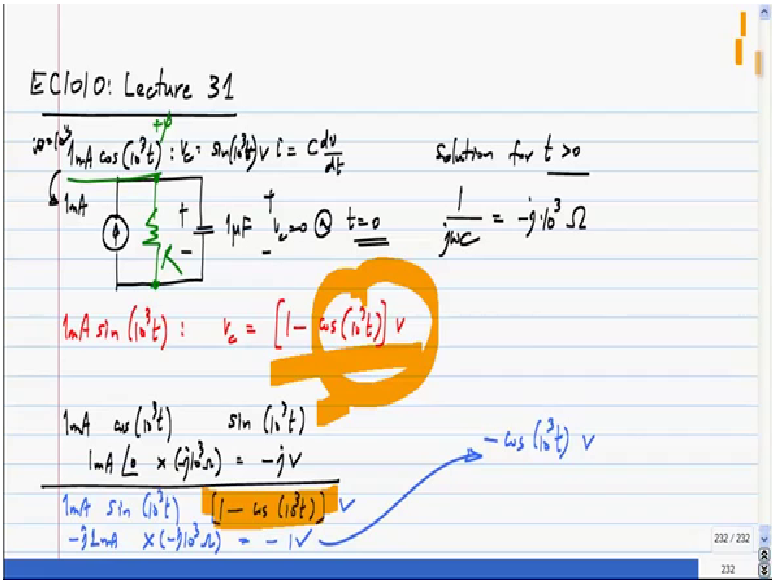
Basic Electrical Circuits
Dr Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture - 147

(Refer Slide Time: 00:07)



In the previous class we learned how to do analysis of circuits with phasors, phasors are this complex multiples of exponential geometry and from that you can get the sinusoidals study state response. So, I looked all lines and simple compared to solving differential equations, but we did running to a problem, but this particular circuit. So, we calculated it in the time domain we got something and we calculated using phasors and we got something else.

So, what is the reason in fact the depending on the input sometimes it works some times its does not work it is pretty horrible like it, why does this happen, what is the answer we are getting answer we are using phasors, what is the answer Jayshreenath, what is the question, then will get to the answer, what we get when we calculate using the face of technique, no I mean finally, I want the answers in the time domain I want the voltages as the function of t that of course, this phasors way some short cut way of doing the same thing.

So, what is the time domain I voltage I get when I solve this circuit using the phasors technique, what I may getting its all on the board should I tell you, which color it is in its in blue, what is the answer I am getting using phasors minus, minus cos omega t. And what is the answer I get from direct time domain calculation, which we know cannot be wrong, because that is the basic I v relationship, what is that 1 minus cos omega t.

So, what is gone wrong here we get something there and we get something here no that is I mean in the other case we got it. So, this phasor analysis is useless I mean depending on the initial condition no that is right I mean shifting of origin I mean this is not quiz right I mean or I just shifted the origin give me half mark no that is not the case this is wrong it is wrong it is different is not it. The whole point of using phasors analysis was; that means, the some circumstances it should give you guaranteed right answer. So, it is giving wrong answer, why is it giving the wrong answer have we gone wrong somewhere.

So, we cannot use it for reactive elements is that the conclusion I mean at least when we came up with the method there seem to be no such restriction, what is going on, what is this analysis called we had another name for it, what study state sinusoidal study state. So, it is sinusoidal no right it is sinusoid is it study state it is not why not no current through a capacitor in study state for d c excitation right if you have sinusoid you can have very much have current through a capacitor you will, but there is some certain issue here.

What is the time constant of this circuit? 0 why, how do you find the time constant of first order circuit find the time constant of first order circuit find the resistance across the capacitor. So, what is the resistance, what is your name what what is the resistance across the capacitor I do not know, what is inside the current source what is the thevenin resistance across the capacitor in the circuit come on infinity. So, what is the time constant of the circuit infinity.

So, end of the steady state never, so this is simple example where you have a natural response plus steady state response the steady state response is identified correctly the natural response never dies out. So, this is just to illustrate that this applies only to the steady state part of it. Now, you have any resistance across this it is does not matter any finite valued resistance R and you calculate the steady state the natural response will die out and in this particular case the natural response does not die out.

