

Basic Electrical Circuits

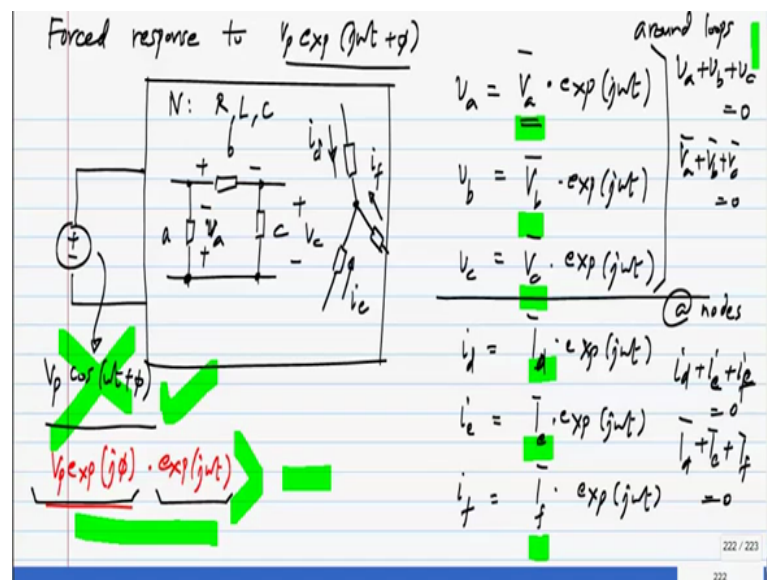
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Lecture - 146

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So, now, let say you have some arbitrary network in which had everything R L C and you can also include control sources if you wish, but that to R L C and the input is $V_p \cos \omega t + \phi$ I could do this exploration s t, but I do not want do it right now. In fact, in networks and systems that is what you will do and every component will have a value, which is the functions just like here for capacitor we had that $1/s$, which look like some resistance. So, will have that, but in this case will do everything with sinusoidal are complex is financial, so I have the frequency is imaginary.

So, this is the circuit, but I want to calculate the solution for solution for meaning may be I want find every branch voltage and every branch current. So, how do we do it first of all use the standard trick of a instead of using this as the input will use $V_p \exp(j\phi) \exp(j\omega t)$ that is in our standard think to do. Now, because we only real value component inside I calculate the solution to this and let say I have some branch current and branch voltage I take the real part of that, that will be the solution do that.

This part of everybody clear about we got that simply by super position I mean we did not do any arbitrary think like just taking the real part or something we super post $\cos \omega t + \phi$ and $\sin \omega t + \phi$ with different waiting factors. So, that become this the only reason to use this is the solution to this is not easier calculate, then that one, so then we find the real part.

So now, let say I have some elements let us not worry about exactly, what they are right now always I will loop in the circuit, let me call them element may be elements a b and c. So, now, one be the general form of the voltage V_a for this input constant times, what is the general form of the voltage V_a for this case with this complex exponential as the input it will be some V_a will be some complex number I will denoted like that or may be assume do this.

But, let say I use lower case V_a to denote the voltage and that will be this is just a complex constant times exponential geometric it is going to be like this for sure why because it is a linear network and it satisfy some linear differential equation it could be ((Refer Time: 03:34)) complicated. But, whatever it is if the forcing faction exponential geometric, then the every exponential geometric will satisfy the different equation with some scaling factor we won't worry about, what that is we just denote the amplitude by some complex number, this is the post response this only the post response the post response the start depend on initial condition is that, that is why I clarify that earlier.

The natural response will depend on the forcing function, but the force response by definition does not depend on the initial condition if the input is exponential geometric there every voltage and current will be exponential geometric time something is that. So, that is all I am saying; that is that will calculate later that will be the case, because if the input is some exponential geometric all the voltages will be something times of that exponential geometric and the Fahrenheit scale with V_p comes from linearity that also I mentioned earlier the force response scales with the forcing function.

