Basic Electrical Circuits
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Lecture – 51
Nodal Analysis with VCCS

Till now we have studied nodal analysis with both types of independent sources for current source it is strait forward, for voltage sources we have to define a super node and go ahead with the analysis. Now we will look at how to formulate nodal analysis equations when we have controlled sources. The first type of control source, we will consider will be voltage control current source. In fact, this will be look similar to resistor; a resistor also can be thought of as voltage control current source with the controlling nodes and the control nodes being shorter together. We have seen this earlier when we synthesized resistor using a voltage controlled current source, but some features of the conductance matrix will be different from what we had when we had only resistors. So, we will look at those things.

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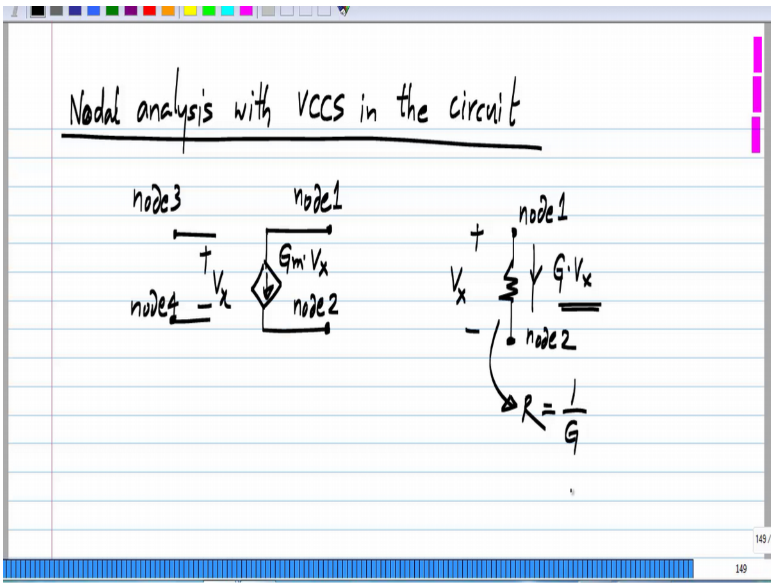

A voltage current source, it is connected between some nodes that let me call these node 1 and node 2, and the controlling nodes as node 3 and node 4. If there is V x between them, the current between node 1 and 2 two will be G m V x. Now when I said it similar to resistor this is what meant if a resistance connected between node 1 and node 2, and the voltage difference between node 1 and node 2 is a V x the current through the resistor will be G times V x, where G is the conductance of the resistor; the resistance R and the conductance is G.

So, you can see that between node 1 and node 2 a certain current is flowing that propositional to V x; in case of the resistor V x is the voltage between nodes 1 and 2; in case of a control source V x is the voltage elsewhere in the circuit.

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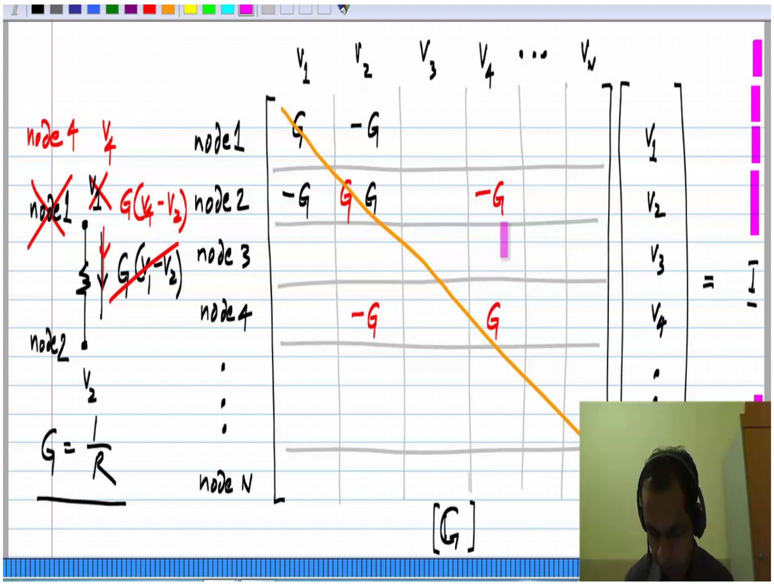

What I have put down here is the general structure of the nodal analysis equations for a resistor. I will filling the entries what I have is that the equation as you all know is conductance matrix G times the unknown vector V equals the source vector I. Now I have shown node one, node two etcetera and here corresponding to each row you know that each rows of this corresponds to one of the equations. The first row corresponds to Kirchhoff's current law node one; the second one to Kirchhoff's current law node two and so on up to node N, there are total of N plus one nodes in the circuit excluding the reference node, we will have N nodes, but I also shown these columns and you know that the entry in this column multiplies V 1 the entries in the second column multiply V 2 and so on. So, I have shown the V 1, V 2, V 3, V 4 on top just for easy readability.

