

Basic Electrical Circuits
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Lecture - 45
How Elements Appear in the Nodal Analysis Formulation

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$$\begin{bmatrix} G_{11} + G_{12} + G_k & -G_{12} & 0 \\ -G_{12} & G_{12} + G_{22} + G_{23} & -G_{23} \\ 0 & -G_{23} & G_{23} + G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix}$$

Now, we will take a circuit and its nodal analysis equations then made changes to the circuit and see what changes occur in the nodal analysis equations. And in this lesson, I would strongly encourage you to a pause the video, after I make the change to the circuit, figure out what change happens to the nodal analysis equation setup by yourself and then compare into what I show you as the answer. This is the better way of learning than simply looking at the answers that I provide. I will take the same circuit that has been used so far. This is I_1 , R_{11} , R_{12} , R_{23} , R_{22} , R_{33} . And this of course is the reference node and this is I_3 ; by now it must be clear why I choose the subscripts the way I did. R_{11} is basically the resistance connected to node 1 and the reference node; R_{12} is between node 1 and 2; R_{13} is between nodes 2 and 3; and similarly R_{22} is from node 2 to ground, and R_{33} is from node 3 to ground and so on, and I_1 is the current source being pushed into node 1, and I_3 is the current being pushed into node 3.

Now, we also have the nodal equation setup for this which is $V_1 \ V_2 \ V_3$ equals $I_1 \ 0 \ I_3$. Now what we will do is we will make various changes to the circuit and see how

this nodal analysis equations setup is affected. As I said earlier please work out the effect of the changes yourselves and compare into what I work out. So, first what I will do is let me add another resistance here R a. So, please work out, what happens to the nodal analysis setup because of this; in this case, it is very obvious, the resistance R a is in parallel with a resistance R 1 1, so the conductance simply add up. And also if you remember what I said earlier the diagonal elements consist of the total conductance connected to node 1, so simply G a will appear here. So, this parallel combination tells you that wherever you had G 1 1 earlier, now you should have G 1 1 plus G a and that is exactly what happens and that is also consistency with the diagonal element being the sum of the total conductance connected to a particular node.

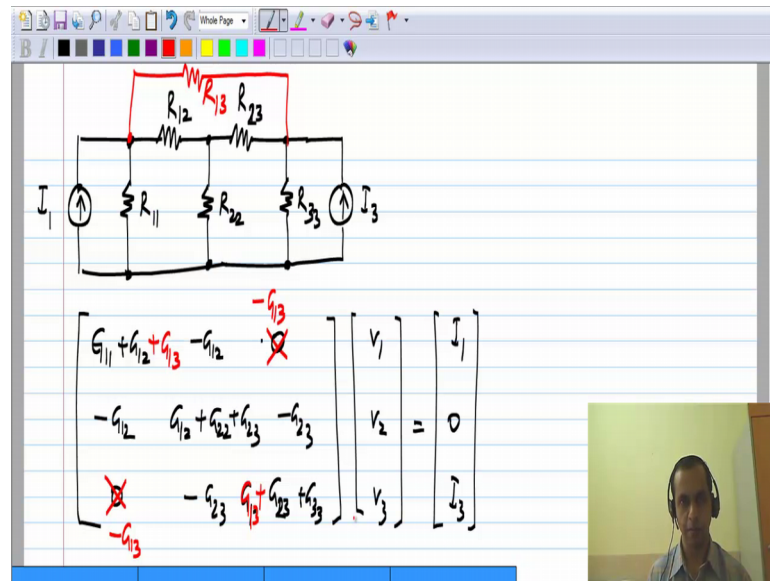
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The slide displays a circuit diagram and its nodal analysis equations. The circuit consists of three nodes. Node 1 is on the left, connected to a current source I_1 pointing upwards. Node 2 is in the middle, and Node 3 is on the right. Resistors R_{12} and R_{23} are connected between nodes 1 and 2, and 2 and 3 respectively. Resistors R_{11} , R_{22} , and R_{33} are connected to nodes 1, 2, and 3 respectively. A current source I_3 is connected to node 3, pointing downwards. A pink highlight is under the bottom wire. Below the circuit is a matrix equation:

$$\begin{bmatrix} G_{11} + g_{12} & -g_{12} & 0 \\ -g_{12} & g_{12} + g_{22} + g_{23} & -g_{23} \\ 0 & -g_{23} & g_{23} + g_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ -I_3 \end{bmatrix}$$

So, now let me remove this. Now let us consider another change. So, what I will do is let me get it out of this current source I 3 and connected it in the opposite direction from what I had earlier. So, please work out what is going to happen to the equations because this we have made changes to the independent source. So, this will change only the right hand side of the equation, and again the answer is pretty clear; this element, which was the total current being pushed into node 3, it was I 3, and now it has become minus I 3 that is all and that is any way petty obvious because we simply reverse the direction current source. So, this becomes minus I 3.

