Basic Electrical Circuits

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Lecture - 43
Setting up Nodal Analysis Equations

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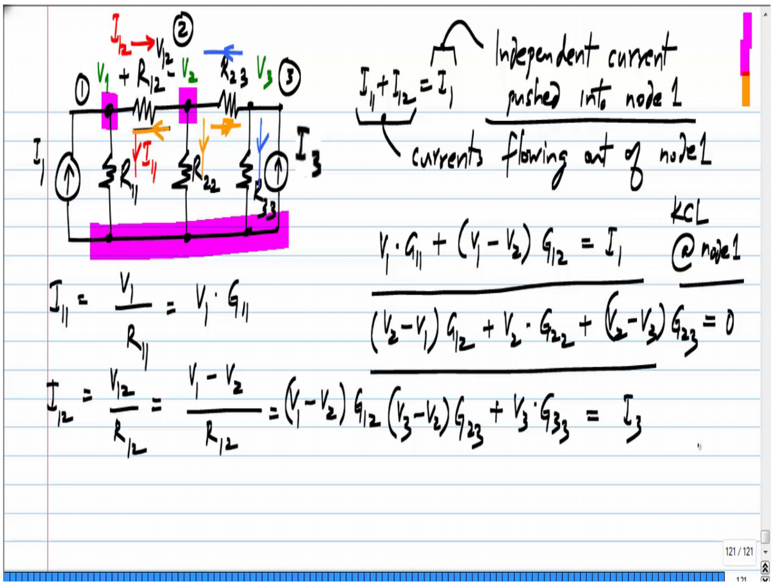

So, let me redraw the circuit. I had we define V 1, V 2 and V 3 with respect to the reference node which is this. So now I have to write I 1 1 plus I 1 2 equals I 1; I 1 1 plus I 1 2 equals I 1. And these are currents flowing out of node 1, this is node 1, this is node 2, this is node 3, and this is the independent current pushed into node 1. Now what is I 1 1, I 1 1 is nothing but V 1 divided by R 1 1. So, I 1 1 is V 1 divided by R 1 1, which is V 1 times G 1 1, so I have to write V 1 times G 1 1

I 1 2 is nothing but the voltage across this resistor if I call the voltage across this resistor V 1 2, it will be V 1 2 by R 1 2, but I will chosen the node voltage as V 1, V 2, V 3 with respect to the reference node as my primary variables, and I will write everything in terms of those variables. So, V 1 2 that is the voltage across this is nothing but V 1 with respect to the reference node minus V 2 with respect to the reference node this is apparent from Kirchhoff’s voltage law. So, this is V 1 minus V 2 by R 1 2 which is V 1 minus V 2 times G 1 2. So, I have V 1 time G 1 1 plus V 1 minus V 2 times G 1 2 and the whole thing equals the independent current source flowing into node 1 and that is basically I 1, so that is my equation for node 1 this is KCL at node 1. So, the equation represents Kirchhoff’s current law and the individual terms in the equations have been written using the element relationships in this case of a resistor, when I write the current here to be V 1 time G 1 1 I have use the element relationship already. And similarly when I write the voltage across R 1 2 to be V 1 minus V 2, I have used KVL and V 1 minus V 2 times G 1 2 is the current flowing in R 1 2. I know this because of the V-I relationship of the resistor. So, this is how I set down the equations.

Now, let us go ahead with the other two nodes. So, let me do it for node 2. So, again my convention is to always fix the current flowing away from node 2. So, I have to take the sum of the current in R 1 2 R 2 2 and R 3 3 in this direction flowing away from the node to be 0, because there is no independent current source connected to this nodes we have only resistor. So, what is the current through R 1 2 in this direction flowing from right to left it will be V 2 minus V 1 divided by R 1 2 V 2 minus V 1 divided by R 1 2 which of course, is V 2 minus V 1 times G 1 2. For nodal analysis, it is lot more convenient used conductance instead of resistances while writing down the equations that is what I am going to use and by definition G 1 2 is 1 by R 1 2 G 1 1 is 1 by R 1 1 and so on. The current through R 2 2 is nothing but V 2 times G 2 2 because V 2 appears across R 2 2. And finally, the current through R 2 3 is V 2 minus V 3 times G 2 3.