So, the total response that you calculate includes the natural response which does not die it that is why the, the answer from phasor analysis, which is calculating only the sinusoidal study state part does not match with this one. But, once you have some resistance or basically any circuit in which the natural response dies out this is still useful, because you have to wait long enough for you to do die out, but once it dies out the total response equals the study state response.

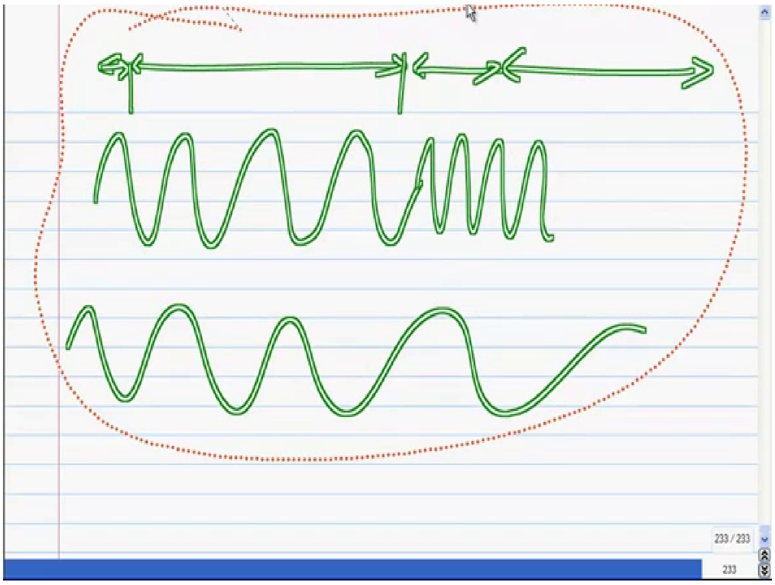
So, actually lots of circuits are like that, but this is just to illustrate some certainties that may be behind the analysis yes no know what happens is the natural response you know that the coefficient of the natural response depends on the forcing function you are talking about the case, where I excited it with cos sin instead of sin. So, it just, so happens that for that the coefficient of the natural response is 0 if I take any other phase.

Let say some phi you will get some other value here I think instead of one you will get some cos phi other value here I think instead of one you will get some cos phi or something like that. So, you can always choose the input, so that the coefficients of the natural responses are 0. So, in that case I mean the natural response essentially you can say dies out I mean or it is not even there from the beginning, but that is a very special case I mean that does not happen for arbitrary inputs.

So, this circuit is basically some pathological circuit in which, time constant is infinity and the natural response never dies out. Similarly if you analyze a analyze an inductor with sinusoidal voltage source across it you will get the same answer similar behavior. So, all this is just a dry homed point that, what we are calculating using a phasors method is the study state response for stable circuits after you wait a long time the total response equals the study state response.

So, it is quite useful, but you can always run into circuits, where the natural response does not die out like in this case it pursues that one it stays constant forever or even worse than some unstable circuits, where it blows up any equations or what anything we did this fact.

(Refer Slide Time: 07:52)



So, you had already learn phasor analysis before this course, where for jee did you have those questions in the exam no. Now, this phasors analysis as you can see is extremely convenient although I said that there are some cases, where it does not apply for the cases where it does apply I mean instead of solving in differential equation you are solving in algebraic equation it cannot get any better than that. And we saw that we could set up the nodal analysis or any of the analysis method we used for resistive circuits.

Now, you can use here also except that there we always had for any resistor we had positive coefficients or maybe you can even solve resistors with negative resistances, but the point is v by i was some constant number, now also v y is a constant number, which can be complex that is all. So, as know as you know how to manipulate complex numbers you can do the sinusoidal study state analysis using phasors very easily. Now the other restriction of course, is that all the excitation should be at the same frequency.

