Basic Electrical Circuits
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Lecture - 32
Power and Energy in an Inductor

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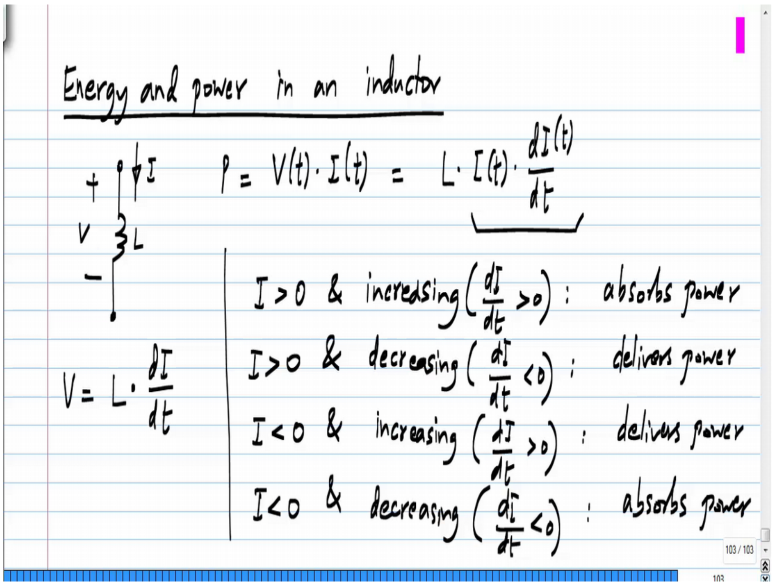

Now, we will look at power and energy in an inductor. We have already done this for the capacitor. And as you know from voltage current relations, capacitors and inductors behaves similarly in a mathematical sense with the roles of voltage and current interchanged. And we will say a similar thing as far as energy is concerned as well. We have an inductor L with a voltage V across it; and the current I through it, and the power just like it is for any two terminal element is the product of voltage and current; but in an inductor, we have the voltage to be the inductance L times dI by dt, the time derivative of the current I. Substituting that here, we get L I of t times the time derivative of the current. So as before, this looks a little complicated, but the important thing is to realize that this can also be either positive or negative.

So, if you look at this product, we have these possibilities actually, let say the current is greater than zero and increasing which means its derivative is also greater than 0, then the inductor absorbs power or power is being delivered into the inductor. Similarly if I is more than zero and decreasing that is the time derivative is less than zero then it delivers power. And I is less than zero and increasing that is its becoming less negative, the time derivative will be more than zero and it delivers power, because the product of a negative current and a positive derivative will yield a positive number here. And finally, for negative I and decreasing at absorbs power. Now for the case of the capacitor I have some wave forms and showed you how this can happen. So, you can do a similar thing, but for the current wave form instead of the voltage wave form as in case of the capacitor. So, by now it should be pretty clear that the inductor also can either absorb power or deliver power. Now how do we decide whether it is active or passive, again as will the capacitors we look at the energy starting from zero states for the inductor whether it can absorb or deliver energy.

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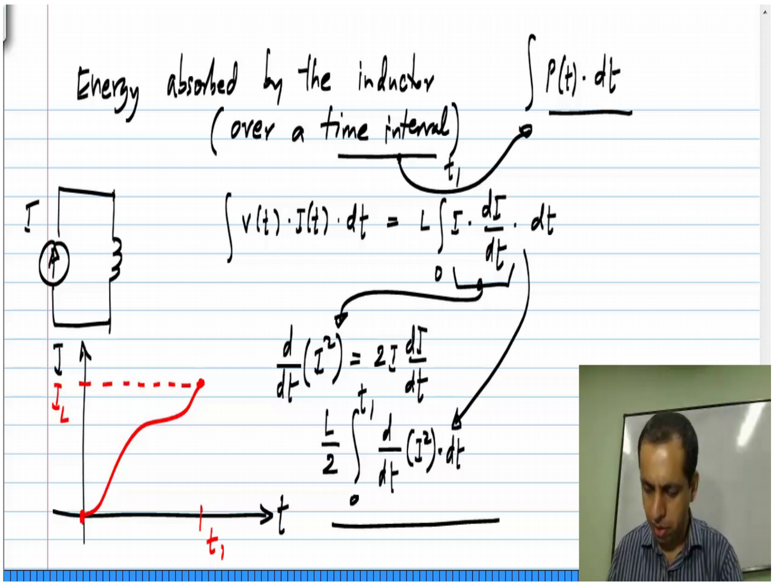