So, again we are looking at current flowing away from node 2; please be mindful of the direction of currents it is always current flowing away from particular node equals the independent current source being injected into that node the independent current source being injected into node 2 is 0. So, the sum of all of this is just 0. And finally, for node 3, we have to sum these 2 currents current flowing in to R 2 3 current flowing into R 3 3 sum of those two equals I 2; the current flowing R 2 3 in this direction will be V 3 minus V 2 times G 2 3 the current flowing in R 3 3 is simply V 3 times G 3 3 and it equals the independent current flowing into node 3 which is I 3 .

Now I will rearrange these into a nicer form. What I will do is I will collect terms containing a particular voltage that is V 1, I collect all of the terms containing V ,1 in this case I have V 1 times G 1 1 here and V 1 time G 1 2 over there. So, I will write V 1 times G 1 1 plus G 1 2, and I will take the terms containing V 2, in this case there is only 1 of them. So, I have minus V 2 times G 1 2. And I do not have any terms involving V 3 at all; these currents V 1 times G 1 1 plus G 1 2 minus V 2 times G 1 2 equals current I 1. And similarly I collect terms containing V 1, I have only one of it in the second equation minus V 1 times G 1 2 and terms containing V 2 there are 3 V 2 times G 1 2 V 2 times G 2 2 and V 2 times G 2.

Finally, terms containing V 3 and there is only one of them I have minus V 3 times G 2 3 this whole thing equals zero. And finally, for the last one I will do the same there are no terms with V 1 minus V 2 times G 2 3 plus V 3 times G 2 3 plus G 3 3 this equals I 3 I have 3 equations and I have 3 variables to solve for V 1, V 2 and V 3. And from that I can easily figure out what happens in the rest of the circuit. So, this set up the set of three equations, and for an N node circuit N minus 1 equation, these are known as nodal analysis equations.

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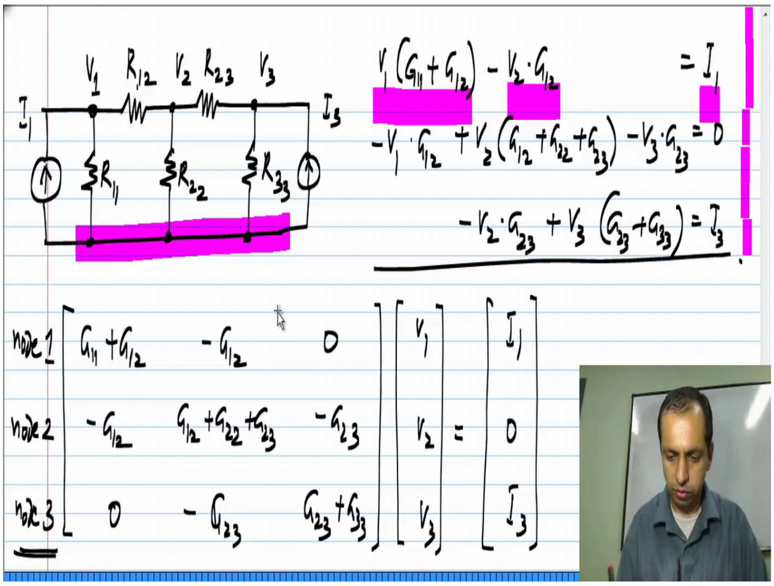

So, for this particular circuit I 1 at R 1 1, R 1 2, R 2 2, R 2 3, R 3 3, and I have got my equations which are in terms of these node voltages V 1, V 2, V 3 with respect to the reference node. So, what did I have at node 1 I had V 1 times G 1 1 plus G 1 2 minus V 2 times G 1 2 equals I 1 at node 2 I had minus V 1 G 1 2 plus V 2 times G 1 2 plus G 2 2 plus G 2 3 minus V 3 times G 2 3 equals zero. And finally, at node 3 I had minus V 2 times G 2 3 plus V 3 times G 2 3 plus G 3 3 equals I 3, and this is usually written down in a matrix form.