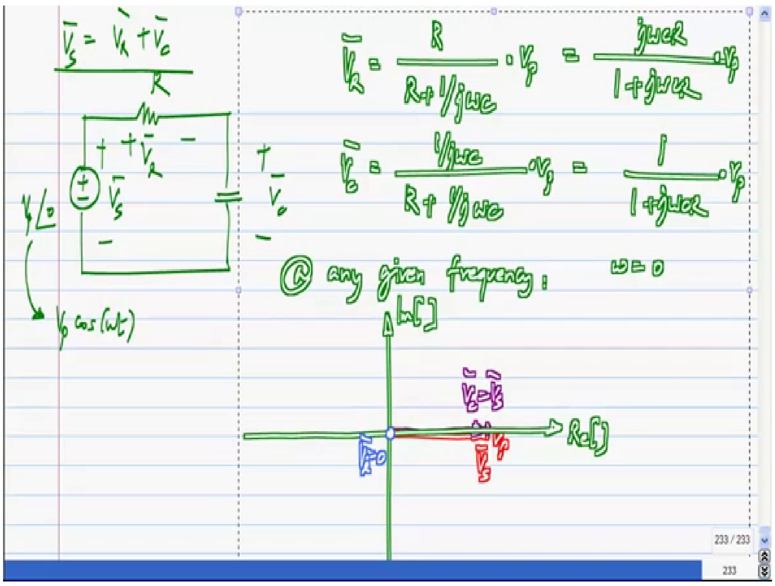
But, if we have excitation of multiple frequency is it is not the big deal I mean you can do it one frequency after another and superpose the time domain results. But, keep in mind that what you get out of this is only the study state response not the natural response frequency of changing with time that will don not deal with here. Because, what do I mean you are thinking of, let say the source that is at 1 kilo hertz for 1 minute and may be, in the next minute is 2 kilo hertz that is not easy to analyze with this if first of all you cannot use phasors for this one.

What you can do is? Let say you are circuit reaches a study state well within that 1 minute you can compute the study state part. And in after that it goes to 2 kilo hertz you compute this study state part of that, but what happens when it switches from 1 to 2 kilo hertz you will get some natural response components that you can predict with this anywhere. And actually I mean technically speaking such a source if a source has something like this, let say after this it becomes higher frequency this is not a sinusoidal source at all this is some other complicated source.

So, sinusoidal source by definition means that it maintains the same frequency from minus infinity to infinity. So, that way I mean it does not apply, but you can apply in some once you are familiar enough with that, let say the this part is study state and this part is natural response you can find the response in this part using phrasal analysis. And then, again for some more time you cannot calculate because there we some natural response, but after that you can calculate this study state for this frequency and so on.

So, you can approximate it, but source whose frequency is changing is not a real sinusoidal source, where as all the techniques all the discussion we had applies only to a purely sinusoidal source any other questions.

(Refer Slide Time: 10:51)



So, now, let us go back to our simple circuit and let say here I apply V p angle 0, what this really means is in the time domain I have applied V p cos omega t. So, now, what is V R and what is V c what is V R, so this is R by j omega C times V p, which is the same as j omega C R by 1 plus j omega C R times V p and this V c is the compliment of that, which is basically.

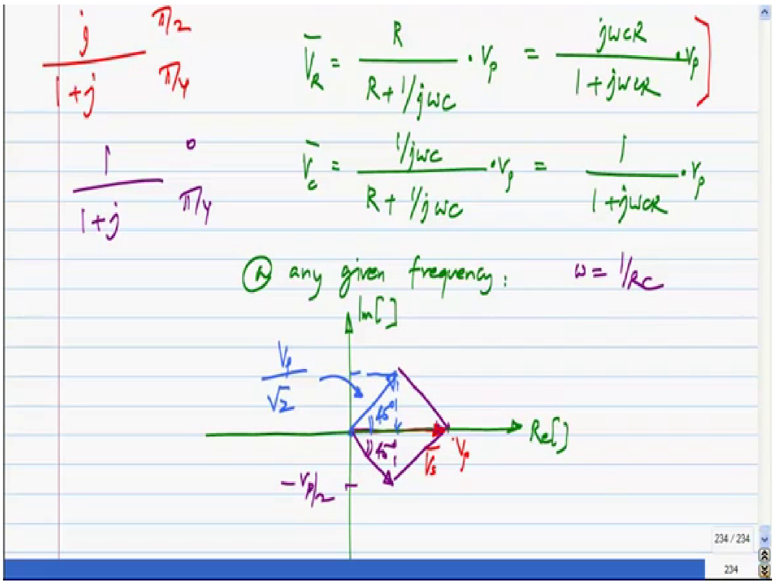
So, now at any given frequency V R and V c are some complex numbers and just like you sometimes represent complex numbers as vectors in the two dimensional plain you can do the same here you can represent phasors in the two dimensional plane and such a diagram is knows as a phasor diagram phasors diagram is just picture of phasors in the x y plane, where the x axis is the real part and y axis is the imaginary part.