So, that is also will be the case, what is we are looking for is the easy way to calculate V_a without going into the differential equation, no no this is we are looking at the solution to this part. So, I am not looking at you no worry about this anymore, because I know how to do that if we I have $V_p \cos \omega t + \phi$ I will calculate the solution to $V_p \cos \omega t + \phi$. And I calculate any quantity I want any branch voltage or current and I will take the real part of it that read taking the real part comes at the end.

So, this V_a is, so let me we express it force response to V_p exponential $j\omega t$ plus ϕ , which I have written as some scaling factor some complex scaling factor time exponential $j\omega t$ you see very soon why that, why I did that. And, what will be the form of V_b , what is that some complex number times also exponential $j\omega t$ and V_c always we expect the same thing some other complex number we do not know, what this complex numbers are, but it will be of this form it is confusion or not is it fine. Similarly if I have a node will, let say I will take three elements connected and I take i_d and i_f i_d will be some complex number exponential $j\omega t$ i_e will be some other complex number exponential $j\omega t$ and i_f will be yet other complex number exponential $j\omega t$ this is.

Everything will be exponential $j\omega t$ time something under something can be a complex number that is all I am written, so far that is fine. So, this is true of every voltage and every current in the circuit it will be some complex number time exponential $j\omega t$. Now, clearly around every loop KVL is obeyed, so nothing controversial over that, so V_a plus V_b plus V_c equals 0 this is we are talking about only the force response, why it is not obeyed for force response.

Now, the natural response will have exponential minus t by RC or something right the natural modes of the system the exponential $j\omega t$ can never cancel that KVL has to wait for the force response and the natural response individually is not it. First just simplify the situation, now let us assume that the natural responses guide out every circuit for, which this calculation is useful will see that calculating only the force response is useful that is because the natural response will die out after sometime.

So, the natural response everywhere will die out and you will be left out only with the force response. So, you can imagine that, so you can imagine that we are calculating it in that situation, where the natural response will become negligible. Now, the other point is that the force response will have exponential $j\omega t$ the natural response will have the natural modes of the system, let say exponential $p_1 t$. So, V_a will have some exponential $j\omega t$ and some other part with exponential $p_1 t$ see this exponential $j\omega t$ can never cancel that exponential $p_1 t$.

So, around each loop the part of the natural response have to balance themselves and the parts of the force response also have to balance themselves. So, that is not a problem, but if the second thing as I said is little complicated do not worry about it just assume that

the natural response has died out. So, $V_a + V_b + V_c = 0$, so that actually means that these complex numbers $V_a + V_b + V_c$ themselves are 0, because this time exponential $j\omega t$ plus that exponential $j\omega t$ plus V_c exponential $j\omega t$ is 0, but exponential $j\omega t$ are common factor to all the voltages in the loop.

So, if you just look at to this complex factors multiply each voltage in the loop they also sum to 0 there will be some quantities were the dimension are voltage and they obeys Kirchhoff voltage law. Similarly, the currents at every node, so this is around loops and this is at nodes $i_d + i_c + i_e = 0$ so; that means, that $i_d + i_e + i_f = 0$ $i_d + i_e + i_f$ these complex numbers, which have multiplied exponential $j\omega t$ they themselves are 0.

So, you can think of these complex numbers complex multiples themselves as currents remember the actual currents are i_d times exponential $j\omega t$, but because of comes out of the common factor we can remove that all together is it.

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$$\begin{matrix} \text{node 1} \\ \vdots \\ \text{node n} \end{matrix} \begin{bmatrix} \text{Matrix of} \\ \text{complex numbers} \end{bmatrix} \exp(j\omega t) = \begin{bmatrix} V_p \exp(j\omega t) \\ 0 \\ \vdots \\ 1 \end{bmatrix} \exp(j\omega t)$$

MNA setup

So, clearly now you can imagine writing, let say you are doing nodal analysis writing KCL equation at every node. What you will have? We will have some matrix of entries corresponding to KCL at node 1 and node n and everyone of these entries will have exponential $j\omega t$ every current will have exponential $j\omega t$. So, I can just take that thing outside and this will be just a matrix of complex numbers.

