

**Basic Electrical Circuits**  
**Dr Nagendra Krishnapura**  
**Department of Electrical Engineering**  
**Indian Institute of Technology Madras**

**Lecturer - 53**  
**Nodal Analysis with CCVS**

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Nodal analysis with CCVS \* Form a supernode & write KCL eq.

The circuit diagram shows three nodes: Node 1 (top left), Node 2 (top middle), and Node 3 (top right). Node 1 is connected to ground through a current source  $I_1$  and a resistor  $R_{11}$ . Node 2 is connected to Node 1 through a current-controlled voltage source (CCVS) with value  $R_m I_x$  and to ground through a resistor  $R_{22}$ . Node 3 is connected to Node 2 through a resistor  $R_{23}$  and to ground through a resistor  $R_{33}$  and a current source  $I_3$ . The current  $I_x$  is defined as the current flowing from Node 2 to Node 3 through  $R_{23}$ .

Equations shown on the slide:

$$V_1 - V_2 = R_m I_x$$

$$I_x = \frac{V_2 - V_3}{R_{23}} = (V_2 - V_3) G_{23}$$

$$V_1 + V_2 (-1 - G_{23} R_m) + V_3 (G_{23} R_m) = 0$$

Now, we will see what happens in case of current controlled sources; first will considered current controlled voltage source. So, let me say take this circuit which has a current controlled voltage source. Again you will recognize the similarity of this circuit to the circuit I considered for the independent voltage source and the voltage controlled voltage source. And in this case, the current controlled voltage source is connected between nodes 1 and 2; and its values given by some  $R_m$  - a proportionally constant times  $I_x$ , and  $I_x$  is defined that way. So, now, what we do, as usual when you have a voltage source the problem is that you do not know the current through it meaning, you cannot write the expression for the current as a function of the voltage across the voltage source. The voltage across the voltage source is defined independently of whatever current is flowing through it.

So, we eliminate those currents all together by forming a super node as usual when we form a super node and write down the Kirchhoff's current law the entire super node. So, as you know any completely close surface, whether it contains one node oR many nodes will have the algebraic sum of all the currents flowing out of it to be equal to zero. When

this case, when you write it for the super node any current flowing between the nodes will not appear because that current is not crossing this surface. So, following the usual procedure, you form a super node and write KCL equation.

Now, because you combined two nodes into a super node, you have lost one equation, but you have definitions of the voltage source. So,  $V_1 - V_2 = R_m I_x$ ;  $I_x$  itself is some unknown, and  $I_x$  is the current flowing through this resistor. We know that  $I_x$  is  $V_2 - V_3$  divided by  $R_{23}$  which of course is same as  $V_2 - V_3$  times  $G_{23}$ . So, I will put that in that equation and write  $V_1 - V_2$  to be  $R_m$  times  $G_{23}$  times  $V_2 - V_3$ . Now as with the voltage control voltage source, the right hand side also consists of variables. So, we take them to the left hand side and write  $V_1 + V_2$  times  $-1 - G_{23} R_m + V_3$  times  $G_{23} R_m$  to be equal to 0. So, this is how we write the equation and you see that it has actually come out quite similar to the case with the voltage control voltage source the only difference is you have to go through an extra step to identify, what is one the right hand side, because the current is unknown and the primary variables and nodal analysis are the node voltages. So, you have to write the current in terms of node voltages.

So, when the controlling current is flowing through a resistor, but you have a current controlled voltage source or a voltage control voltage source, you analyze it similarly. So, finally, you write the control voltage source in terms of these voltages  $V_2$  and  $V_3$ , where  $I_x$  flowing in the resistor between  $V_2$  and  $V_3$ . So, that is what we will have for a current controlled voltage source. Now there is a particular case which I have not considered, let us say that we had slightly different circuit, and  $I_x$  was not over there, but  $I_x$  was through another voltage source. It could be a dependent voltage source or it could be an independent voltage source, what I am saying is we have a current controlled voltage source over there, and the voltage depends on a particular current that current itself is not current through a resistor, but current through a voltage source; whether it is controlled or independent.

Now there is a problem, because you do not know what the current through the voltage sources. And earlier we formed a super node and ignore that current all together, but here that current is controlling another voltage source. So, this case is a bit more complicated and I would not deal with that here. You can of course, always solve for the circuit in more ad hoc way and there are systematic ways of handling this also by defining auxiliary variables, which is known as modified nodal analysis we will not be dealing

with that here. So, as for as the scope of this course is concerned will be considering only currents through resistors as controlling currents, we would not consider currents through voltage source as controlling currents, because in that case nodal analysis cannot be used in the form that I have described.