Now, from this I am assuming that you are quite familiar with calculating the and going between different representations of complex numbers between rectangular and polar forms and also rationalizing complex numbers when the denominator is complex etcetera etcetera, so I won’t go into any of that. So, this is the real part and this is the imaginary part and I have to take some frequency. So, first of all let me take omega equal to 0, what will the phasors will look like and let me call the source voltages V s.

So, clearly V s should be V R plus V c in this circuit at omega equal to 0, what will the phasor diagram look like of V s V R and V c, what is the phasor corresponding to source voltage V s mayug, what is the phasor corresponding to the source voltage V s on the real axis what is the length V p. So, this is is V s where this point is V p and at omega equal to 0, what is the phasor corresponding to V R, what is it at omega equal to 0 on the real axis.

What is the length? V p 0 I mean that you can see from expression here it is proportional to omega at what it means is that at 0 frequency which is d c what is the study state current 0, so there is no voltage dropped across the resistor. So, V R would be just 0 and V c, V c equals V p; obviously, it has to be that is also V p.

(Refer Slide Time: 15:41)



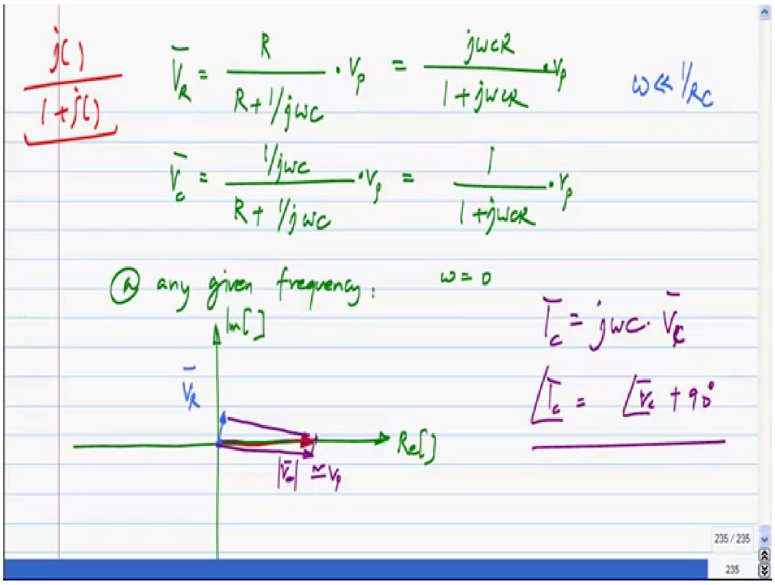
In general if the frequencies are very low first of all let me take another frequency omega equals 1 by R c, what happens in this case sai sathvik not here. So, what are the phasors V s of course, is the same, because I have not changed the applied voltage. So, V s will be this, what are the phasors V R and V c louder V R is on the real axis at omega equal to 1 by R c, what is the value of this at omega equal to 1 by R c j by 1 plus j.

So, where is that vector on the complex plane 45 degrees, which way upwards or downwards upwards, upwards 45 degrees are you sure, what is the angle of that j by 1 plus j. So, the angle of the numerator is 5 by 2 and then, angle of denominator is 5 by 4, so the angle is plus 5 by 4and V R is. And, what is the magnitude of this, so the length of this is basically V p by square root of 2. So, the real part is V p by 2 and the imaginary part is also V p by 2 and what about V c Deepak.

Student: 45 degrees.

So, it will look like this and this also is 45 degrees, so the angle is 1 by 1 plus j. So, the angle of the numerator is 0 the angle of the denominator is 5 by 4, so 0 minus 5 by 4 minus 5 by 4and the real part is V p by 2 and the imaginary part is minus V p by 2. And; obviously, these two should add up to V s V R plus V c equals V s sit down sit down this is fine. So, at omega equal to 1 by R c the magnitude of voltages across the capacitor and resistor are equal to each other, but of course, they will be in 90 degrees out of phase of each other.

