Basic Electrical Circuits
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Lecture - 36
Number of Independent KCL Equations

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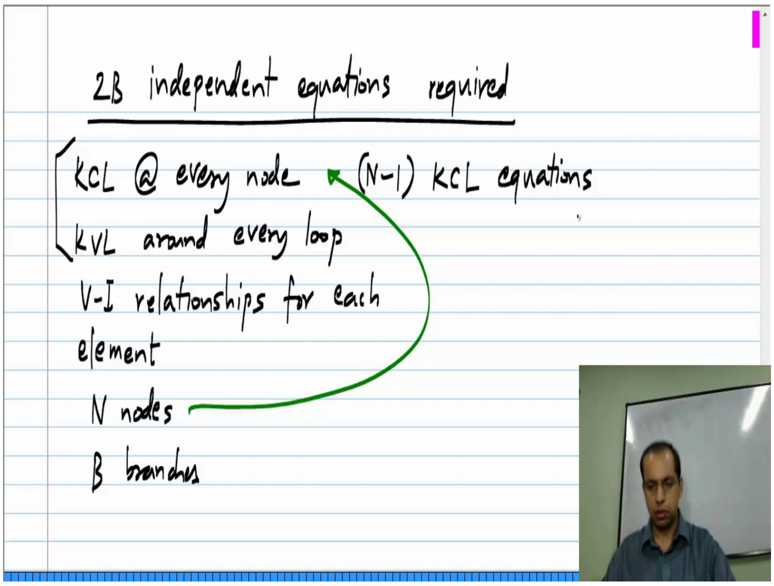

Now how many equations does each of these things give us of course, it depends on the circuit itself, the configuration of the circuit, but if we have N nodes and B branches, these N nodes will give you N minus 1 Kirchhoff's current law equations.

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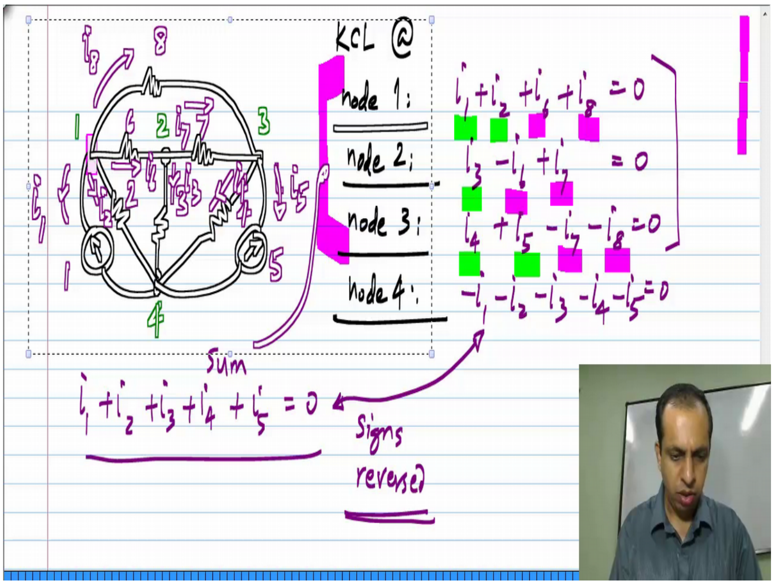

Let us considered the same circuit again; I will illustrate with this example, but this is true of every circuit with N nodes. So, we have nodes 1, 2 and 3 and branches 1, 2, 3, 4 5 6, 7, 8; we also have node 4 over here. Now I will just label the currents in each branch in some arbitrary direction it does not matter i 1, i 2, i 6, i 3, i 4, i 5, i 7, i 8. Now we can write Kirchhoff's current law equations at node one, what do we have the sum of i 1, i 2, i 6 and i 8 to be 0. In fact, I would encourage you to pause the video at this point and the write the KCL equations at every node by yourselves. And then restart the video and verify that whatever you have got is correct, this is just to get some additional practice in keeping all the science in the KCL equations correct.

