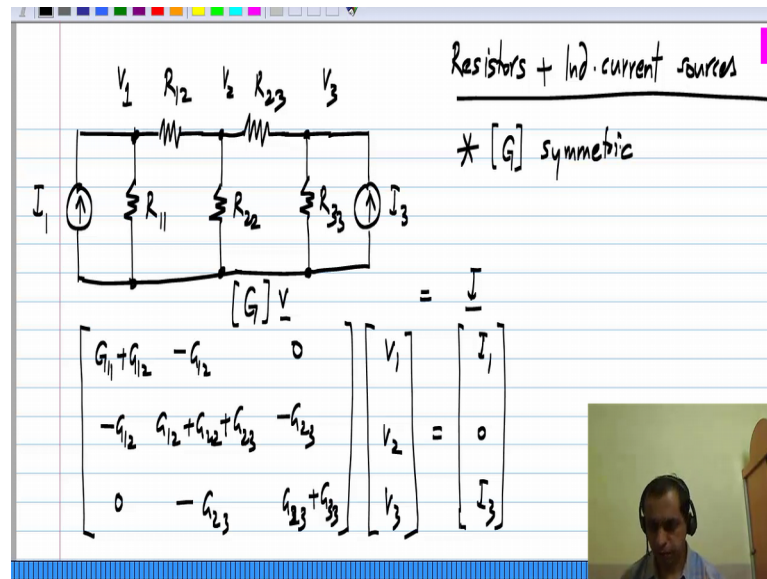


**Basic Electrical Circuits**  
**Dr Nagendra Krishnapura**  
**Department of Electrical Engineering**  
**Indian Institute of Technology Madras**

**Lecture – 55**

We have studied nodal analysis and seen how to use it in different situations.

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Here is a summary of nodal analysis and its equations under different circumstances. I will go through all the cases for which we carried out nodal analysis. So, I will show the circuit, I shown here and also I have written down the nodal analysis equations in matrix form. I am not going to go through the derivation, we have already done that, but what I suggest you do is for each of these circuits, you pause the video, you look at the circuit, write down the nodal analysis equations and compare it to what I have given here. So, that will give you some practice in carrying out nodal analysis for all situations.

The first case we took was the simplest one for nodal analysis; this was when we had only resistors and independent current sources in the circuit. Here, I have the circuit we studied earlier, now in this case when we have only resistors and independent current sources. We have set up the equations as usual as G matrix times the unknown vector of voltages being equal to the vector of sources, the G matrix is symmetric. So, that is the most important feature and by solving this equation basically by inverting the G matrix, you can find all the node voltages and from there, you find everything else.

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+ Ind. Voltage sources

\* Form a supernode  
\*  $[G]$  asymmetric

$$[G] \underline{V} = \underline{I}$$

Supernode

$$\begin{bmatrix} G_{11} & G_{22} + G_{23} & -G_{23} \\ 1 & -1 & 0 \\ 0 & -G_{23} & G_{23} + G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ V_0 \\ I_3 \end{bmatrix}$$

Voltage Source

Next we also included independent voltage sources, the complication in this case is that you do not know what current flows through the voltage source, that is you cannot relate the current through the voltage source to the voltage of the voltage source, it has to be determined by other constraints from the rest of the circuit. So, we bypass this problem by forming a super node, if a voltage source is connected two nodes, we collect those two nodes into a super node and write a single nodal equation for the whole super node.

We have lost one of the equations when we formed the super node, but we have the constraint of the voltage source. So, we still have three equations in three variables and we can solve for the node voltages. The first one is for the super node, the second one is for the voltage source. And in this case, form a super node and we have the equation for the voltage source and finally, these G matrix, where this is G, this is the unknown vector V this one equals the source vector I. Now, the source vector consists of both independent voltage and current sources G matrix is asymmetric.