So, let say this is the modified nodal analysis set up I will show it in abstract we will see how we can make more concrete this is fine and this will be equal to the source vector

and what is that in this particular case I will put the source vector in the top, but you can put anywhere I mean. So, I will assume there is only one source, but you can have multiple sources. So, let us assume just a single source. This is understood by the currents we have. So, let me not worry about how to do this. This is a linear set of linear equations. I will have it like this. (Refer slide Title)

Node 1: $(\bar{i}_a + \bar{i}_b + \dots) \exp(j\omega t) = 0$

V_{01}

Forced response to $i_p \exp(j\omega t + \phi)$

$v_a = \bar{V}_a \cdot \exp(j\omega t)$
 $v_b = \bar{V}_b \cdot \exp(j\omega t)$
 $v_c = \bar{V}_c \cdot \exp(j\omega t)$

$i_d = \bar{i}_d \cdot \exp(j\omega t)$
 $i_e = \bar{i}_e \cdot \exp(j\omega t)$
 $i_f = \bar{i}_f \cdot \exp(j\omega t)$

$v_a + v_b + v_c = 0$
 $\bar{v}_a + \bar{v}_b + \bar{v}_c = 0$

$i_d + i_e + i_f = 0$
 $\bar{i}_d + \bar{i}_e + \bar{i}_f = 0$

@ nodes

Let me make this current source. So, let me make it $i_p \cos \omega t + \phi$, which makes it $i_p \exp(j\phi) \exp(j\omega t)$.

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Node 1: $(\bar{I}_a + \bar{I}_b + \dots) \exp(j\omega t) = 0$

Node 2: $(\bar{I}_p + \bar{I}_y + \dots) \exp(j\omega t) = I_p \exp(j\phi) \cdot \exp(j\omega t)$

Diagram showing a resistor R with voltage $\bar{V}_R \exp(j\omega t)$ and current $\bar{I}_R \exp(j\omega t)$. Below it, the phasor relationship $\bar{V}_R = \bar{I}_R \cdot R$ is shown, along with the real-time voltage $\hat{V}_R \cos(\omega t + \phi)$ and current $\hat{I}_R \cos(\omega t + \phi)$.

For some particular node, where the current source is connected, let me call it node 2 may be I will have something. So, I_x plus I_y plus some other currents times exponential $j\omega t$ were all these are the just currents connected to the second node $I_p \exp(j\phi) \exp(j\omega t)$. So, entry on the left hand side will have the exponential $j\omega t$ as common factor and every source element will also have exponential $j\omega t$ other as a common factor.

So, you can get rid of this all together and write these KVL or KCL relationship only in terms of complex multiple or exponential $j\omega t$ this is fine and d is complex multiples of $j\omega t$ these are what are known as phasors ((Refer Time: 15:05)) as we gone it is quite common to say the voltage across these two node is 1 plus a 1 something, but you have to understand, what that really means what does it mean real part of what.

So, what it means is if you apply the input and wait for the long time if you apply the sinusoidal input let say $V_p \cos(\omega t + \phi)$ and wait for a long time the actual voltage across that will be 1 plus time exponential $j\omega t$ and real part of that, so that is what it mean. So, you should that although we will do all the manipulation in terms of phasors and also the voltages are some complex constants this is fine.

The reason this is valid is because every voltage around the loop every voltage around every loop has a exponential $j\omega t$ as common factor and every current at a node arriving at a node exponential $j\omega t$ as a common factor. So, in circuit analysis we can do all our calculation without this exponential $j\omega t$ this is fine next, what we

have to do is to find the element relationship. Because, how we do any circuit analysis with KCL, KVL and element relationship what we have shown, so far is that in all our KCL and KVL equation we can just remove this exponential $j\omega t$ and then move out.

