Basic Electrical Circuits
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Lecture - 49
Nodal Analysis with Independent Voltage Sources

So far we have studied nodal analysis, which is a systematic way of setting down the equation for the circuit, and using matrix inversion to solve for the node voltage variables in the circuit. Now the limitation is that whatever we have studied so far applies to circuit with independent current sources and resistors. Of course, we can have the other component in the circuit such as independent voltage sources, and also controlled sources. So, what we are going to do now is to adopt nodal analysis for use in those cases. The case of circuits with resistors and only independent current sources is easiest for nodal analysis, but it can be used for others with little bit of tweak.

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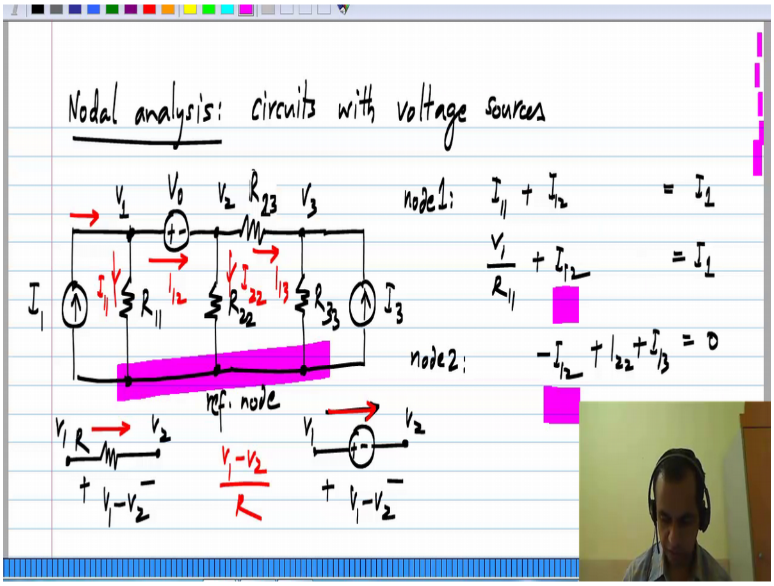

So, let us take this circuit which has a voltage source. It is very similar to the circuit we used earlier except that one of the resistors is replaced by a voltage source. So, what is that we did when we carried out nodal analysis, we took every node; as usual, I will define this to the reference node and these voltages serve V 1, V 2 and V 3. Now we looked at every node and that node we wrote KCL equations showing the currents flowing out of that node being equal to the independent current being injected into that node. Now what is the problem when we have a voltage source, clearly we have to write I 1 1 plus I 1 2 to be equal to the current being pushed here which is I 1. The problem here is that we cannot relate this I 1 to the current through voltage source to voltage of the voltage source. Now if we have a resistor and let say I have V 1 and V 2 on either end of the resistor. So, the voltage across the resistor is V 1 minus V 2, this is enough to tell me what the current is by ohms law that current will be V 1 minus V 2 divided by R where R is the value of the resistor.

On the other hand, if you have a voltage source with V 1 and V 2 at either ends, the voltage of the voltage source of course is V 1 minus V 2 it has to be for these two voltages to be V 1 and V 2. Now there is no relationship between this current here through the voltage source and the voltage across the voltage source. In fact, the definition of the voltage source is that it maintains the given voltage regardless of what current is flowing through it. So, you cannot determine the current through a voltage source from the voltage of the voltage source. The current in a voltage source is determined by whatever it is connected to an external constraint.

So, when I write my equation for node 1, I would write I 1 1 plus I 1 2 equals I 1. I 1 1 of course, is V 1 divided by R 1 1, but I 1 2, I do not know we cannot express it in terms of voltages and element relationship. So, I have to keep I 1 2 as it is; this is still a variable. And same problem occurs with node 2 basically the voltage source is connected between node 1 and node 2. So, again at node 2, I have to write I 2 2 plus I 1 3 minus I 1 2 because I am taking all currents to be flowing away from the node to be equal to zero. So, I have to write minus I 1 2 plus I 2 2 plus I 1 3 to be equal to 0, because no independent current source is injecting into node 2. Again I do not know the value of minus I 1 2; I 2 2 and I 1 three I can relate to the older resistance resistor values. This is the essential problem when we have voltage sources in a circuit and we have to carry out nodal analysis, The current in voltage source cannot be related to the voltage across voltage sources.

So, what is it that we do, so you noticed that again the voltage sources connected between node 1 and node 2. And in node 1, this I 1 2 appears with the positive sign and in node 2 it is appears with negative sign. It does not matter in which direction we take I 1 2, we could have taken this way or leftwards, but in one of the node equations, it is getting pushed into the node and it will be subtracted from the left had side; and for the other node, it will be pulled away from the node it will be flowing away from the node and it will be added to the left hand side. So, clearly, we cannot use this I 1 2 in our equations. So, we have to eliminate it and that can we do by combining the equations for nodes 1 and 2.