Now let us see if resistances connected between nodes one and two, what happen it how does it contribute to this conductance matrix. So, there will be a current and the voltage at node one is V 1, the voltage at node two is V 2. So, the current will be g times V 1 minus V 2 where G again is 1 by R, and we saying repeatedly some times its convenient to use resistance in the formulation of equations; sometimes it is more convent used conductance.

Now this current flows from node one to node two, so obviously, entries corresponding to this resistor will appear in this equation corresponding to node 1 and this equation corresponding to node 2. And Secondly, the current itself depends on V 1 and V 2. So, it will appear in this column corresponding to V 1 and this column corresponding to V 2. And we already work this out; you already know how to analysis circuits with resistors and control sources. So, you know these things, I am trying to show in a general way how each resistance of its conductance matrix; current flowing away from node 1 is G times V 1 minus V 2. So, in the equation for node one will have g times V 1 minus G times V 2. Remember this G will multiply that and this minus G will multiply that one as shown of top.

The current flowing away from node 2 is G times V 2 minus V 1. Remember in each node we group the currents which are flowing away from the node. So, the current flowing away from node 2 is G times V 2 minus V 1, and we will have g times V 2 minus G times V 1 of course, in these column and these rows there will be other terms corresponding to other component connected to this node, but I am talking about now the contribution only this particular resistor.

Now let me take a different case, where this was not node one, but let us say node 4, and this would not be V 1, but it would be V 4. And the current of course, would be G times V 4 minus V 1 flowing in that direction - downward direction. So, what could happen in that case, the resistance term would appear in the equations for node 4 over here and node 2 over there. The current flowing away from node four as G times V 4 minus V 1. So, we would have G times V 4 minus G times V 2 current flowing away from node 2 is G time V 2 minus V 4. So, I will have instead of this G, I will show it in red, it will be G time V 2 minus G times V 4. The symbol shown in black and red represent two different cases. What I want to you appreciate now is the fact that in either case, you look at these four black entries there symmetric about the diagonal of the matrix. And you look at these four red entries, there also symmetric about the diagonal of the symmetric. So, in either case, we will have a symmetric matrix as we already seen.

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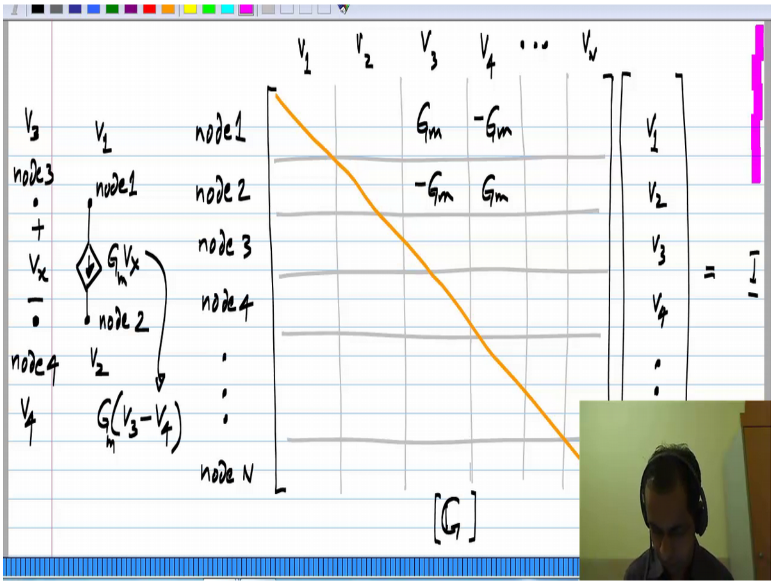

Now let us say what happens when we have a voltage control current source. I have a voltage control current source here. It has a current flowing from node 1 to node 2, but the current itself is proportional to V x which is the voltage between node 3 and node 4. So, this current is nothing but g times V 3 minus V 4; the voltage at node 1 is V 1; the voltage at node 2 is V 2; the voltage at node 3 is V 3; voltage at node 4 is V 4. So, how does this appear in the conductance matrix, first of all the current is flowing from node 1 to node 2, which means that the equation for node 1 and equation for node 2, these two equations contain the term corresponding to the control source G m. So, the current flowing away from node 1 is G m times V 3 minus V 4. So, it will have G m over here and minus G m over there, plus G m multiplies, V 3 minus G m multiplies V 4. The current flowing away from node 2 is the negative of this, so we will have G m times V 4 minus G m times V 3.

Now you see that in many ways this is similar to the resistors, case the two nodes between which the controlled source is connected there equations contain terms corresponding to the control source, but the important difference is that in general these entries are no longer symmetric about the diagonal of the matrix. So, when you have a voltage controlled current source you can go ahead and write the node equations like before except that the conductance matrix will not be symmetric, so that is the difference in general between these two cases. Now I will show a particular example, usually what I do is I show some particular examples and then generalize. In this case I decided to do it other around because the resistor case is so easy I also wanted to show how the voltage controlled current source is similar to resistor, but the only difference is that the entries appear is symmetrically in the matrix.