If you look at energy absorbed by the inductor, and when we say energy, we have to specify a time interval. We have to integrate the power over that time interval; the limits of integration should be such that it is spans the time interval. Now, let me imagine that the inductor is driven by current source; earlier for the capacitors case, I just showed the capacitor and some voltage v form you can think of that as a capacitor being driven by a voltage source, here I am driving it with the current source I. And it has some variation over time I versus t. Let say that it starts from zero and then at time t 1 it goes to a certain value I L. So, the integral this which gives you the energy is integral V of t I of t dt, which is equal to integral L I dI by dt over time. And the interval of integration is from 0 to t 1. As before this looks a little complicated, but you easily see that this is related to the time derivative of the current square, because d by dt I square is 2 I dI by dt. So, this integral turns out to be L by 2 integral 0 to t 1, the time derivative of I square over time, basically we have the integral of time derivative of the square current over time.

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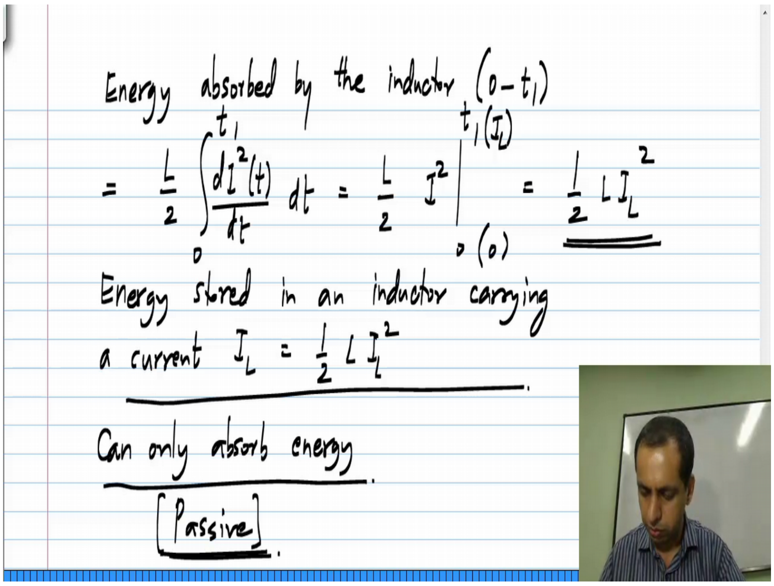

So, the energy absorbed by the inductor over an interval zero to t 1 equals this. This is an integral with respect to time of the time derivative of I square so obviously, it is the function itself which is I square from 0 to t 1 of course, as time goes from 0 to t 1 the inductor current goes from zero to certain value I L, so this number is nothing, but half L I l square. So, what does this result tell us first all if you change the current in inductor from 0 to I l over a time t 1 then the total energy deliver to the inductor will be half L I L squared. Now it does not depend on how the current gets from 0 to I L, it can do it in any which made you will get the same answer. So, from that we can also infer that if an inductor is carrying a current I L, it will have a stored energy of half L I L squared. So, the energy stored in an inductor carrying a current I L is half L I L squared.

Now, as with the capacitor, when you take the inductor current from zero to certain value I L, it absorbs certain amount of energy and after that it can return it all back. So, let say the current goes back to 0, then all of these energy will come back out of the inductor in to the rest of the circuit. But starting from zero, the inductor can only absorb energy, because if you see the expression of energy we have I l square which is always a positive number. So, it always absorbs energy and it is also a passive element just like the capacitor or the resistor. Unlike the resistor, it does not dissipate energy; it only stores energy which can be recovered later. So, the inductor is a passive element.