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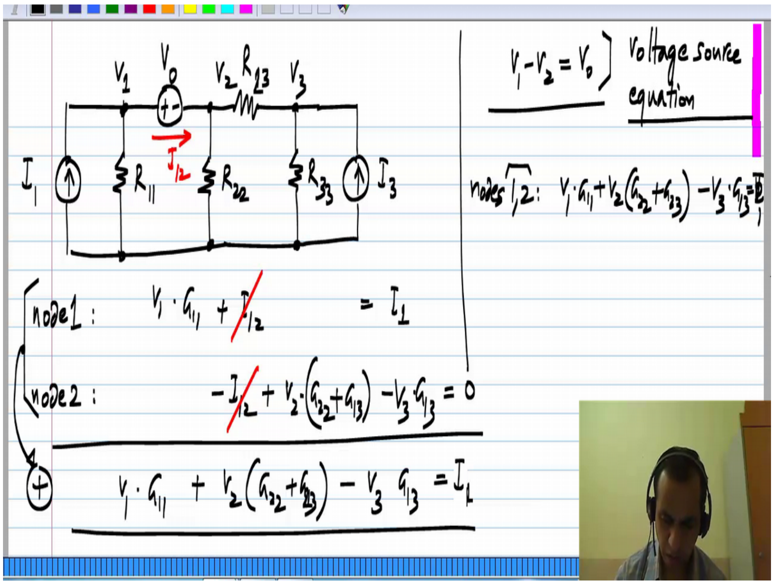

Let us look at that. So, again I will take I 1 2 that way. And for node 1, I would write V 1 times G 1 1 plus I 1 2 equals I 1; for node 2, I would write minus I 1 2 plus the current through R 2 2 which is V 2 times G 2 2 plus the current through R 1 3, which basically gives you V 2 times G 1 3 minus V 3 times G 1 3 equals 0. Now if I sum these two equations, what will I get, this I 1 2 goes away and we will have V 1 times G 1 1 plus V 2 time G 2 2 plus G 2 3 minus V 3 times G 1 3 to be equal to 0. So, we have the combined the equation for nodes 1 and 2 into a single equation. And we do not have this problem of the unknown current in the voltage source, because the unknown current in the voltage source is going from the node 1 to node 2, it appear with the plus sign in the equation for node 1, and minus sign in the equation for node 2, and gets canceled out when I add that 2 equation so that is fine, so we got rid out the unwanted variable.

But now the problem is that we have lost one of the equations. Before I had three variables V 1, V 2, V 3; and I had three equations KCL at node 1, 2 and 3. But now I have combined the equations at nodes 1 and 2 into a single equation. So, instead of two equations I have only one for both nodes 1 and 2, but I do have the constraint of the voltage source so that gives me the missing equation; meaning I lost 1 equation because I added the KCL equation for nodes 1 and 2. On the other hand, I also know that this V 0 will constraint V 1 and V 2; that means, that V 1 minus V 2 equals V 0. So, this voltage source equation, so I have this voltage source equation in place of the missing equations.

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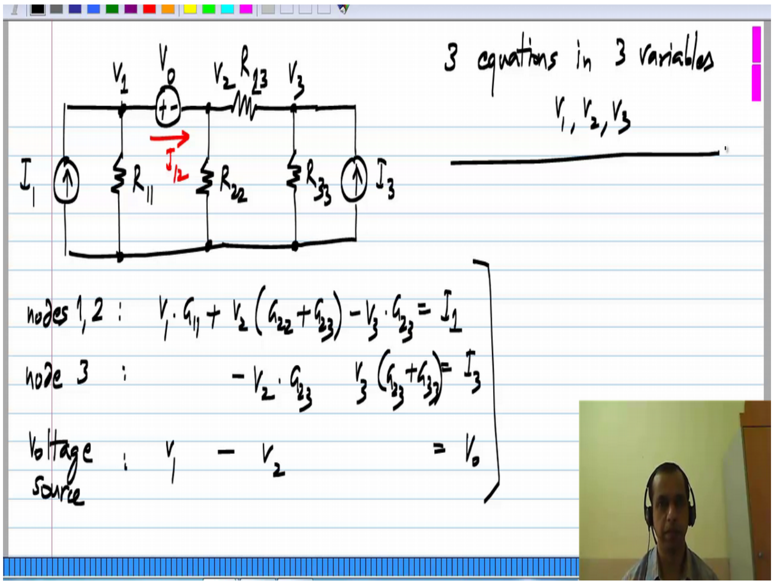

So, what are my three equation, now I have a combined equations for nodes 1 and 2 which are those nodes those are the nodes between which the voltage source is connected. So, I have V 1 times G 1 1 plus V 2 times G 2 2 plus G 2 3 minus V 3 times G 1 3 equals I 1. What are my three equations for nodes 1 and 2 between which the voltage source is connected I have a single equation which is V 1 time G 1 1 plus V 2 times G 2 2 plus G 2 3 minus V 3 times G 2 3 equals I 1; and for node 3, I have the same equation as before. So, V 3 time G 2 3 plus G 3 3 minus V 2 times G 2 3 equals I 3. And finally, I have the voltage source equation itself which is that V 1 minus V 2 equals V not the value of the voltage source. So, again I have three equations in three variables the three variables are V 1, V 2 and V 3 and I can solve for these three variables.