Basic Electrical Circuits
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Lecture - 56

Here I will discuss the questions in the fourth assignments; that is, the assignment on units 7 and 8th on nodal analysis.

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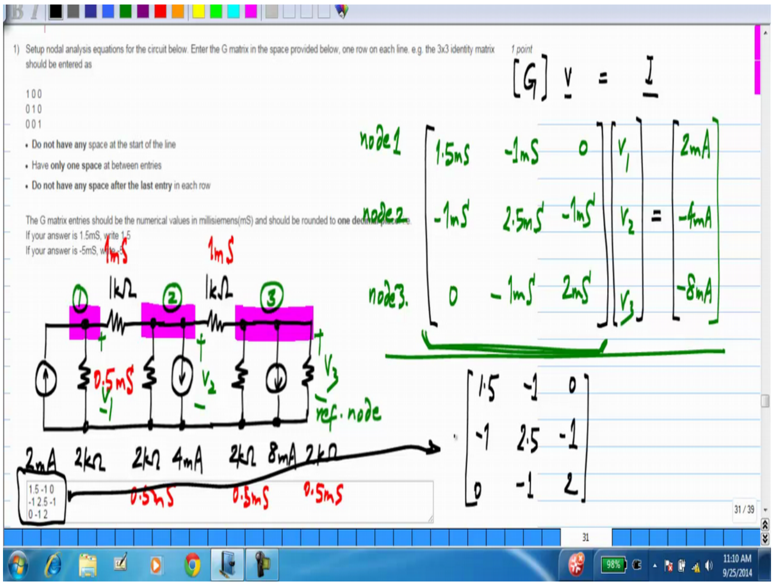

1) Setup nodal analysis equations for the circuit below. Enter the G matrix in the space provided below, one row on each line, e.g. the 3x3 identity matrix should be entered as

1 point

```
1 0 0
0 1 0
0 0 1
```

- **Do not have any** space at the start of the line
- Have **only one space** at between entries
- **Do not have any space after the last entry** in each row

The G matrix entries should be the numerical values in millisiemens(mS) and should be rounded to **one decimal place.**
If your answer is 1.5mS, write 1.5
If your answer is -5mS, write -5

```
1.5 -1 0
-1 2.5 -1
0 -1 2
```

Now, the first question here you are given a circuit and ask to determine the conductance matrix for the nodal analysis. So, this is the easiest of cases for nodal analysis, we have only resistors and independent current sources and there are four nodes as you can see and the reference node is already given to you. These nodes are numbered 1, 2, 3; this is so that you put the entries in the right order and it is easy for us to check answers.

I will setup the nodal analysis equations, we have G times V being equal to I all with the usual meaning of let us start with three nodes, we have node 1, node 2, node 3. So, if we look at node 1 the sum of conductance is at that node is half a Millisiemens here and 1 Millisiemen there. May be it is useful to simply convert all resistance values to conductance's, this is half, this is also half, half and half, whereas this is 1 and 1 Millisiemens.

So, the sum of these two it is a 1.5 Millisiemens, then between nodes 1 and 2 which corresponds to entry G 1 2 we have 1 Millisiemen and you know that, what appears there is a negative of that conductance and between nodes 1 and 3 there is no conductance connected directly. So, we will have 0 then coming to the second node equation, the sum of conductance at note 2 has 1 plus 1 plus half, so we have 2.5 Millisiemens and between nodes 1 and 2 we have this 1 Millisiemen connected, so this is 1 Millisiemen. Already you know that, it has to be symmetrical for this particular case. So, this minus 1 and this minus 1 these two are equal, which is necessary for this matrix being symmetrical and would be nodes 2 and 3 you again have minus 1 Millisiemens.

Finally, if you come to node 3 you see that you have half and half and 1. So, a total of 2 Millisiemens and between nodes 2 and 3 we have 1 Millisiemen which appears as minus 1 in this and we have 0 over there. So, this times the unknown vector which of course, is V 1, V 2, V 3, V 1 is the voltage over there V 2 is the voltage over there and V 3 is the voltage there. They are the node voltages with respect to the reference node and this whole thing equals, we have to compute the independent current sources pushing into each node. We look at node 1, you have 2 milli amps pushing in and node 2 you have 4 milli amps being pulled out of minus 4 milli amps pushed in finally, node 3 we have 8 milli amp being pulled out which means minus 8 milli amp pushed in.

