

**Basic Electrical Circuits**  
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**Lecture – 46**

Earlier, when I introduce the circuit analysis I had said that complete solution to a circuit means finding all the branch currents and all the branch voltages. Now, what we have done so far is to setup nodal analysis equations and by solving this, we get all the node voltages. So, how do we get the complete solution then?

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Nodal equations for a circuit

$$[G] \underline{V} = \underline{I}$$

$$\underline{V} = [G]^{-1} \underline{I}$$

$\underline{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$ 
 $\underline{I} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix}$

$V_1 \cdot G_{11}, V_3 \cdot G_{33}, (V_1 - V_2) G_{12}, (V_2 - V_3) G_{23}$

So, let us take the nodal equations for some circuit and use the example, which we have been using so far, which is a three node circuit with two current sources and five resistors. And we know the nodal equations for this, so it is G times the vector of voltages V equals the vector of sources I and the vector of voltages V consist of three vectors V 1, V 2, V 3, where all these are voltages of node 1, node 2 and node 3 with respect to the reference node.

This is what we solve for, when we invert the matrix and multiply the vector of current sources, which in this cases this vector I 1, 0, I 3. Now, what did we want to solve for? We wanted to solve for every voltage and every current, but by doing this the important part of the work is already done, then all we have to do is to identify between which nodes each component is connected, that with trivial application of Kirchhoff's voltage law will give us all the voltages. I am going to write that down here, first of all let us

write down the voltages across every element. Across  $I_1$ , all we have to do is to note that  $I_1$  is connected between node 1 and 0.

So, across  $I_1$  the voltage is  $V_1$  and  $R_{11}$  is also connected between the node 1 and 0, so across  $R_{11}$  the voltage is  $V_1$ . Similarly, across  $R_{22}$  the voltage is  $V_2$ , across  $R_{33}$  the voltage is  $V_3$  and across  $I_3$  the voltage is  $V_3$ . I mean this looks trivial, but this is just a systematic way of finding out all the branch voltages and which can be applied to large circuits as well. And then, for these components which are connected between two nodes neither of which is the reference node, the voltage across it will be difference between two node voltages  $V_1$  minus  $V_2$ . This is by trivial application of Kirchhoff's voltage law and here, it is  $V_2$  minus  $V_3$ .

So, we can get all the branch voltages easily, once we have the vector of node voltages and then, we have to calculate all the currents, now we use the voltages and  $I-V$  relationships of the device to find the currents. So, again we will do it for all of these, it is trivial of this case, but what I am telling you is the way to systematically generate the complete solution to the circuit. So, if you take the current  $I_1$  in this direction consistent with a passive sign convention given the sign of this  $V_1$  will have this current to be minus  $I_1$ , because this is the current source and the current through  $R_{11}$  would be  $V_1$  times  $G_{11}$  and so on.

So, the current through  $R_{22}$  would be  $V_2$  times  $G_{22}$ , this is the current through  $R_{22}$ . And similarly, through  $R_{33}$  it would be  $V_3$  times  $G_{33}$  and through these resistors  $R_{12}$  and  $R_{23}$ , it can also be equally easily calculated as  $V_1$  minus  $V_2$  times  $G_{12}$  and  $V_2$  minus  $V_3$  times  $G_{23}$ . So, this is how we can get a complete solution to the circuit.

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\* Set up nodal analysis equations  $[G] \underline{v} = \underline{I}$

\* Solve to obtain node voltages

$$\underline{v} = [G]^{-1} \cdot \underline{I}$$

\* Use the node voltages to obtain all branch voltages

\* Use element I-V relationships and node voltages to obtain all branch currents

Labels in the image:  
- Source vector (pointing to  $\underline{I}$ )  
- unknown vector (pointing to  $\underline{v}$ )  
- Conductance matrix (pointing to  $[G]$ )

So, in summary when you want to solve for every electrical variable in the circuit you setup the equations for nodal analysis  $G$  times  $V$  equals  $I$ , where this is the vector of independent sources this is the conductance matrix and this is the unknown vector consisting of node voltages. And you will solve this to obtain node voltages. So,  $V$  will be the inverse of this matrix times  $I$ , I am showing these square brackets to emphasize that it is a matrix.

Then, use the node voltages to obtain all branch voltages and finally, use element  $I-V$  relationships and node voltages to obtain all branch currents. So, at the end of this process you will have all branch voltages and all branch currents of course, the hard part consist of these two steps. So, that is what I have emphasized, so when we say nodal analysis that is, what is meant by nodal analysis although to get all the branch voltages and branch currents you need to do these as well. But, once you have solved for this unknown vector  $V$ , these steps are rather easy.