For convenience and the matrix has a nice structure which once you identify you will be able to put down right away without going term by term as I did in this particular example. So, what I do is I write this set of linear equations this can be done for any set of linear equations as a matrix G 1 1 plus G 1 2 minus G 1 2 0 minus G 1 2 G 1 2 plus G 2 2 plus G 2 3 minus G 2 3. And finally, minus G 2 3 G 2 3 plus G 3 3 this is a matrix times vector of variables V 1 V 2 V 3 and this will be equal to the vector of the independence sources I 1 zero and I 3, there is nothing new in this all I have done is to written down these equations the set of 3 equations in 3 variables in matrix form. So, I have a 3 by 3 matrix times a variable vector equals the independent source vector.

Clearly the first row corresponds to the first equation G 1 1 plus G 1 2 times V 1 which is what we have here minus G 1 2 times V 2 which is what we have here equals I 1 which is what we have over there. So, I can go term by term and identify each equation. So, the first row corresponds to node 1 the second row to node 2, and third one to node 3. So, the nodal analysis equations, which basically are written down as currents flowing out of a particular nodes in resistances equals the independent current source being injected into that node that I have put down in a matrix form.

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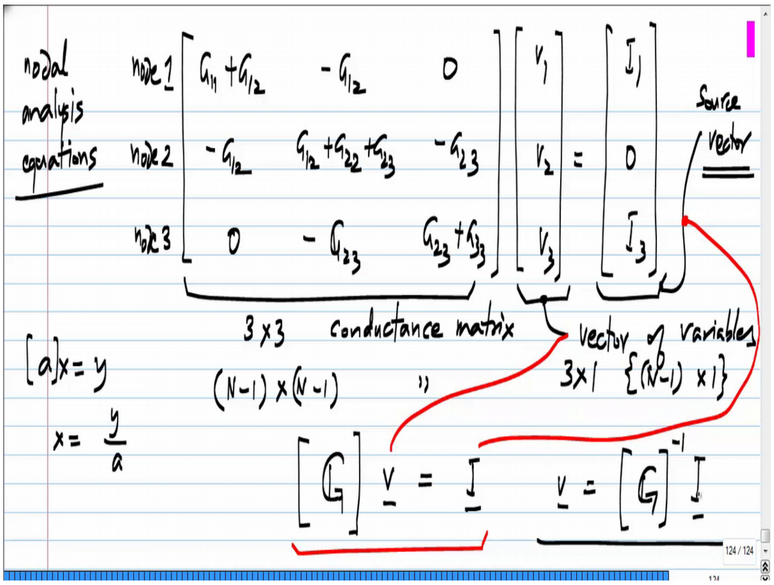

So, these are the nodal analysis equations and I have a 3 by 3 conductance matrix. And in general in an n node circuit it will be n minus 1 times n minus 1 conductance matrix. Now I will denote that like this I will put the square brackets to just show that it is a matrix and what is this, this is the vector of variables and the size of this in this case, this is three, now it is 3 times 1 matrix or in general it will be N minus 1 times 1. So, basically the vector consists of all the node voltages with respect to the reference nodes and there will be N minus 1 of them. And finally, the entry on the right hand side this corresponds to the vector of sources, this corresponds to the vector of sources.

So, what I have is conductance matrix G times the vector of voltage is V being equal to the vector of independent sources I. So, this is this and this is that, so this gives us a compact notation. So, this is like a x equals y, and to solve for x you do x equals y divided by a when a is just a number and when is a scalar, but exactly the same thing will apply here as well. So, to solve for the vector V, you have do G inverse times I. So, this is the solution to the nodal analysis equations.

So, you are first set up the nodal analysis equation, I will point out the structural the conductance matrix. Once you understand it well, you do not need to first write down the equations and then put it in a matrix form, you can put it in a matrix form right away, the matrix equations can be solved for small matrixes usually 2 by 2 or 3 by 3, you can do it by hand; otherwise you resort to a computer. So, there will be a systematic way of putting down the conductance matrix, and therefore a systematic way of solving for a large circuit.