So, these are the nodal analysis equations and you can find V 1, V 2, V 3 and everything else you want in the circuit by solving this. Of course, in this particular question what you ask for is just the G matrix, you should enter this and the formatting is kind of reset. So, you have to enter it like this, you are given some space for answers and we have three rows for three rows node matrix and the matrix entries must be a rounded to one decimal place, when there is a decimal place and left as integer, when it is an integer. So, and the units are of course, given as Millisiemens, so 1.5 minus 1 and we have 0 minus 1, 2.5 and minus 1, 0 minus 1 and 2, so that is what has to be enter in this space.

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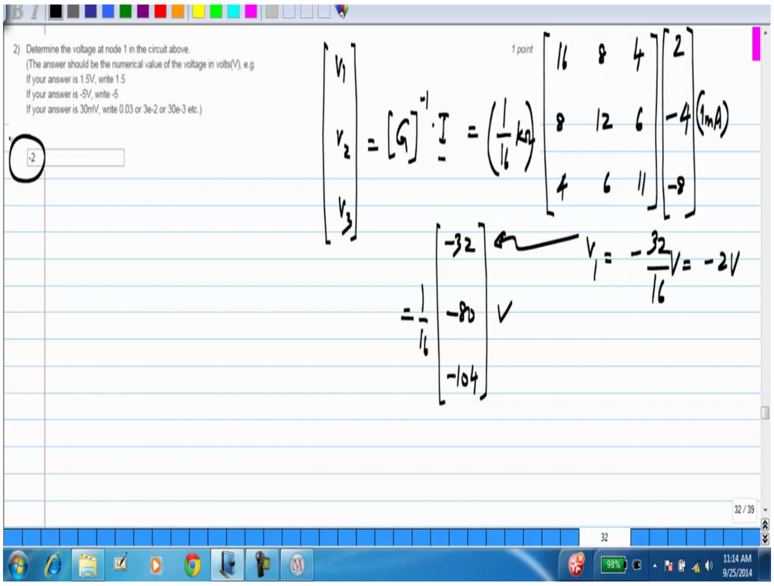

2) Determine the voltage at node 1 in the circuit above.
(The answer should be the numerical value of the voltage in volts(V), e.g.
If your answer is 1.5V, write 1.5
If your answer is -5V, write -5
If your answer is 30mV, write 0.03 or 3e-2 or 30e-3 etc.)

1 point

-2

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = [G]^{-1} \cdot \underline{I} = \left(\frac{1}{16} k\Omega\right) \begin{bmatrix} 16 & 8 & 4 \\ 8 & 12 & 6 \\ 4 & 6 & 11 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \\ -8 \end{bmatrix} (1mA)$$

$$= \frac{1}{16} \begin{bmatrix} -32 \\ -80 \\ -104 \end{bmatrix} V$$

$$V_1 = -\frac{32}{16} V = -2V$$

So, coming to the second question, it is the follow up of the first question, first question you are just asked to set up the conductance matrix. Here you should solve the nodal equations and you are asked for the voltage at node 1 which is basically V 1. So, you have to invert this matrix and then find out the answer. You know that V 1, V 2, V 3 will equal G inverse times I inverse of that G matrix transfer to be 1 by 16 kilo ohms times 16, 8, 4, 8, 12, 6, 4, 6, 11.

So, it is basically 1 by 16 kilo ohms, this comes from all the G matrix entries being in Millisiemens and then you get this matrix and that you have to multiply with the current source vector, which is 2 minus 4 and 8 and I will take this 1 milli amp as the common factor. So, this 1 milli amp multiplies by this 1 kilo ohm here to give you 1 volt and you have just numbers left with units of volts and if you multiply these two matrices, you will get the first row will be minus 32 and the second row will be minus 80, third row will be minus 104, so this many volts.