Then, what you are doing with element relationship in fact that only like two special cases with we have resistor and you have some V_R exponential $j\omega t$ across it, what is the current through it I_R exponential $j\omega t$, but what is the relationship between V_R and I_R V_R equals I_R times R . So, how do we get this actually the actual voltage is some something that is a I call this V_R hat and the current is I_R hat $\cos \omega t$ plus ϕ .

So, this I_R is nothing, but I_R hat exponential $j\phi$ and after that they will have exponential $j\omega t$. So, we know that V_R hat is I_R hat times R , so that is the convention relationship for a resistor, now when you this complex voltages the same relationship will hold and this V_R the complex number is nothing but, V_R hat exponential $j\phi$ and after that you will have exponential $j\omega t$. So, clearly you have the same exponential $j\phi$ here and there.

So, we are this complex number is that times complex number times R . So, similarly you can work out for capacitor and inductor will do that tomorrow, now V_R hat and I_R hat are real they are the actual voltages amplitudes of voltages across the resistors. So, if the voltages across the resistor is some V_R hat $\cos \omega t$ plus ϕ you know that the current is just the voltage divided by the resistor. So, now, whatever calculation we have to do with cosine and sine were too complicated, so we will translate everything into the exponential domain and then, do it.

So, this analysis we will see right now we will assume only one source and this analysis is valid only for that frequency, because all those complex number that we calculate will depend on the frequency as well. So, will continue from here tomorrow and then, look at element relationship and how to solve circuits in general.

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EC/010: Lecture 30. **Phasors** Sinusoidal steady state analysis at a frequency ω (KVL)

Linear circuit R, L, C

$I_p \cos(\omega t + \phi)$

$V_a = \bar{V}_a \cdot \exp(j\omega t)$

$V_b = \bar{V}_b \cdot \exp(j\omega t)$

$V_c = \bar{V}_c \cdot \exp(j\omega t)$

$i_d = \bar{i}_d \cdot \exp(j\omega t)$

$i_e = \bar{i}_e \cdot \exp(j\omega t)$

$i_f = \bar{i}_f \cdot \exp(j\omega t)$

$V_a + V_b + V_c = 0$

$\bar{V}_a + \bar{V}_b + \bar{V}_c = 0$

$i_d + i_e + i_f = 0$

$\bar{i}_d + \bar{i}_e + \bar{i}_f = 0$

(KVL)

(KCL)

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Yesterday we started looking at sinusoidal steady state response of linear circuits in particular, how to calculate it for arbitrary circuits. One method is of course, is to write the differential equation with the correct variable and when you had sinusoidal steady state response you will really find the response for exponential $j\omega t$ and take a real part, but we do not want to be writing differential equation every time.

So, what we do is use the fact that if you have exponential $j\omega t$ as every response in the circuit will be something types exponential $j\omega t$ some complex number times exponential $j\omega t$. So, given that this exponential $j\omega t$ comes out as a common factor. So, that is why I illustrated with some circuit, so this is linear circuit and now, of course, it includes inductor and capacitors and if we have a loop and you have, let say V_a , V_b and V_c and the excitation lecture is a we can take a current or the voltage source it does not matter this is some $I_p \cos(\omega t + \phi)$, which has been substituted by $I_p \exp(j\phi) \exp(j\omega t)$.

So, now, this is sinusoidal steady state analysis at a frequency ω that is crucial we have the exponential $j\omega t$ here or $\cos(\omega t + \phi)$. Now, as I said the steady state response for of any a quantity in the circuit will be of the form of some complex number exponential $j\omega t$. I use this within the over part, let us it is in over part to indicate there it is a complex number. And similarly, if you have a number of currents entering the node i_d , i_e and i_f , then each of those also is of the same general form.