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The diagram shows a circuit with three nodes:  $V_1$ ,  $V_2$ , and  $V_3$ . Node 1 is the reference node. A current source  $I_1$  is connected between node 1 and node 2. A resistor  $R_{11}$  is also connected between node 1 and node 2. A VCCS is connected between node 2 and node 3, with its current being  $G_m V_x$ , where  $V_x$  is the voltage across  $R_{22}$ . A resistor  $R_{22}$  is connected between node 2 and node 3. A resistor  $R_{23}$  is connected between node 2 and node 3. A current source  $I_3$  is connected between node 3 and the reference node. The nodal analysis matrix is given as:

$$\begin{bmatrix} G_{11} & G_m & -G_m \\ 0 & G_{22} + G_{23} - G_m & -G_{23} + G_m \\ 0 & -G_{23} & G_{23} + G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix}$$

Labels in the diagram include:  $+ VCCS$ ,  $* [G]: \text{asymmetric}$ , and node voltages  $V_1, V_2, V_3$ .

Next we look at the case, where we had a voltage controlled current source. Now, in this case there is no problem writing KCL equations at all the nodes. The only thing is that, the G matrix becomes asymmetric, because the current through the voltage controlled current source depends on voltages elsewhere in the circuit. So, you can see for this example that the matrix is asymmetric.

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The diagram shows a circuit with three nodes:  $V_1$ ,  $V_2$ , and  $V_3$ . Node 1 is the reference node. A current source  $I_1$  is connected between node 1 and node 2. A resistor  $R_{11}$  is also connected between node 1 and node 2. A VCVS is connected between node 2 and node 3, with its voltage being  $kV_x$ , where  $V_x$  is the voltage across  $R_{22}$ . A resistor  $R_{22}$  is connected between node 2 and node 3. A resistor  $R_{23}$  is connected between node 2 and node 3. A current source  $I_3$  is connected between node 3 and the reference node. The nodal analysis matrix is given as:

$$\begin{bmatrix} G_{11} & G_{22} + G_{23} & -G_{23} \\ 1 & -k & k \\ 0 & -G_{23} & G_{23} + G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix}$$

Labels in the diagram include:  $+VCVS$ ,  $* [K]: \text{asymmetric}$ , and node voltages  $V_1, V_2, V_3$ . Red annotations in the matrix label the first row as "Super node" and the second row as "Voltage source".

Next we look at including voltage controlled voltage sources into a circuit and carrying out nodal analysis. Now, when we have a voltage controlled voltage source, just like the independent voltage source we have to form a super node and we have to add the equation for the voltage source, this is the controlled source. So, the right hand will be 0

and all the terms appear on the left hand side and again the G matrix is asymmetric.

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+ CCVS

\* [G]: Asymmetric

$$\begin{bmatrix} G_{11} & G_{22} + G_{23} & -G_{23} \\ 1 & -1 - G_{23} R_m & G_{23} R_m \\ 0 & -G_{23} & G_{23} + G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix}$$

Super node

CCVS

Next we add a current controlled voltage source. So, again as with any voltage source we have to form a super node and that equation appears there and the next one is the equation for the current controlled voltage source and as usual, the G matrix is asymmetric and also, because it is a controlled source the right hand side is 0.

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+ CCCS

\* [G]: asymmetric

$$\begin{bmatrix} G_{11} & k G_{23} & -k G_{23} \\ 0 & G_{22} + G_{23} - k G_{23} & -G_{23} + k G_{23} \\ 0 & -G_{23} & G_{23} + G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix}$$

Finally, we look at the last of the controlled sources, which is the current controlled current source. And in this case, we can write KCL at all the nodes and that is what we have done and controlling current is the current through some resistor and the controlling

current has to be expressed in terms of the voltages across the resistor.

So, again we get a G matrix that is asymmetric, I mean both of these cases when we had a current controlled voltage source or a current controlled current source, we assume that the controlling current is flowing through a resistor. So, the controlling current can be expressed in terms of the voltages across the resistor. If the controlling current were flowing through a voltage source, things would get more complicated, but we are not considering that case here.

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