There of course, ask for only V 1, so V 1 is minus 32 by 16 volts is minus 2 volts. Now, what you can of course, do is you can go back to this circuit and evaluate everything by super position and verify that, you have got the same answer. First of all it will give you confidence that your answer is right, it also gives you extra circuit analysis practice.

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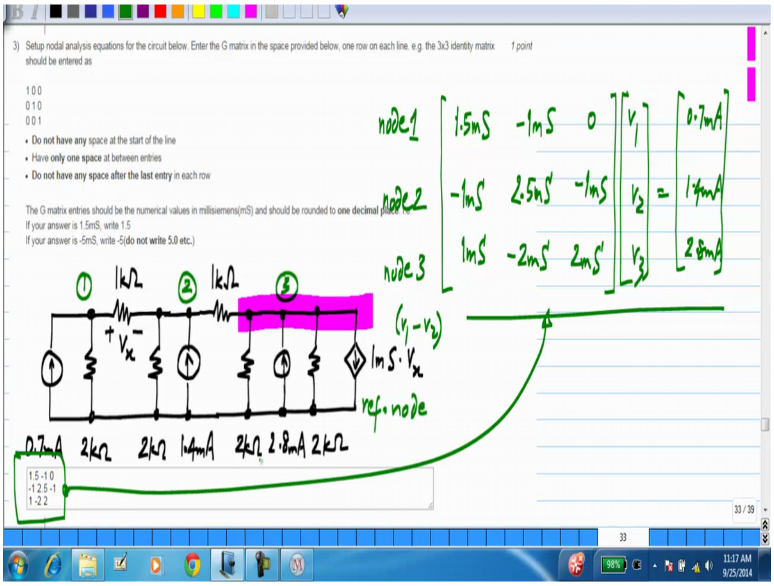

3) Setup nodal analysis equations for the circuit below. Enter the G matrix in the space provided below, one row on each line. e.g. the 3x3 identity matrix should be entered as

1 0 0
0 1 0
0 0 1

- **Do not have any** space at the start of the line
- Have **only one space** at between entries
- **Do not have any space after the last entry** in each row

The G matrix entries should be the numerical values in millisiemens(mS) and should be rounded to **one decimal place**.
If your answer is 1.5mS, write 1.5
If your answer is -5mS, write -5(**do not write 5.0 etc.**)

1.5 -1 0
-1 2.5 -1
1 -2 2

$$\begin{array}{l} \text{node 1} \\ \text{node 2} \\ \text{node 3} \end{array} \begin{bmatrix} 1.5\,mS & -1\,mS & 0 \\ -1\,mS & 2.5\,mS & -1\,mS \\ 1\,mS & -2\,mS & 2\,mS \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0.7\,mA \\ 1.4\,mA \\ 2.8\,mA \end{bmatrix}$$

Coming to the third question, again you are asked to set up the nodal analysis equations; in this case we have this voltage controlled current source. So, again let me start setting up the equations for node 1, node 2, node 3; at the first node we have the sum of conductance's which is 1.5 Millisiemens and between node 1 and 2 1 Millisiemens, node 1 and 3 nothing, also remember this control sources connected to node 3. So, only the equations for node 3 will be effected by this, the rest of it remain the same and if you observe, this circuit was exactly the same as what was in the previous question.

And we have minus 1 Millisiemen, the total conductance there is 2.5 Millisiemens and minus 1 Millisiemens. Now, here if we look at the current flowing out of node 3 of course, you have the conductances over there. But, you also have this voltage controlled current source and this V x is nothing but, V 1 minus V 2. Because, you have 1 Millisiemens multiplying V 1 minus V 2, 1 Millisiemens will get added to the column corresponding to V 1 and minus 1 Millisiemen to corresponding to V 2.

So, we will have 2 Millisiemens over there and normally between node 2 and 3, because there is 1 Millisiemens of conductance we would have had minus 1 and we have this additional minus 1. So, this becomes minus 2 Millisiemens and for V 1 it was originally 0, it becomes 1 Millisiemens. Of course, we have the unknown voltage vector being equal to 0.7 milli amps pushed in 1.4 milli amps pushed in and 2.8 milli amp also pushed in and this answer should correspond to the G matrix and it does as I have shown here.

