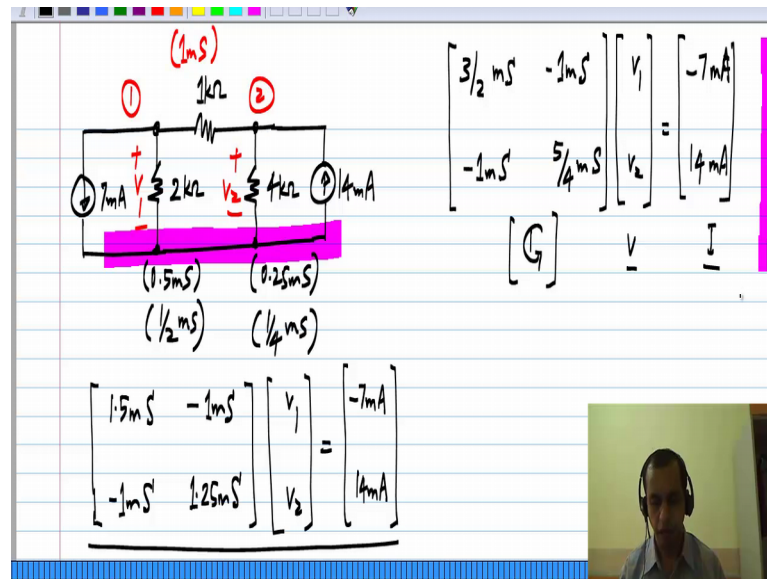


Basic Electrical Circuits
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Lecture – 47

Now, let us do this with the numerical example.

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I have a 7 milli amp current source, a 2 kilo ohm resistor here, a 1 kilo ohm resistor there, a 4 kilo ohm resistor over there and a 14 milli amp current source over there. Now, this is a very simple circuit, you do not need to necessarily go through nodal analysis to find the solution to this. In fact, you can try super position, because it has two sources. I do strongly encourage you to do that and when you solve it in, let us say two or three different ways and find the same solution; you will have a lot of confidence in the answers.

But, we will go through nodal analysis, because that is what we are trying to learn at this point. So, first of all for nodal analysis it is more convenient to take the conductance values than resistance values. So, this 1 kilo ohm resistor here, this is 1 kilo ohm resistor corresponds to a conductance of 1 Millisiemens and this 2 kilo ohm resistor is half a Millisiemen or 1 by 2 Millisiemens and this 4 kilo ohm is 0.25 Millisiemens or 1 4th of Millisiemen.

Now, we need to setup nodal analysis equations and let us choose this node to be the reference node and I will solve for these voltages V_1 and V_2 at nodes 1 and 2 with respect to the reference node. So, what is my set of nodal equations? I have the G matrix, which will be 2 by 2 matrix in this case I have two variables V_1 and V_2 and that will be equal to the vector of independent current sources pushing current into nodes 1 and 2.

So, first of all what is this entry? The very first entry here that is the sum of conductance as at node 1, which is 1 Millisiemen plus half Millisiemen which corresponds to 1.5 Millisiemen. And this one which corresponds to the conductance between node 1 and 2 is minus 1 Millisiemen. Similarly, this one which is also the conductance between nodes 1 and 2 is minus 1 Millisiemen. You know any way it had to be like this, because we know that the conductance matrix is symmetrical and finally, the last one is the sum of conductance as node 2 which is 1 Millisiemen plus 0.25 Millisiemen, which is 1.25 Millisiemen.

On the right side, we have the vector of currents being pushed into nodes 1 and 2, what is being pushed into node 1 is minus 7 milli ampere, because 7 milli ampere is being turn out of it. So, this is minus 7 milli ampere and what is being pushed into node 2 is plus 14 milli ampere and we have to solve this. Now, let me rewrite this fractional entries in a different form, so that it is easier to calculate the inverse. This can be written as 3 by 2 Millisiemen minus 1 Millisiemen minus 1 Millisiemen and plus 5 by 4 Millisiemen.

This is not strictly necessary, you can of course, work with decimal numbers, but here I am trying to do it without having to use calculators and so on. I will put it down this way minus 7 and plus 14 milli amps, so this is G, this is V and this is I.

