business as usual no change from before. And finally, we have the voltage controlled current source and this analogous to a current controlled current source with k replaced by $G_m R_{13}$. So, that is all it illustrates and again, you see that the resistance matrix here is hybrid, it has both quantities with dimensions of resistance on the first two rows and the dimensionless quantities in the third row.

Now, I went through mesh analysis a little quickly, because we have studied nodal analysis extensively and you can make analysis between each case of mesh analysis and some particular case of nodal analysis. So, please study these correspondences as well, so that you can learn both nodal analysis and mesh analysis really well.