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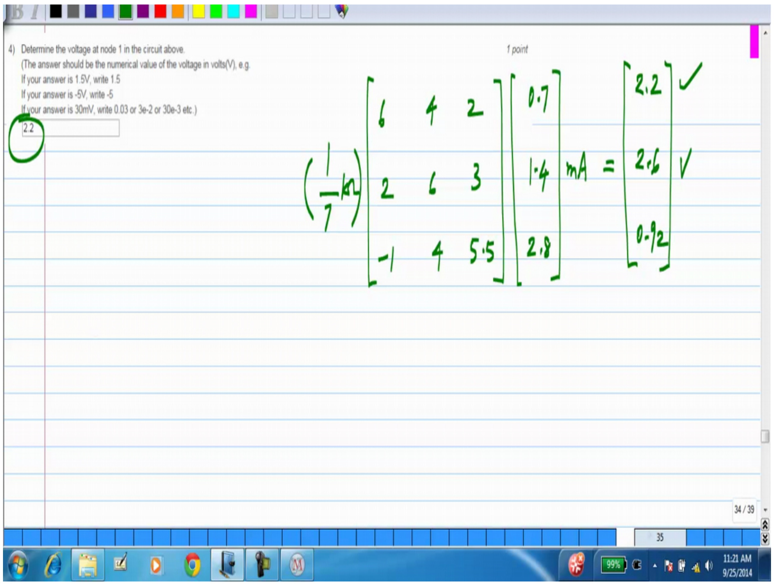

4) Determine the voltage at node 1 in the circuit above. *1 point*

(The answer should be the numerical value of the voltage in volts(V), e.g.

If your answer is 1.5V, write 1.5

If your answer is -5V, write -5

If your answer is 30mV, write 0.03 or 3e-2 or 30e-3 etc.)

2.2

$$\left(\frac{1}{7}k\Omega\right)\begin{bmatrix}6 & 4 & 2\\ 2 & 6 & 3\\ -1 & 4 & 5.5\end{bmatrix}\begin{bmatrix}0.7\\ 1.4\\ 2.8\end{bmatrix}mA=\begin{bmatrix}2.2\\ 2.6\\ 0.92\end{bmatrix}\begin{matrix}\checkmark\\ V\\ \end{matrix}$$

On this problem again refers to the solution of the previous problem, turns out that if you compute the inverse of the G matrix you will get 1 by 7 kilo ohms times these numbers in the matrix and inside you have 0.7, 1.4 and 2.8 milli amperes. Again milli amperes times kilo ohms give you volts and this answer turns out to be, these voltages turn out to be 2.2, 2.6 and 0.92 volts. You are asked for the voltage at node 1, which is that you have 2.2 volts over there.

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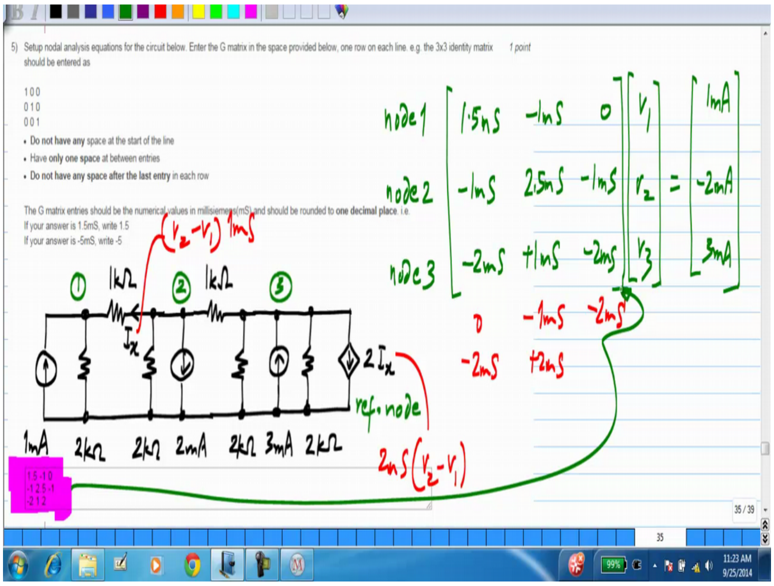