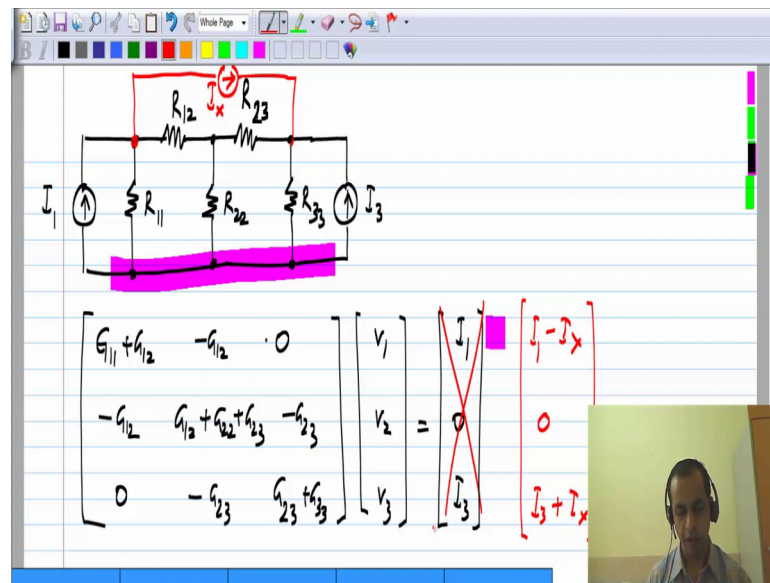
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So, now let us consider another change; let me add a conductance between nodes 1 and 3. So, what does this change, please work it out. Now when I have a conductance; obviously, the total conductance at those nodes to which I add the conductance will change; in this case, nodes 1 and 3 that is where I added the conductance, so the total conductance at these nodes will change. So, the diagonal elements corresponding to these nodes will change. So, we have node 1 here before the total conductance was G_{11} plus G_{12} , now it will also have G_{13} . And this is for node 3, so again earlier we had G_{23} plus G_{33} . Now we will have G_{13} as well so G_{13} plus G_{23} plus G_{33} .

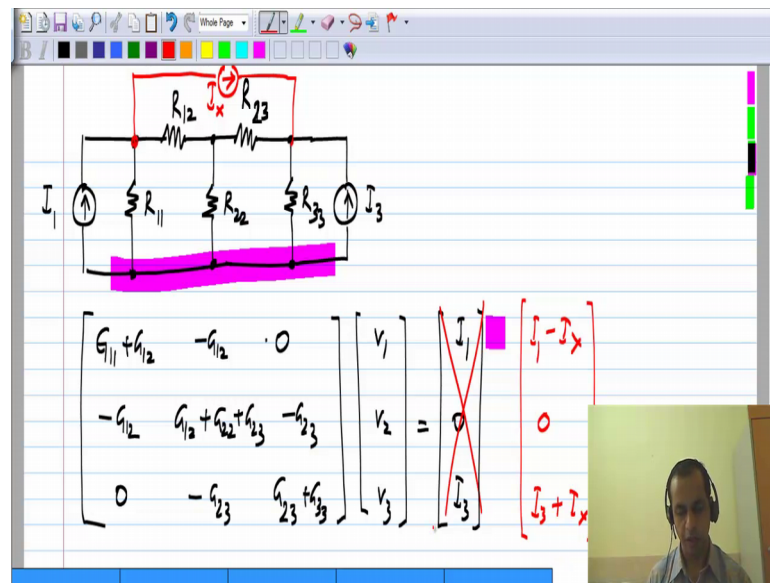
Also another thing that changes is the half diagonal element because this conductance is connected between two nodes neither of which is a reference node. So, this element here which was earlier zero, because no conductance was connected between nodes 1 and 3 now becomes minus G_{13} . And similarly here this becomes minus G_{13} the matrix is symmetric, because we still have a circuit consisting of only conductance and current sources the matrix is symmetric and this is what happens to the nodal analysis setup.

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Now let us look at another change. So, let me add a current source I_x between nodes 1 and 3 in this direction. What changes do you expect from this, I made changes to the independent sources connected to the circuit, so this will change the right hand side of the circuit. Now what is this element, the first element of the source vector, it is the total independent current being pushed into node 1. Now it was originally I_1 , now it will be I_1 minus I_x , because of the polarity of a I_x which is connected. So, instead of I_1 , I will have I_1 minus I_x ; now node 2 has been untouched, so that will still be 0 and node 3, it will become I_3 plus I_x , because I_x if you see it is being pulled from node 1 and being pushed into node 3. So, these are the changes that occur to the nodal analysis setup.

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Let me take one final example, what I will do is let me connect a current source like this I_x between nodes 1 and 2 instead. So, again I_x is being pulled from node 1, so this becomes I_1 minus I_x and I_x is being pushed into node 2, so it becomes plus I_x and I_3 will remain as it is. So, the source vector changes whenever you change the sources which are current sources in the present case. So with this, hopefully the structure of the G matrix and the source vector, they are clear.

Again the solution of course, involves matrix inversion you can solve for it, but the idea of this nodal analysis is to have a systematic way of setting up equation for arbitrator large circuits. So, it is very important to understand the structure, because normally when you start off with circuit analysis you look at the circuit and then may be identify something as the important loop for node 2 start off with or may be its whatever is convenient for you, you start writing the equation is some ad hoc way and finally, when you have enough equations, you simultaneously solve them. But for large circuit this would not work, we have a certain number equations we have to setup and we have to do it systematically. So, this nodal analysis is one way of doing that.