So, now,; that means, that every branch holder and every branch current in the circuit has the exponential circuit has a exponential geometric angle of the expression. So, instead of writing KVL around $V_a + V_b + V_c = 0$ this is of course, correct we can instead use only this part of it, because the rest of this is the common factor any way. So, we can write the sum of these complex number $V_a + V_b + V_c = 0$ and these multipliers these complex numbers, which are multiplying the exponential complex number $j\omega t$ are what are known as phasors.

So, this \bar{V}_a is the phase corresponding to the voltage V_a \bar{V}_b \bar{V}_c and so on and similarly for current. And of course, now for currents we have to write KCL, which would have been $i_d + i_e + i_f = 0$ instead we can simply write the sum of these complex numbers to be 0. So, we can write KVL and KCL just in terms of this complex numbers instead of using loop time domain voltages, any questions about this these are the phasors this is how they come over.

We will see we will, because these are complex numbers they have some amplitude and angle we can make useful relationships on a phasor diagram a complex number is nothing but, the two dimension vector write. So, you can on a two dimensional plane plot these things $V_a + V_b + V_c = 0$ means that these are the part of a diagram that is fine.

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The image shows handwritten notes on a blue-lined background. At the top, there are three circuit diagrams: a resistor R , a capacitor C , and an inductor L . Each diagram shows a voltage v across the component and a current i flowing through it. Below each diagram are the corresponding phasor equations:

	$v_R = \bar{V}_R \cdot \exp(j\omega t)$	$v_C = \bar{V}_C \cdot \exp(j\omega t)$	$v_L = \bar{V}_L \cdot \exp(j\omega t)$
Voltage	$i_R = \bar{I}_R \cdot \exp(j\omega t)$	$i_C = \bar{I}_C \cdot \exp(j\omega t)$	$i_L = \bar{I}_L \cdot \exp(j\omega t)$
Current		$i_C = C \cdot \frac{dv_C}{dt}$	$v_L = L \cdot \frac{di_L}{dt}$
	$\frac{v_R}{i_R} = R$		
	$\frac{\bar{V}_R}{\bar{I}_R} = R$	$\bar{I}_C \cdot \exp(j\omega t) = j\omega C \cdot \bar{V}_C \cdot \exp(j\omega t)$	$\frac{\bar{V}_L}{\bar{I}_L} = j\omega L$
		$\bar{V}_C / \bar{I}_C = 1/j\omega C$	

So, now, that simplifies the life somewhat, so we can get rid of exponential $j\omega t$ and differential equation and so on. But, the reason first of all where and why we are had

differential equation in the first place is, because inductors and capacitors current voltage relationships that have derivative in them. So, now, what we do about those, so let us see what happens in the phasor domain as we call it.

So, let say the voltage across this is V_R and the current is I_R and V_R is some complex number exponential $j\omega t$ and I_R is some complex some other complex number exponential $j\omega t$. So, now, ohms law of course, says that V_R by I_R equals R if this resistance is the value R and that consequently also means that the ratio of the complex number V_R bar divided by I_R bar equal also R its extremely simple this is for the resistor.

Now, let us take a capacitor I have V_C and i_C and this is the capacitor C . So, again V_C is some V_C exponential $j\omega t$ and i_C is i_C bar exponential $j\omega t$, what is the relationship. So, from this, what do we have, what is it that we get out of this i_C bar exponential $j\omega t$, which is the current is $j\omega C V_C$ bar exponential $j\omega t$. So, clearly the ratio of voltage to current phasors will be 1 by $j\omega C$.

So, the key point here is that, what is the integral or the derivative relationship has become an algebraic relationship just like ohm law that is the advantage of doing this. What is the limitation here? Can we use this everywhere, what is the context in which, we can treat the voltage current ratio of a capacitor like this like a number, what exponential this is only for sinusoidal steady state it is true for exponential also.

But, this particular thing is for a sinusoidal steady state at the frequency of ω if you have the arbitrary wave form for V_C and i_C you have to relate them using the integral. Now, it is sinusoidal that makes that there is a sinusoidal exponential frequency ω and its steady state; that means, that the natural responses are let out, so that is the context in this, this whole thing is useful. And similar for an inductor v_L and i_L and this is an inductor of L .