5) Setup nodal analysis equations for the circuit below. Enter the G matrix in the space provided below, one row on each line. e.g. the 3x3 identity matrix should be entered as *1 point*

1 0 0
0 1 0
0 0 1

- **Do not have any** space at the start of the line
- Have **only one space** at between entries
- **Do not have any space after the last entry** in each row

The G matrix entries should be the numerical values in millisiemens(mS) and should be rounded to **one decimal place**. i.e.

If your answer is 1.5mS, write 1.5

If your answer is -5mS, write -5

1.5 -1 0
-1 2.5 -1
-2 1 2

$$\begin{array}{c}\text{node 1}\\ \text{node 2}\\ \text{node 3}\end{array}\begin{bmatrix}1.5mS & -1mS & 0\\ -1mS & 2.5mS & -1mS\\ -2mS & +1mS & -2mS\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}=\begin{bmatrix}1mA\\ -2mA\\ 3mA\end{bmatrix}$$

$0 \quad -1mS \quad -2mS$

$-2mS \quad +2mS$

$(v_2-v_1)\,1mS$

$2mS(v_2-v_1)$

Now, we have again the same circuit different values of current sources and also we have a current control current source instead of a voltage controlled current source as in the previous problem. Now, this is connected to node 3, so it will only effect the equation for node 3, we can write the once for nodes 1 and 2 as before by looking at the sum of conductance's and conductance's connected to the other nodes, the circuit is the same. So, the first two rows will remain the same.

Now, here if you just look at the conductance's and ignore the control source we would have these entries 0 minus 1 Millisiemen and minus 2 Millisiemen, because of this control source this is two times I x and I x is in this direction. So, I x itself will be V 2 minus V 1 times 1 Millisiemens. So, this becomes 2 Millisiemens times V 2 minus V 1, so 2 Millisiemens get added to the second column corresponding to be 2 and minus 2 Millisiemens to the first column corresponding to V 1.

So, we have minus 2 Millisiemens and plus 2 Millisiemens which are the contributions of the controls source. So, the third row will be minus 2 Millisiemens plus 1 Millisiemens and minus 2 Millisiemens and this times V 1, V 2, V 3 will be equal to the current sources being pushed in which are one milli amp minus 2 milli amp and 3 milli amp and of course, the nodal analysis matrix this corresponds to whatever is inside the...

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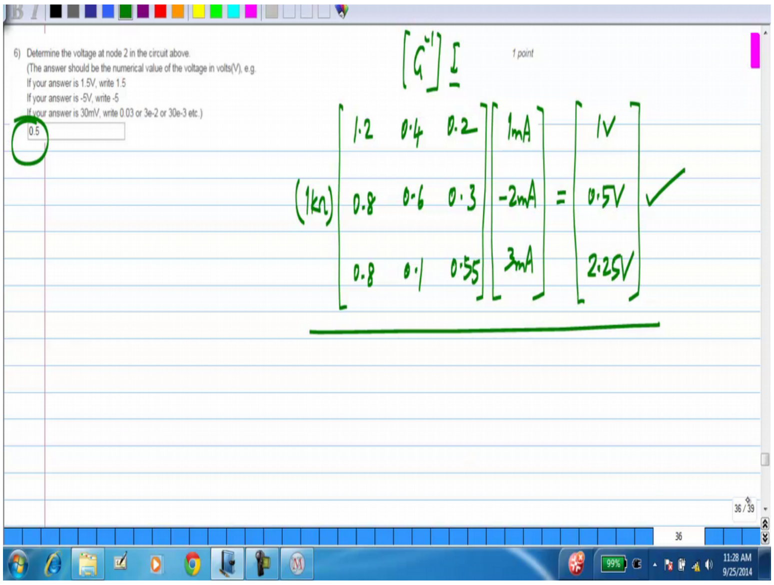

6) Determine the voltage at node 2 in the circuit above.
(The answer should be the numerical value of the voltage in volts(V), e.g.
If your answer is 1.5V, write 1.5
If your answer is -5V, write -5
If your answer is 30mV, write 0.03 or 3e-2 or 30e-3 etc.)

