Basic Electrical Circuits
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Lecture - 31
Power and Energy in a Capacitor

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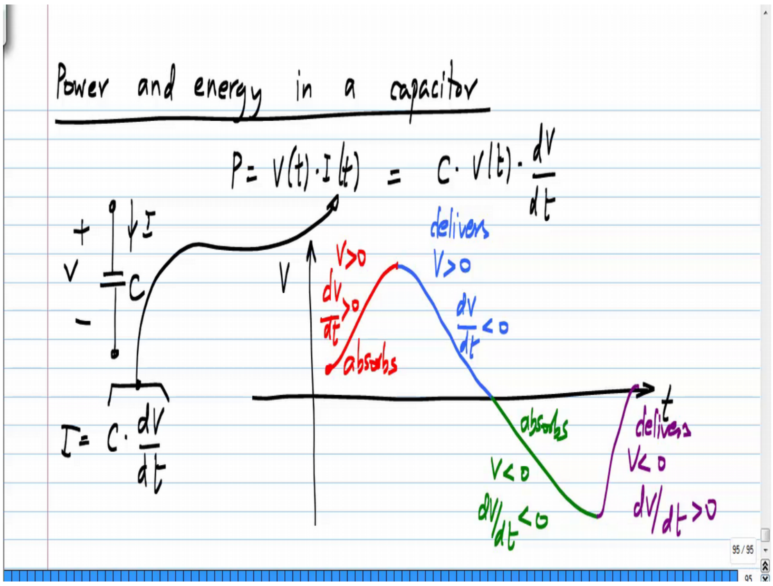

Now I will consider the power and energy in a capacitor. As usual, we take the voltage and current I at the appropriate passive sign convention then the power just like for any two terminal element is the product of voltage and current, but of course, the characteristics where capacitor is a that it enforces this relationship I equals C times the time derivative of voltage between the current and voltage. Of course, we have to substitute this current into this expression to give us C times V of t times dV dt. First of all this expression looks a little more complicated than that for a resistor, but the important point here is that this is the power delivered to the capacitor and this can be either positive or negative, because let us considered some voltage wave form and voltage could change in this way versus time. Clearly here the voltage is positive and its derivative is positive, it means that there is a positive voltage here on the upper plate there is positive charge and that is increasing with time. So, current has to flow in the upper terminal. So, the current is also positive.

So, clearly this product here is positive and the capacitor absorbing power. The other way around is also possible lets imagine that the voltage does that where the voltage is positive, but its time derivative is negative. Here this product will be negative and the capacitor will be delivering power it is absorbing negative power then we continue on that way where V is negative and where time derivative is also negative. So, the product is positive and in this case, again the capacitor absorbing power. And finally, we can do that where V is negative whereas the derivative is positive because the voltage is increasing and the capacitor is delivering power. So this is possible because depending on the voltage value and its rate of change we can have the capacitor to be either absorbing or delivering power. So, here it absorbs power here, it delivers power and here again where I have shown the green part of the wave form it absorbs power and in this part it delivers power. So, all this thinks are possible.

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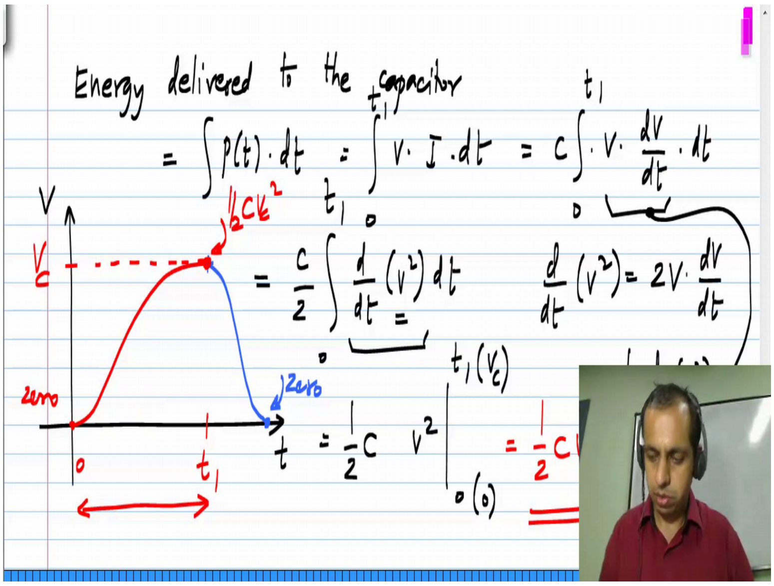