So, v_L is v_L bar exponential $j\omega t$ and i_L i_L bar exponential $j\omega t$ and using the same relationship v_L is L times $d i_L / dt$ we get, what is the ratio of v_L by i_L $j\omega L$ is not it. So, now, the advantage is that all of these especially the more complicated elements like capacitor and inductor also have a voltage current, which are simply related by some ratio constant ratio. So, as far as the analysis is concerned you could treat them just like resistors this is fine.

So, this is our I thing. You probably you already familiar with phasors and how to use them for calculation, but this is how they come over. Because, this the sinusoidal with sinusoidal excitation you will get all of the wave form everywhere in the circuit to be sinusoid of the same frequency. So, now if you represent that I mean if you do the usual super position and use the exponential $j\omega t$ instead of $\cos\omega t + \phi$ everywhere you will have this exponential $j\omega t$ as a common factor.

So, you can drop that exponential $j\omega t$ from every voltage and current in the circuit and pretend the these numbers which are phasors themselves obey KVL and KCL they do, but and also as I said as we drawn i keep saying that oh this v_r is the voltage and $I R$ is the current and So on. But, what is it really mean what is the voltage across the resistor. So, it is a if you apply the sinusoidal frequency ω ; that means, that the voltage across the resistor will be this complex number times the exponential $j\omega t$ the real part of that 1.

So, that is the meaning of this whole thing, so you should not loose connection to this although this is used, so widely that after a while you use this using this exponential $j\omega t$ will become second. So, this is used everywhere in communication systems in circuits. So, if you use here the exponential $s t$ instead of ωt this $j\omega$ will be replace by s . So, that is with and s can be a general complex number it does not have to be purely imaginary the exponential $s t$ verses purely imaginary that corresponding to is the combination of this sine cos, where as if you have a complex exponential you have some exponential module as soon as well.

So, you can use that in fact, that is the basis for this Laplace transform analysis that you will see in the next semester. So, always and capacitor can be replaced by an no inductor capacitor can be replaced not a resistor it is some.

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The image shows handwritten notes on a digital whiteboard. At the top, it lists the impedance formulas for three components: Resistor ($Z = R$), Capacitor ($Z = \frac{1}{j\omega C} = -\frac{j}{\omega C}$), and Inductor ($Z = j\omega L$). Below this, it lists the admittance formulas: Resistor ($Y = \frac{1}{R} = G$), Capacitor ($Y = j\omega C$), and Inductor ($Y = \frac{1}{j\omega L} = -\frac{j}{\omega L}$). In the center, a circuit diagram shows a resistor R and an inductor L connected in series between two terminals. The voltage across the terminals is V and the current entering is I . To the right of the diagram, the total impedance is given as $Z = \frac{V}{I} = R + j\omega L$. Red annotations identify R as 'resistance $Re\{Z\}$ ' and $j\omega L$ as 'reactance $Im\{Z\}$ '. Below the diagram, the admittance is calculated as $Y = \frac{I}{V} = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2} = G + jB$. Blue annotations identify G as 'Conductance $= Re\{Y\}$ ' and B as 'Susceptance $= Im\{Y\}$ '. The real part of the admittance is labeled 'resistance $Re\{Z\}$ ' and the imaginary part is labeled 'susceptance $Im\{Y\}$ '. The page number '226 / 226' is visible in the bottom right corner.

The generic term for this it is known as impedance, impedance is basically by voltage by current phasor for a resistor; obviously, the impedance is R itself because whatever the value of whatever the voltage is in current you have the ratio is always R you have and for a capacitor it is one over j omega C or minus j over omega C and for the inductor it is j omega L and it is common to represent this impedance by Z .