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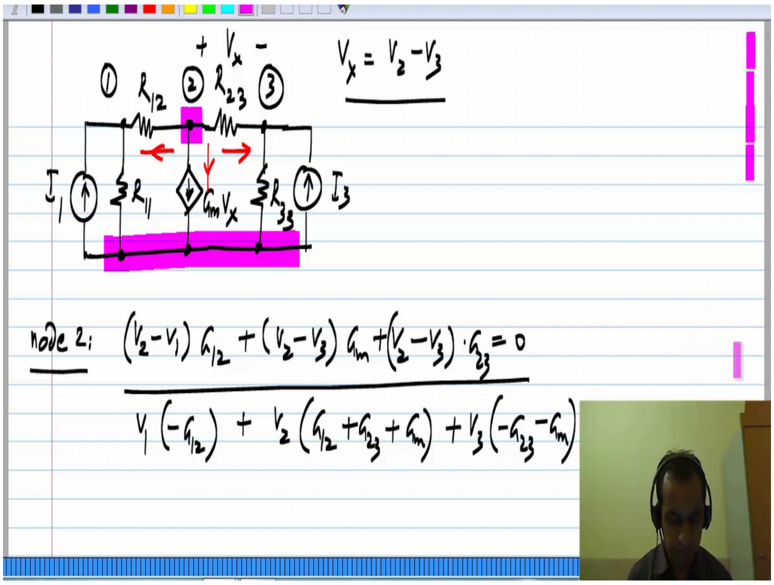

So, let say we take a circuits a similar to what we going to consider all along, but one of the resistors is replaced by a voltage controlled current source this is I 1, I 3, R 1 1, R 3 3 R 1 2, R 2 3. And we have only three nodes in here 1, 2 and 3. And let say that this current source is G m times V x, where V x this one, so V x is basically V 2 minus V 3. So, first of all, it is obvious that this current source appears only in the equation for node 2, because it is connected between node 2 and the reference node. I have chosen the same reference node as a previously done. So, the equation for nodes 1 and 3 will be exactly the same as before.

So, if I write the equation for node 2, what will I get the total current flowing out of this which is the sum of this current that current and that current will be equal to zero, which means that the first one them of is V 2 minus V 1 times G 1 2 and current through G m is basically V 2 minus V 3 times G m, and current through this is basically V 2 minus V 3 times G 2 3, and the whole thing equals 0. And if I group the coefficient of each variable together, I will have V 1 times minus G 1 2 plus V 2 times G 1 2 plus G 2 3 plus gm plus V 3 times minus G 2 3 minus G m equals 0, so this is what I will have. Now let me write down the entire conductance matrix, I will copy this circuit over.

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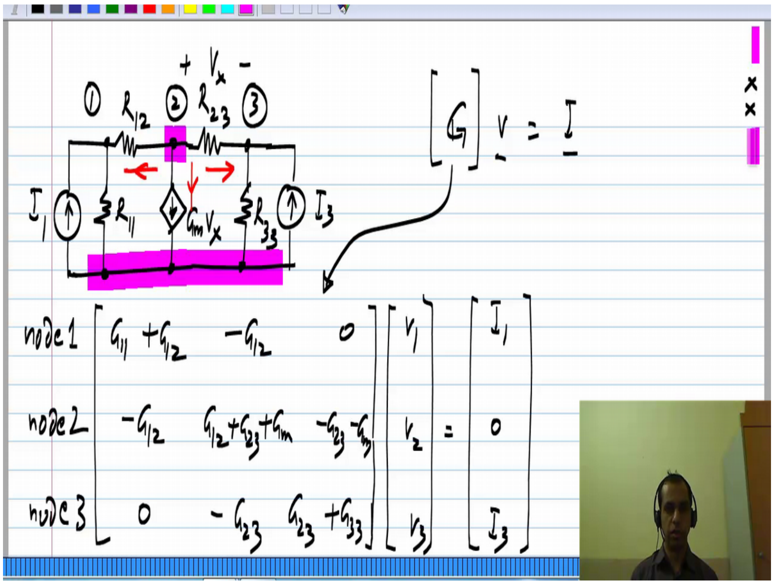

As I said the equations from node 1 and node 3 are exactly the same as before. So I will have G 1 1 plus G 1 2 over there, minus G 1 2 there and 0 for the first row. And for the node 3, G 2 3 plus G 3 3 the total conductance minus G 2 3 over there, and 0 over there; and of course, the special case is node 2 which gives you minus G 1 2 and for this we will have G 1 2 plus G 2 3 plus G m. And the last one will be minus G 2 3 minus G m. So, it is still of the form G times V is I and this G matrix contains the conductance and the trans conductance or the coefficients of voltage controlled current sources in the circuit. And also you can see that the structure is not symmetric, but other than the fact that the G matrix is a symmetric, it is not very different from the case of having only resistors. So, it turns out this is the simplest of the control sources for other control sources, we will have to do things like we did in case of voltage source using super nodes and so on.