0.5

1 point

$$[G^{-1}]\,\underline{I}$$

$$(1k\Omega)\begin{bmatrix} 1.2 & 0.4 & 0.2 \\ 0.8 & 0.6 & 0.3 \\ 0.8 & 0.1 & 0.55 \end{bmatrix}\begin{bmatrix} 1mA \\ -2mA \\ 3mA \end{bmatrix} = \begin{bmatrix} 1V \\ 0.5V \\ 2.25V \end{bmatrix} \checkmark$$

This question all we have to do is to invert the matrix and solve for it, in this case you are asked for the voltage at the second node, node 2. But of course, if you solve for that unknown vector we will have all the quantities you want we compute the inverse of the matrix it turns out of this of course, with units of kilo ohms I will put it on this side 1 kilo ohms times is whole thing and the current source vector is 1 milli amp minus 2 milli amp and 3 milli amp and this gives you transfer to be 1 volt 0.5 volts and 2.25 volts, you are ask for the voltage and node 2 which is that one, so answer is 0.5.

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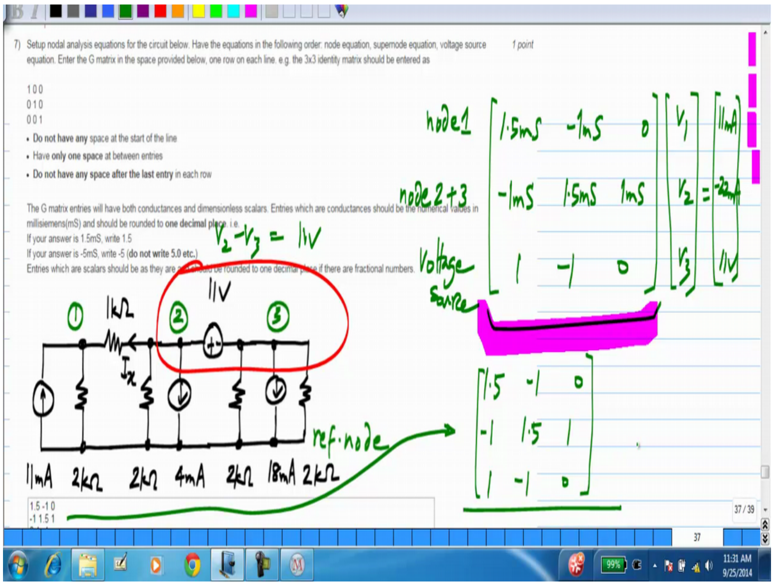

7) Setup nodal analysis equations for the circuit below. Have the equations in the following order: node equation, supernode equation, voltage source equation. Enter the G matrix in the space provided below, one row on each line. e.g. the 3x3 identity matrix should be entered as 1 point

```
1 0 0
0 1 0
0 0 1
```

- **Do not have any** space at the start of the line
- Have **only one space** at between entries
- **Do not have any space after the last entry** in each row

The G matrix entries will have both conductances and dimensionless scalars. Entries which are conductances should be the numerical values in millisiemens(mS) and should be rounded to **one decimal place**. i.e.
If your answer is 1.5mS, write 1.5
If your answer is -5mS, write -5 (**do not write 5.0 etc.**)
Entries which are scalars should be as they are and should be rounded to one decimal place if there are fractional numbers.

```
1.5 -1 0
-1 1.5 1
```

Here, we have again a circuit with an independent voltage source. So, this means that we have to use the super node, we have to combine nodes 2 and 2 into a super node. So, for node 1 we have the normal stuff which is 1.5 Millisiemens minus 1 Millisiemens and 0 the total conductance for node 2 is 1.5, the tool conductance from node 3 is 1, the total conductance at node 2 is 1.5 Millisiemens at node 3 it is 1 Millisiemens. We have only one conductance from the super node to some other nodes that is this 1 Millisiemens conductance, so we have minus 1 Millisiemen over there.