(Refer Slide Time: 18:45)



Now, what you think happens for low frequency by low frequency whenever I have any quantity with dimension I have to say low compared to what. So, let say omega must less than 1 by R C, but not 0, what will the picture look like, what is the phasor V R karthik Jayachandran yes omega very small like much smaller than 1 by R C, but not 0. So, approximately, what is the angle of the phasor 10 to 0, why yes ashwak what will tend to 0 no, no the magnitude of V R is of course, very small.

Because, j times omega c R is very small and the denominator the magnitude is almost one right one plus j omega c R, but what is the angle of this angle will be 0 5 by 2 it will be almost perpendicular to we have j times some small number divided by 1 plus j times or some small number. So, the angle of this complex number is almost 0 degrees the angle of this is 90 degrees. So, it is almost 90, but will it be more than 90 or less than 90 slightly less than 90.

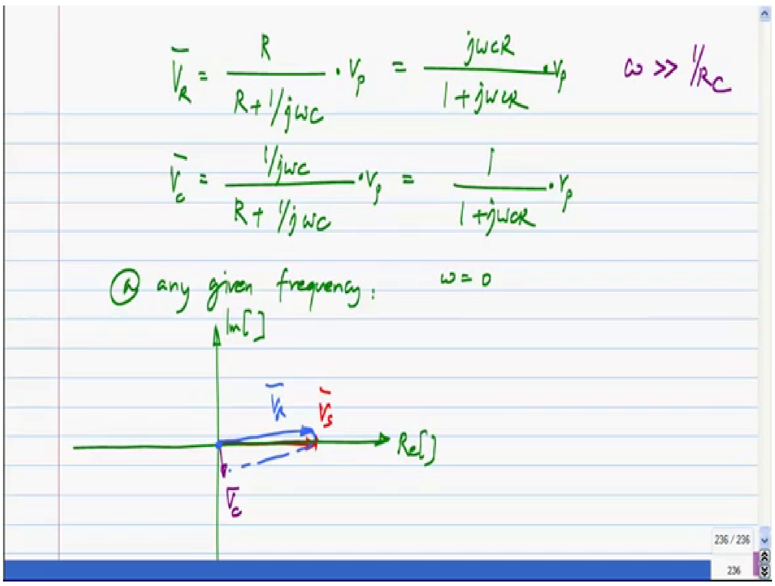
So, I will show it as some small thing like this it is almost 90 degrees, but not quiet ninety will certainly will less than 90 and V c what is the magnitude of V c it will be almost V p, because this omega c R is very small. So, the magnitude of the denominator is almost 1 and what is the angle of this merely 0, but is it negative is it in the first quadrant or the fourth quadrant fourth quadrant and of course, these should always add up to this one.

So, what happens is that if you start with 0 frequency the voltage across the resistor is 0 and all of the input voltage is across the capacitor that is if you apply d c voltage. As you increase the frequency of the sinusoid there will be some current flowing through the capacitor the current is very small. So, the voltage drop across the resistors still quiet small, but it is there nonetheless and the almost all of the voltages still the capacitor. And also the voltage across the resistor and capacitor in this particular case will always be at 90 degrees to each other.

Because, the current in the capacitor and the voltage across the capacitor are 90 degrees apart, which is leading, which one is leading, leading meaning whichever is more counter clock wise V c. So, angle of i c is angle of V c plus 90degrees, so the current leads the voltage by 90 degrees I think this fact you probably is knew even from high school or something. So, now, the voltage across the resistor is like the is the current in the capacitor scaled by some number.

So, this is leading that by 90 degrees, but the magnitude is very small as you go on increasing the frequency, what happens is that the resistor phasor rotates counter clock wise and also its magnitude goes on increasing the capacitor phasor also rotates counter clock wise, but its magnitude goes on decreasing.

(Refer Slide Time: 23:30)



So, now, you should be able to quickly tell me what happens at very high frequency and again high frequency meaning omega much more than 1 by R C. So, this 1 by R C is some characteristic frequency of this this network, because you see that its omega time C R that matters whether omega time C R is much more than 1 or much less than 1 tells you the behavior of each of these phasors there are some question, which 1 V of.