So, at node 1, we will have i one plus i 2 plus i 6 plus i 8 to be 0; at note 2, I will again have the sum of currents leaving the node to be 0, which is i 3 minus i 6 plus i 7 equal 0; node 3, i 4 plus i 5 minus i 7 minus i 8 to be zero; and finally, at node 4, we will take the sum of currents leaving the nodes which is minus i 1 minus i 2 minus i 3 minus i 4 minus i 5 to be 0. So, it looks like an every node we can write a KCl equation, but in this case, let me consider the sum of these 3 equations, I sum these 3 equations what will I get, you can see that i 6 will cancel with minus i 6, i 7 will cancel with minus i 7, and i 8 will cancel with minus i 8, and I will have all these other things which are left out i 1 i 2 i 3 i 4 i 5. So, if I some all these equations, I will get i 1 plus i 2 plus i 3 plus i 4 plus i 5 to be 0. And you notice that this is an exactly the same as the equation for node 4, but with the signs reversed. Basically the 4th equation is dependent on the first 3 equations; we have only 3 independent equations. So, in general, in a circuit with the N nodes, we will have only N minus 1 independent Kirchhoff's current law equations. Now this is not a particular property of this circuit, this is not a property of this particular circuit, but true for all circuits, I will quickly show why that is the case.

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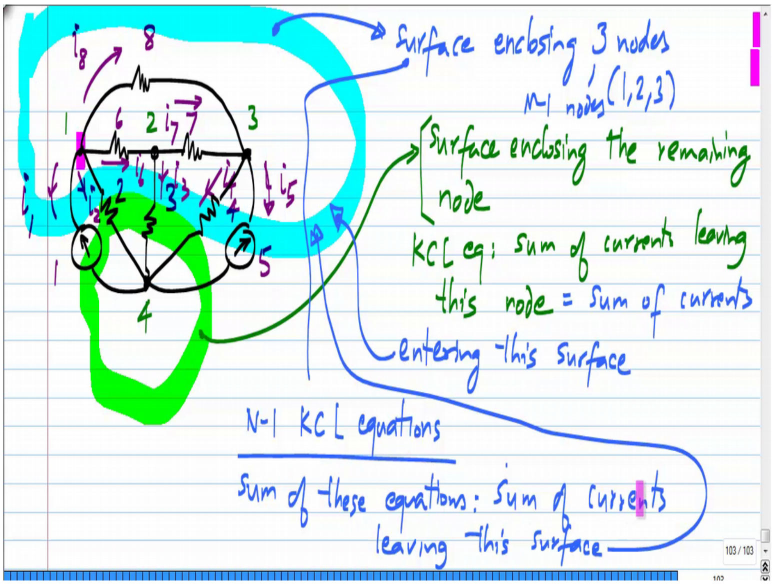

Let me copy over this circuit. Now let us consider a particular node for instance node 4. So, I will draw a surface enclosing node 4. Now let me also draw a surface enclosing all the other 3 nodes. So, this one, this one is a surface enclosing 3 nodes 1, 2 and 3. So, in general, if you have an n node circuit, you imagine a surface inclosing N minus 1 nodes. And this one here is a surface enclosing the remaining one node. Now let us write KCL equations, if we do that what we will be writing is the total current leaving this node in this case the KCL equation for the last node KCL equation, what does here should a some of currents leaving this node.

Now, clearly one of the surfaces encloses N minus 1 nodes, the other surface encloses the remaining node. So, the sum of currents leaving this node is exactly the same as the sum of currents entering the other surface. So, sum of currents leaving this green surface here is obviously equal to the sum of currents entering the blue surface, because blue surface encloses the rest of the circuit. So, this also equals some of currents entering this surface. Now when we write Kirchhoff's current law for the first N minus 1 nodes, what are we really doing, we are taking the sum of currents leaving this node and all currents leaving that node and all currents leaving that node and all currents leaving that node and so on. And if you sum all of those equations together what we get is exactly the sum of all currents leaving this blue surface.

So, let us considered the N minus 1 KCL equations then the sum of these equations is basically the sum of currents living this surface by this I mean the blue one, so that is because the N minus 1 KCL equations will have currents going between nodes inside the same surface those will get cancelled out, because when you write it for node one, it will appear with one sign; when you write it for node 2, it will appear with another sign. So, when you sum all the equations, the only things that will be left are the actual currents going out of this surface. So, now, you clearly see that the KCL equation for the last node, which denotes the currents leaving the green surface and entering the blue surface, is obviously the exact opposite of the sum of the first N minus 1 KCL equations, which denotes the sum of all currents leaving the blue surface. So, there are N minus 1 KCL equations; I hope that is convincing.