On the right hand side of course, we will have current source being pushed in to the first node which is 11 milli amperes and the second one it should contain the total current being pushed into the super node, which is minus 4 minus 18 minus 22 milli amperes. And finally, we have the equation for the voltage source which is that V 2 minus V 3 equals 11 volts. So, 1 minus 1 0 and here we will have 11 volts, observe that this matrix no longer has all entries is conductance's. So, it has some high brides stuff these are in Millisiemens and these are dimensional less.

Similarly, the source vector has currents and voltages, now in this case you are asked for the, this matrix in a particular format, which is that the conductance's should been Millisiemens and the dimension less quantity should be ask there. So, the answer is 1.5 minus 1 0 minus 1 1.5 1 1 minus 1 0. So, this is what you should have over there.

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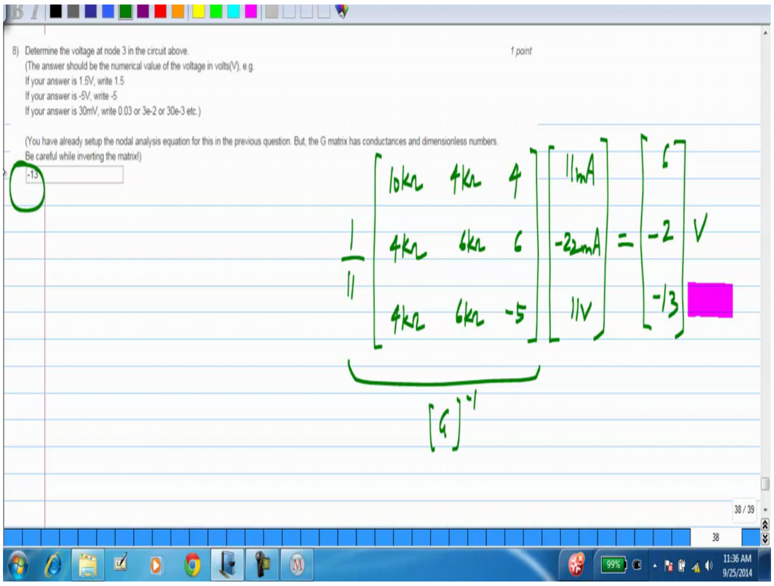

8) Determine the voltage at node 3 in the circuit above.

(The answer should be the numerical value of the voltage in volts(V), e.g.

If your answer is 1.5V, write 1.5

If your answer is -5V, write -5

If your answer is 30mV, write 0.03 or 3e-2 or 30e-3 etc.)

1 point

(You have already setup the nodal analysis equation for this in the previous question. But, the G matrix has conductances and dimensionless numbers. Be careful while inverting the matrix!)

-13

$$\underbrace{\frac{1}{11}\begin{bmatrix} 10k\Omega & 4k\Omega & 4 \\ 4k\Omega & 6k\Omega & 6 \\ 4k\Omega & 6k\Omega & -5 \end{bmatrix}}_{[G]^{-1}} \begin{bmatrix} 11mA \\ -22mA \\ 11V \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ -13 \end{bmatrix} V$$

This is based on the solution to the previous circuit, the inverse of the G matrix transfer to be 1 by 11 times 10 kilo ohm 4 kilo ohm 4 4 kilo ohm 6 kilo ohm 6 4 kilo ohm 6 kilo ohm and minus 5, this is G inverse and because the G matrix had conductance's and dimensional these entries, the G inverse also has the same and be very careful while computing inverses of these matrices to keep the units and numbers consistent. And we had the source vector which is 11 milli amp minus 22 milli amp and 11 volt and you can calculate all the quantities, this transfer to be 6 minus 2 and minus 13 volts your ask for voltage and node 3 which is that one, so the answer is minus 13.

