Basic Electrical Circuits
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Lecture - 44
Structure of the Conductance Matrix

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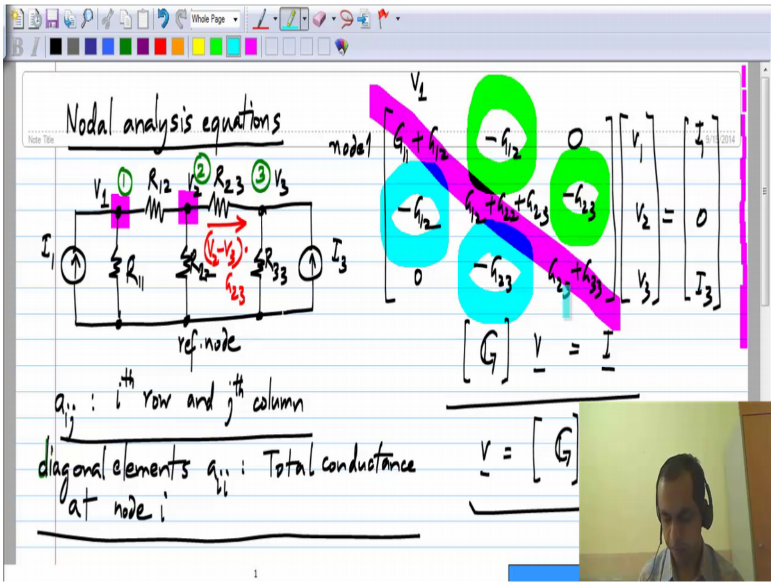

We have looked at how to set up nodal analysis equations. We write down Kirchhoff current law equations at N minus 1 nodes of the circuit; and in each term of the equation, we use voltages which are refer to the reference node of the circuit, and element relationships from the resistors in the circuit. And also we saw how to put it in matrix form. For this particular circuit, we have got the nodal analysis equation to be times V 1, V 2, V 3 where this is the reference node; and this is nodes I have V 1, V 2, V 3 by the respect to the reference node equals the source vector I 1 0 and I 3. And of course, this can be written as the conductance matrix G times the vector of node voltages V equals the vector of independence sources I. So, this is the nodal analysis equations setup and of course, we can solve for V by using matrix inversion with which we are familiar G inverse times I.

Basically, this is saying that we have three simultaneous linear equations, and three variable V 1, V 2, V 3 and we solve for it. And we solve for it, we are basically doing matrix inversion. Now we will briefly look at techniques and matrix inversion later that is only for hand analysis, things that you all ready know. The important part a setting up of the equations, there are many techniques for matrix inversion and especially for a large circuits lot of work has been done to figure out efficient ways of inverting matrix on the computer. So, the important part for us now is to understand the structure of the G matrix and be able to set of the equation for circuits of any size. Of course, for circuits of small sizes such as once with may be 3 or 4 nodes, you should be able to invert the matrix by hand and solve the circuit.

Let us look at the structure of this conductance matrix. First let us focus on the diagonal elements of the matrix. What do you see let me remind you that the first row is node 1 and the first column here corresponds to whatever is multiplying V 1. If you look at the element a 11 of the matrix where use the standard notation a i j corresponds to ith row and j th column, we have G 1 1 plus G 1 1 2. Similarly if we look at a 2 2, that is the second row and second column, we have G 1 2 plus j 2 j 2 3, and finally, a 3 3 is G 2 3 plus G 2 3. You easily notice that the diagonal elements are basically some of conductance connected to particular nodes. The first row correspondence to node 1 and the diagonal element in that element a11 is basically the total conductance connected to node 1, so this is node 1, we have 2 resistance R 1 1 and R 1 2 connected to it. So, the total conductance would be the conductance of this G 1 1 plus the conductance of that G 1 2.

Similarly, if you look at node 2, there are resistors connected G 1 2, G 2 3 and G 2 2. So, all of them are summed up and appear as diagonal element in the second row. And finally, node 3 has G 2 3 and G 3 2 connected to it and the sum of that appear all there. So, the diagonal elements, if you look at a i i that is diagonal elements in the i th row then this has the total conductance at node i so that should be pretty obvious. Now if you look at the half diagonal elements this is element a 1 2, so if you look at element a 1 2 what is that we have node 1 over here and node 2 over there, and we have a resistance R 1 2 connected between them the conductance of that is G 1 2.