So, this numerator is 90 degrees the denominator is some small angle. So, it is 90 minus some small angle that is all. So, omega much greater than 1 by R C mahesh kumar about, which point about which plane, so that is correct this is V s and V c will be almost minus 90 degrees, but very small magnitude and V R would be almost 0 degrees and magnitude almost equal to V p and of course, the sum will be V s, now what happens is at very high frequencies the capacitor is almost short circuit.

So, almost all of the applied voltage appears across the resistor that is what this is saying. So, you can determine the same thing in many ways the impedance of capacitor and the impedance of the resistor is basically an impedance divider just like resistive divider at omega equal to 1 by R C, what is the impedance of the capacitor its j times R it is the same magnitude as the resistor, but 90 degrees with 90 degree phasor.

Now, at very high frequencies that is omega much more than 1 by R C the impedance of the capacitor is much smaller than the impedance of the resistor. So, all the voltages across the resistor similarly, when omega is much less than 1 by R C that is at very low frequencies the impedance of capacitor is much higher than the impedance of the resistor. So, all voltages across the capacitor when these phasor diagrams are very useful to visualize phasor relationships and also some times to do designs may be you want to designs circuits, where you want certain phase relationship between certain quantities and so on.

Now, in the tutorial there many more examples of this probably more complicated circuits, but the principal is the same the phasor diagram is simply the representation of phasors on the complex flame the only thing is that; obviously, our voltages and current obeys Kirchhoff's current law. So, if you have a number of voltages around a loop that should form some closed polygon they should form polygon. And similarly, if you have number of entering node they should also form polygon.

So, that is all are there to it any questions about this drawing phasor diagrams first of all with phase I mean you can always claim that 90 degree is leading is 270 degree is lagging. So, everything is modular three sixty degrees, but it is common to if you lesser restrict you are angles to plus or minus 180 in that range then is i c is this part you understand that is. So, that is mean that if you have the voltage way form like this across the capacitor current way form may be on different scale would look like that.

So, this sinusoid is leading the other sinusoid that is the or I mean it is obvious from the angles the angle of the current is 90 degrees more than the angle of the voltage. So, that is I mean that is what is referred to has leading angle, but I mean if you call this two seventy degrees lagging you won't be wrong and similarly no it is just visual representation of things that is all. So, I mean many times either same advantage you have drawing figures like to visualize phasors between different voltages and so on.

So, it is useful will seen I mean we kind expand this to some more stuff, but it is what can I say I mean there are some applications where you would want to be visualize phasors between different things and when we come to three phasors you will see when we come to three phasor system its becomes quite useful that correct you can you can, but that is I mean that is not what you normally say I mean you can certainly say that you will not be wrong, but I mean it is just contrary to convention and that is all.

Because, phase is modular 360 degrees right like if I say that sin is cos of sin theta is cos theta minus ninety degrees or cos theta plus 270 degrees is both the correct. So, one of them expresses the feeling that or the statement that sin is lagging. cos by 90 degrees other one says sin is leading cos by 270 degrees they are both correct. But, normally when you say leading or lagging you restrict the angles you specified to plus minus 180 degrees that is on just KVL no certainly not I mean you have to that that rule applies to specific circuits.

So, in the same circuit if you have another current source V c plus V R after will be V s because the loop is the same, but for arbitrary circuits you may first of all have lot more than two quantities no you may want it in some cases. Now, it is hard to explain it to, now again when we come to three phase we have number of examples of this you may want the certain phasor relationship between two components.

Now, when you come to let say communication systems you will have you heard of modulation, what is it you multiply some low frequency signal with high frequency carrier to translate it to high frequencies. So, normally what you do is you also translate you have different versions of the low frequency signals some 0 degree version and plus ninety degree version and you multiply the 0 degree version with the cos and plus 90 with minus sign and so on and then, you add them up.

So, in analyzing those systems which useful to know the phase relationship it is in fact, to necessary to know the between different parts of the circuit now; obviously, there is no more information in this diagram than in these expressions. But, as you say the picture is worth thousand words may be in this case only 10, but it is useful sometimes. So, phasor diagram just drawing that I mean this what is involved you may have lot more complicated circuits last time we analyzed we had the I think the two node circuits with the two capacitors and inductor we have phasor diagram for different quantities. And then, draw them; obviously, these I mean you would want to draw them if you have currents and voltages you should draw them on either different diagrams or scale the access appropriately and so on.