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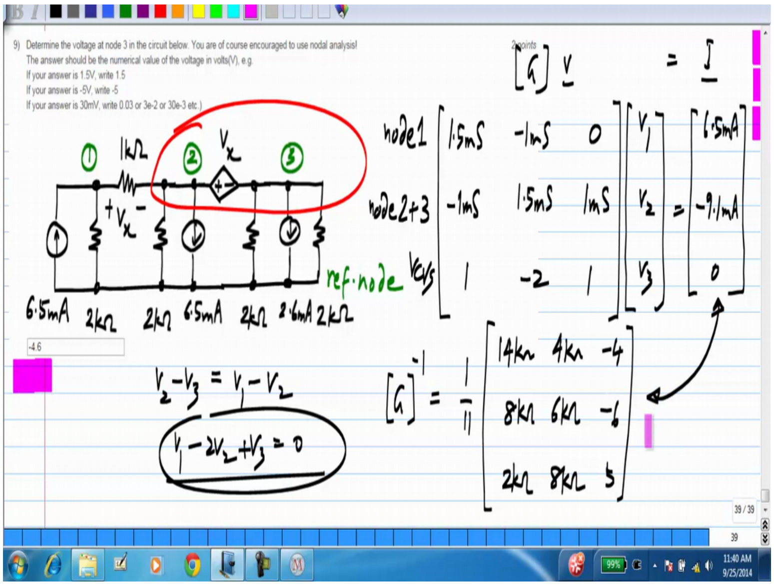

9) Determine the voltage at node 3 in the circuit below. You are of course encouraged to use nodal analysis!
The answer should be the numerical value of the voltage in volts(V), e.g.
If your answer is 1.5V, write 1.5
If your answer is -5V, write -5
If your answer is 30mV, write 0.03 or 3e-2 or 30e-3 etc.)

$$[G]\,\underline{v} = \underline{I}$$

$$\begin{array}{l} \text{node1} \\ \text{node2+3} \\ \text{Vcvs} \end{array} \begin{bmatrix} 1.5mS & -1mS & 0 \\ -1mS & 1.5mS & 1mS \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 6.5mA \\ -9.1mA \\ 0 \end{bmatrix}$$

$$V_2 - V_3 = V_1 - V_2$$

$$\boxed{V_1 - 2V_2 + V_3 = 0}$$

$$[G]^{-1} = \frac{1}{11} \begin{bmatrix} 14k\Omega & 4k\Omega & -4 \\ 8k\Omega & 6k\Omega & -6 \\ 2k\Omega & 8k\Omega & 5 \end{bmatrix}$$

We come to the last problem, we have a voltage control voltage source in this and you are simply ask to solve for this and give the value of voltage at node 3 of course, you can do this in many ways super position or nodal analysis, in this exercise it is implied that you will do it by nodal analysis, but of course, you can get the same answer by and doing it in any other way. I will show the nodal analysis set up again.

Because, we have a voltage source, we have to combine nodes 2 and 3 into super node and the set up looks like this 1.5 Millisiemens minus 1 Millisiemens and 0 that is for node 1 which is not changed at all and for this super node consisting of nodes 2 and 3, you have the total conductance add node 2 to be 1.5 Millisiemens node 3 to be 1 Millisiemens and conductance's from the super node to other nodes, these only one which is minus 1 Millisiemens from node 1.

Finally, we have the voltage control voltage source equation, which says that the voltage between node 2 and 3, this is where we have connected the control voltage source equals V x, which is V 1 minus V 2. So, if you simplify this we will get V 1 minus 2 V 2 plus V 3 to be 0, so you have 1 minus 2 and 1 and we have the unknown vector and the source vector over here 6.5 milli amps, the total current being pushed into the super node which is 6.5 plus 2.6 minus 9.1 milli amp.

Finally, the last one is far the voltage control voltage source equation that we will just be 0 that is the right hand side from here. So, as usual we have G matrix times the unknown vector being equal to the source vector and G inverse in this case transfer to be, this is G inverse and multiplying with source vector we can get all the node voltages and in this case you are ask for the voltage at node 3 and it will turn out that will be minus 4.6 volts.