Now, if the power delivered to the capacitors can be both positive and negative how do you tell if the capacitor are passive element or not for that we considered the energy delivered to the capacitor this of course, is the integral of p which is the integral of V I dt and let us imagine that the voltage across the capacitor the voltage across the capacitor changes from zero to some value vc in a time zero to t 1. So, the horizontal axis is the time axis and we want to find the power delivered to the capacitor in this interval zero to t one. So, we have to carry out this integral from zero to t 1, this is nothing but 0 to t 1 C times V time derivative of V with respect to time. This was very complicated V itself has some kind of variation we had not specified what that is its time derivative looks complicated and finally, the integral of that how we evaluated, but it turn out its very simple and it does not depend on any of the details of the wave form you realize that as V times time derivative of V is nothing, but it is something related to the time derivative of V square the time derivative of V square is 2 V time derivative of V. So, this part is half of time derivative of V square. So, basically the energy delivered to the capacitor nothing, but C divided by two from here times the time integral of the time derivative of V square we have the time derivative of V square and we integrated with respect to time.

So obviously, we just get this function itself V square over the appropriate limit from zero to t one. So, which means that the voltage goes from over a time interval zero to t one the voltage goes from zero to V C and thus give us finally, the expression for energy delivered to the capacitor over this interval which is half C V C square . So, if you have a capacitor starting from zero volts and you changed its voltage from zero to a voltage V C in some way it does not matter what wave form is used the total energy delivered to the capacitors have C V C square, now of course, this gives you the formula for the capacitor energy when its voltage is V C because if the capacitors voltage is V C you can imagine that it is taken from zero to V C in some way it does not matter how you have taken.

So, if you do that you have delivered the energy of half C V C square to the capacitor and this energy half C V C square will be stored in the capacitor because if you now take it back to zero what I just showed you this energy is the energy in the capacitor over here lets say after that the capacitor voltage goes to zero like this and we know that in this interval when the voltage is positive and the time derivative is negative it delivers power, but also we know that basically the energy here will again be zero. It is starts with zero energy and its goes to half C V C square and it comes down to zero. So, this energy that you delivered this half C V C square does not go anywhere. So, during the charging process you delivered energy to the capacitor and during the discharging process as the voltage falls from e C to zero you can recover all of that energy it delivers that much energy. So, a capacitor stores energy unlike the resistor were dissipates the energy that is delivered to it, but the one thing is that the net energy delivered to the capacitor is always positive that is you assume that a capacitor starts from zero volts it has zero energy and after that you change the voltage in whatever way you want and take it to some voltage V C the energy delivered to it will be half C V C square because of the square term the energy this quantity will be positive so that means that a positive energy is delivered to the capacitor . So, once a capacitor is charged for some voltage you can have the capacitor delivered the energy to the rest of the circuit, but starting from zero volts a capacitor always absorbs energy a capacitor is also a passive element.

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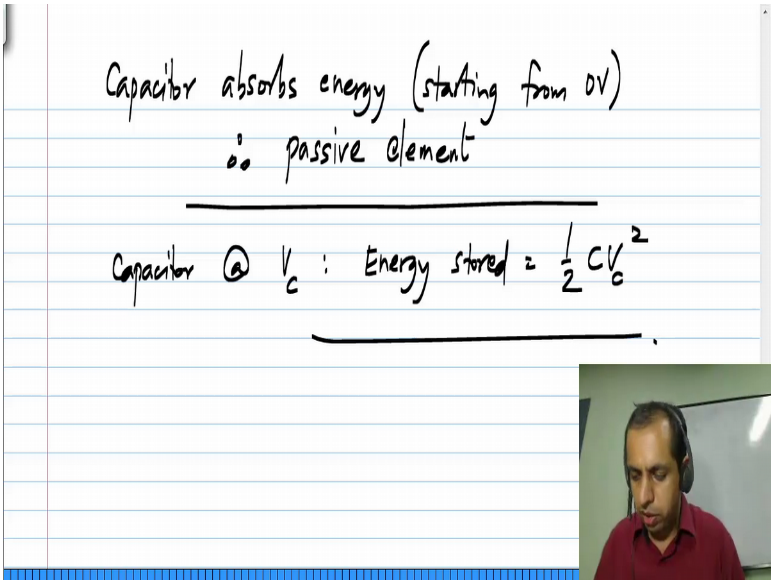

So, like I said you can charge it to a certain voltage we see and then have the capacitor delivered energy to the rest of the circuit or basically capacitor absorbs negative energy, but to get the this starting point were the discharge to be see you have to supply energy. So, if you take the net energy absorb by the capacitor it will be positive. So, the capacitor absorbs energy and it is passive. Of course, this is the useful application of a capacitor you can charge it to the certain voltage and use it as a source of energy that is what a capacitor is used for in many applications. So, the summery is that capacitor absorbs energy and also a capacitor at a voltage V C that is when a capacitor has a voltage V C it has energy stored in it which is half C V C square it is clear.