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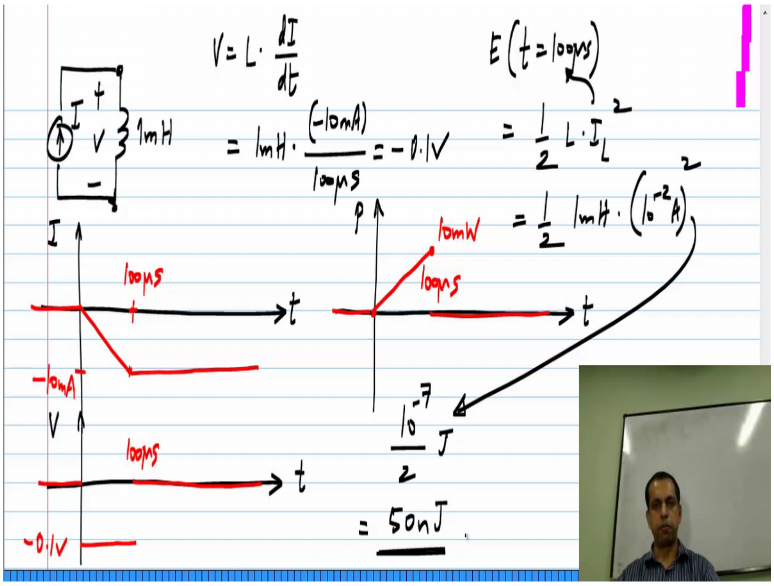

So, in many respect its similar to a capacitor, in case of the capacitor, I went into more detail the inductor I treated a little more quickly because mathematically there are very similar. So, now, I will quickly take numerical example for the energy in an inductor. So, let say we have 1 milli Henry inductor and it is driven by current source like this. And the current has a certain wave form. And just for fun, let me say that the current is zero before to equal to 0, and it goes negative. And it stays at this value after certain time. So, let me say that the current changes from 0 to minus 10 milli ampere over an interval of 100 microseconds and it stays at minus 10 milli ampere after t equals 100 microseconds.

So, we can do all sorts of calculation with this; first, let us calculate the voltage across the inductor in this polarity we know that v is l times the time derivative of five. So, clearly before to equal to zero, the current is changing the voltage is 0; after t equals 100 microseconds when the current is again a constant, the voltage will be 0. And between these two from 0 to 100 microseconds, we have a constant slope which corresponds to a constant current in the inductor. And how much is that current, it will be remember I have taken these to be straight line segments. So, it is easy to calculate the slope, it will be 1 milli Henry times the change of minus 10 milli amp over an interval of 100 microseconds; this will turn out to be minus 0.1 volts. So, this is minus 0.1 volt.

We can also sketch the instantaneous power versus time. So, what happens now, clearly before t equal to 0 or after t equals 100 microseconds, the voltage will be zero. So, the product of voltage times current will also be 0. And between these two between 0 and 100 microseconds, it is the product of this constant and the straight line, so it will be a straight line which has positive values because both the current and voltage are negative. And this value is the product of these two which happens to be 10 milli watts.

As usual if you want to calculate the energy you can integrate the power over time over whatever interval you want, but of course we know that there is an easier way to do that. So, let say we considered the energy in a inductor at t equals 100 microseconds, all we have to do is calculate half L I L square where I L is the current t equals 100 microseconds. And this comes out to be half times 1 milli Henry times I L which is 10 to the minus 2 ampere square. So, this number is 10 to the minus 7 by 2 joules or this is basically 50 Nano joules of energy, so that is the energy stored in the inductor. Now you should be able to carry out the same calculations of certainly the voltage from the current as well as the power and energy for arbitrary variations of current in the inductor.