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The image shows a digital whiteboard with handwritten mathematical equations. At the top, a matrix equation is written:
$$\begin{bmatrix} 3/2 \text{ mS} & -1 \text{ mS} \\ -1 \text{ mS} & 5/4 \text{ mS} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -7 \text{ mA} \\ 14 \text{ mA} \end{bmatrix}$$
 This is followed by the equation
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 10 \text{ k}\Omega & 8 \text{ k}\Omega \\ 8 \text{ k}\Omega & 12 \text{ k}\Omega \end{bmatrix} \begin{bmatrix} -7 \text{ mA} \\ 14 \text{ mA} \end{bmatrix}$$
 Below this, the inverse of the conductance matrix is calculated. A bracketed expression $\left(\frac{3}{2} \cdot \frac{5}{4} - 1 \right)$ is shown, which is then multiplied by $\text{k}\Omega$ to give $\frac{1}{7}$. This is equated to the inverse of the matrix $\begin{bmatrix} 10 \text{ k}\Omega & 8 \text{ k}\Omega \\ 8 \text{ k}\Omega & 12 \text{ k}\Omega \end{bmatrix}$, which is labeled as $\begin{bmatrix} G \end{bmatrix}^{-1}$. A small video inset in the bottom right corner shows a person wearing a headset.

So, now, I have to invert this and that is quite easy. I am assuming you know how to do this, in another lesson I will discuss matrix inversion briefly, but for now I will assume you know how to do this. So, the inverse of this will be 1 over the determinant of this matrix and for inverting a 2 by 2 matrix, we do this and the unit of this of course, will be kilo ohms that is I have not put any units over here, but the units of all the numbers will be kilo ohms, that is why I put kilo ohm outside, that is because the unit of each of these entries is Millisiemens, the inverse of that is kilo ohms or you can put it inside as well.

The unit of the determinant will be Millisiemens square and the unit of each of these will be Millisiemens and Millisiemens divided by Millisiemens square will give you kilo ohm. And if you expand this whole thing you will get 1 by 7 times 10, 8, 8, 12 all kilo ohms. So, this is the inverse of the G matrix. Now, the unknown vector V_1, V_2 is the inverse of the G matrix times the current vector minus 7 milli amp 14 milli amp.

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The image shows a handwritten nodal analysis problem. At the top, a matrix equation is written on lined paper:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 10k\Omega & 8k\Omega \\ 8k\Omega & 12k\Omega \end{bmatrix} \begin{bmatrix} -7mA \\ 4mA \end{bmatrix} = \begin{bmatrix} 6V \\ 16V \end{bmatrix}$$

Below the equation is a circuit diagram. It features a top node and a bottom reference node. A 10mA current source is connected to the top node from the left. A 2kΩ resistor is connected between the top node and the reference node, with a 3mA current arrow pointing downwards. A 1kΩ resistor is connected between the top node and the reference node, with a 10mA current arrow pointing to the left. A 4mA current source is connected to the top node from the right. A 4kΩ resistor is connected between the top node and the reference node, with a 4mA current arrow pointing downwards. The node voltages are labeled as 6V and 16V.

This you can easily see is 6 volts and 16 volts. So, that two node voltages turn out to be 6 volts and 16 volts, like I said you can do it by an independent methods such as super position and verify that this is indeed the case. So, now, we have set up the nodal analysis equations and solve for the unknown vector of node voltages, the rest of it is pretty simple and usually in assignment problems and so on, you will not be asked to calculate every branch voltage and every branch current, but some specific ones which you can calculate using the voltages V_1 and V_2 which you already solved for.

So, what I now have is that this is 6 volts with respect to the reference node, which means that this voltage is 6 volts and similarly this voltage is 16 volts. Now, it is quite common in circuits to 0.1 node and say the voltage at that node is something, let say 6 volts. In those cases it is understood that the other node is some implicit reference node usually the reference node chosen for nodal analysis or what is indicated as ground in the circuit. A voltage is always measured between two points, so you have to be very clear about, what the two points are when you specify a voltage.

Now, the rest of it is very simple, the current source currents we already know and the current through this resistor will be 3 milli ampere, the current through that one will be 16 by 4 which is 4 milli ampere and the current going in that way will be 10 milli ampere and you can quickly check Kirchhoff's current law at each node and it will be valid. 14 milli amps is being driven from here, 4 goes this way and 10 goes that way and out of that, 3 goes this way and 7 goes that way. So, this way you can solve for any variable you want in the circuit using nodal analysis.