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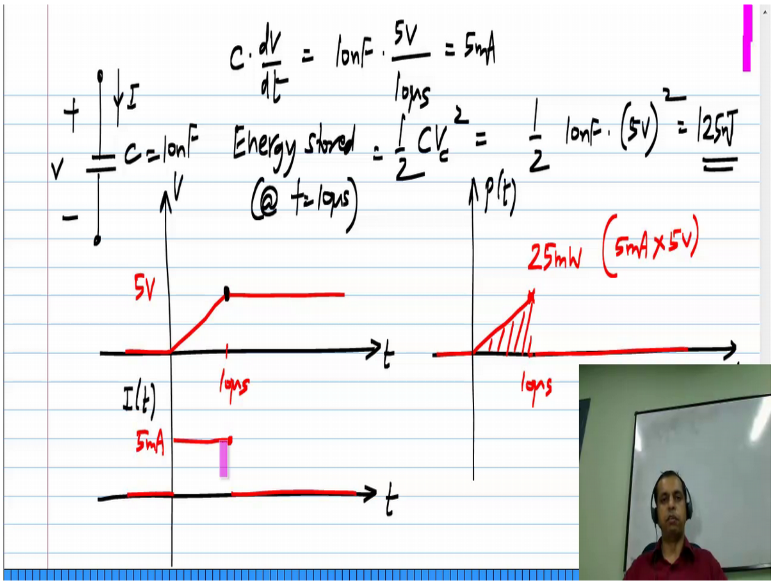

Let us take a quick numerical example, I have a capacitor C then voltage V and a current I let me define the capacitor wave form. Let say it a zero for t less than zero then its raises to five volts over a interval of ten micro seconds and I assume that it will increase in a straight line and then it stays constant at five volts and let say the capacitor value is 10 nano farads. Now I will plot the current through the capacitors we know that before t equal to zero the voltage is constant. So, the current is zero and about t equals ten micro second the voltage is again constant. So, the current is again zero, but between these two intervals the current which is C dv dt is 10 nano farads times 5 volts divided by 10 micro seconds will be 5 milli amp. So over this interval because I assumed a straight line variation in voltage we have constant current of five milli amp otherwise it is zero.

Now what can we calculate from this first of all we can calculate the power which is nothing but the product of voltage and current. So, before t equal zero both voltage and current are zero. So, the power is zero and after t equals ten micro seconds voltage is five volts current is zero. So, the power is again zero and in between these intervals we have a straight line like this multiplied by a constant. So, this is a similar straight line the value it reaches here at t equal ten micro second is the product of this value five volts and this value five milli amps. So, this is 25 milli volts, this is five milli amp time five volts. So, this is how the power in the capacitor power observed in the capacitor changes with time. Now what is the energy stored in the capacitor the energy stored in the capacitor at any point can be obtained by integrating the power curve up to that point. So, let say we take this particular point.

Then we have to compute this area of course, we do not have to do it like this, because we already know that if the capacitor voltage certain value we see the energy the energy stored in it will be half C V C square we started from zero volts it goes to V c. So, the energy observed by the capacitor is also the same thing. So, the energy absorbed up to this point t equals ten micro second or equivalently the energy stored at t equals ten micro second is half C V C square where I have to take value of the capacitor voltage at ten micro second is half 10 nano farads five volt square, basically we are looking at this point here which 125 nano joules. Of course, if you take a trouble to integrate this curve you will find exactly the same volume.

So, the reason I showed just that first of just as an example calculation involving power and energy in capacitors. Now the energy is very easy you know the voltage across the capacitor you know the energy stored in the capacitor yes perhaps less usual to calculate the power in a capacitor, but it can be done as I showed here and the other reason. I showed is also that I find many students are not comfortable with drawing wave form or amount used to it, but I personally find that drawing wave form are very good wave getting intuition on circuits. They are not for precisely calculation for that you use the calculator or the computer, but to get a sense of which way thing are varying it is very good to draw the wave forms. And the kind of examples I solved here and also the kind of problems that are in the assignments where the waveform of is linear is very easy to do. So, I strongly encourage you to draw the wave forms at least for some cases to get an idea of what is going on in the circuit. So, especially things like capacitors where the time derivative enter the picture you may not always be able to calculate things exactly, but you can get a sense of variation while stretching V and the time derivative of V and so on.