So, basically the element a 1 2 of the matrix is nothing but the negative of the conductance connected between node 1 and node 2. So, you can go through the remaining entrees of the matrix and easily verify that is a case. For instance the element a 2 3 is the negative of conductance connected between nodes 2 and 3. And there is a reason for it which is pretty obvious also; when you write Kirchhoff current law equation here and look at the term for the current through this resistance R 2 3, what we have we are writing currents and flowing away from this node. So, current flowing away from this node 2 or 2 3 would be V 2 minus V 3 times G 2 3. So, obviously we will have V 2 times V 2 3 that appears in the diagonal, and then minus V 3 times V 2 3 and this happens for every node.

So, if you look at any half diagonal term, it will have basically negative of the conductance connected to that node and that is exactly what we have. And also another thing that you observe is that matrix is symmetric; clearly we have minus G 1 2 here and minus G 1 2, there minus G 2 3 there and minus G 2 3, and similarly zero and zero this zero is because there is no conductance directly connected between node 1 and node three. So, there is zero conductance connected between them. Now it is also obvious why do is matrix because this is element a12, which should have negative of conductance between nodes 1 and 2, and this is element a 2 1 which should have the negative of conductance between nodes 2 and 1 which is the exactly the same as the conductance between nodes 1 and 2. So, the matrix is symmetric as well.

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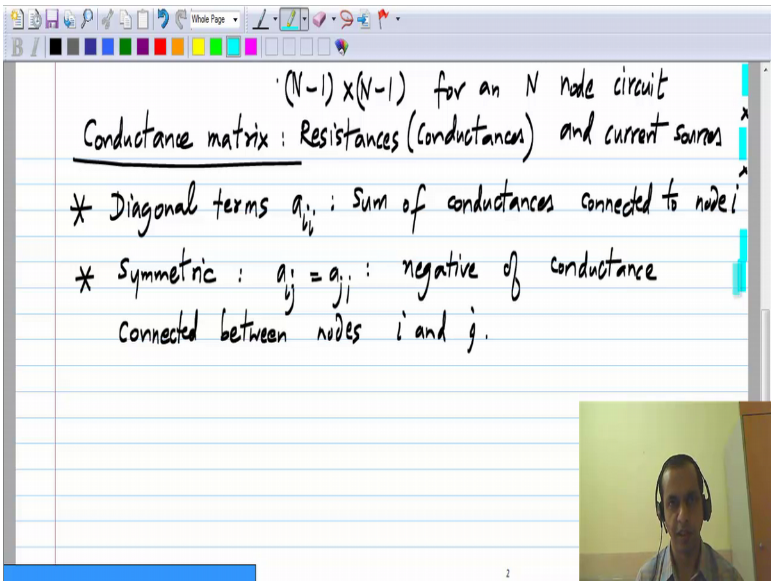

So to just summarize the structure of the conductance matrix, firstly, we will have the diagonal terms a i to be the sum of conductance’s connected to node i, and it is quite clear why this happens, because again we are summing all of the currents flowing out of a let say node 2, now there are three currents here through G 1 2 through G 2 3 and through G 2 3. Now all of them will have V 2 in them the current trough G 1 2 is V 2 minus V 1 times V 1 2; currents V 2 2 is V 2 minus zero times V 2 2 and current through G 2 3 is V 2 minus V 2 3 G 2 3. So, all of them will have V 2. So, the sum of all conductance to connected to a node will appear in the diagonal. And of course, the matrix itself is symmetric and the terms aij or aji which will be equal to each other corresponds to the negative of conductance connected between nodes i and j.

So, this is the structure of the conductance matrix. Let me also add that for an N node circuit, the conductance matrix will be N minus 1 times N minus 1, it is a square matrix. And also these properties, we have written, they are for the specific kind of circuit we have been considering, which consist of only resistances or conductance, and current sources. As we add more elements, we see that some of these properties may be modified. So, we will look at those things later. This structure will be obvious when we consider examples of what happens when we make some changes to the circuit, so that is the structure conductance matrix for a circuit consisting only resistors and current sources. This will become clearer when I take an example circuit and then modify into certain ways and see how the conductance matrix changes.