The inverse of this that is known as admittance, which is y and that is i by V it is a analogous quantity to conductance and for this it is G , which is 1 by R and this is j omega C and 1 by j omega L or minus j by omega C this is fine. Now, we have only looked at resistor capacitor inductor you can define the same, now for any voltage and current that is, let say you have some any network with two terminals you can take the voltage across it and the current going through it.

So, earlier we did this we took a circuit whose two terminal for exposed we applied a voltage and form the current or somehow we know the voltage and the current the ratio of the voltage to current was the equivalent resistance. Now, equality the equivalent impedance similarly i by V would be the equivalent admittance. So, now let say inside this I have a series connection of R and L , what is the equivalent impedance R plus j omega L .

So, you just add up the impedances for series components similarly for components in parallel you add up the admittances and so on, whatever we would take in resistance the same thing apply to impedances whatever you take with conductance's. Now, it applied

to admittances only the here is that the resistances and conductance's are always real numbers and for a physical resistance and conductance there always a positive numbers, where as this Z and Y the impedance and admittance are complex numbers.

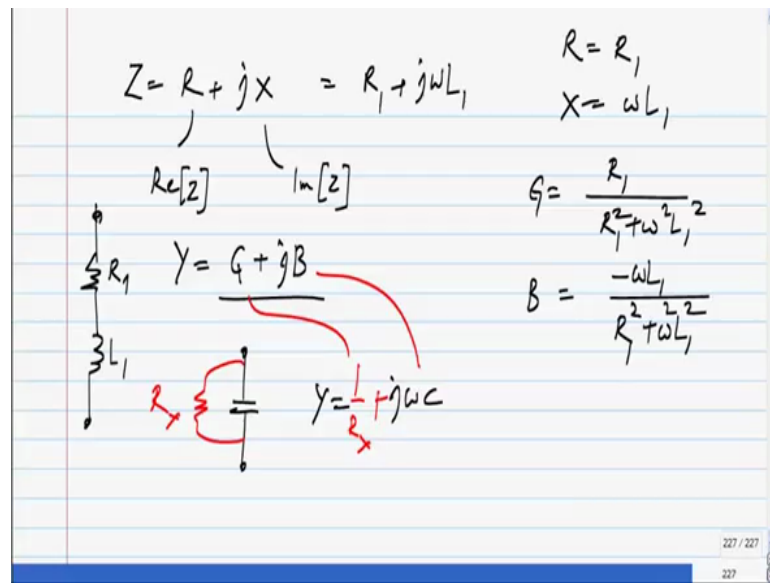
So, this impedance could have a real part as well as an imaginary part and, where as the capacitor and inductor have a purely imaginary impedance. But, in general for a complicated network you could have both real and imaginary parts and the real part of this impedance is called resistance and the imaginary part of this is called the reactance. So, it is common to denote the reactance by x .

What is the admittance of? This is the reciprocal of $R + j\omega L$ I will write it as $\frac{1}{R + j\omega L}$ I will maximize it and write it as $\frac{R - j\omega L}{R^2 + \omega^2 L^2}$ Now, this in the form of $G + jB$, where G is the conductance, which is the real part of the admittance and this B , which is the imaginary part of the admittance this is known as susceptance. So, in this case this part ωL by $R^2 + \omega^2 L^2$ is B .

So, in general this quantity is R or $L \times G$ and B can all be frequency dependent. So, in this case you can see that R is G is frequency dependent it depends on frequency is it no their resistance there resistance is frequency independent the resistance of some complicated circuit that is not frequency independent resistance by definition is the real part of the admittance. The real part of admittance is not necessarily frequency independent the resistance is just a resistor of course, it frequency independent.

So, there their resistance will be 0, what is that no Z is $R + jx$, what R and x are depends on what circuit is inside I mean I put an resistor and inductor in series as I put something like this, what is it you want to get some complex quantity. So, the real part of Z is the resistance of that circuit it is not the resistance that physically inside. So, maybe I confuse the issue by starting with the example, which are R and L series.

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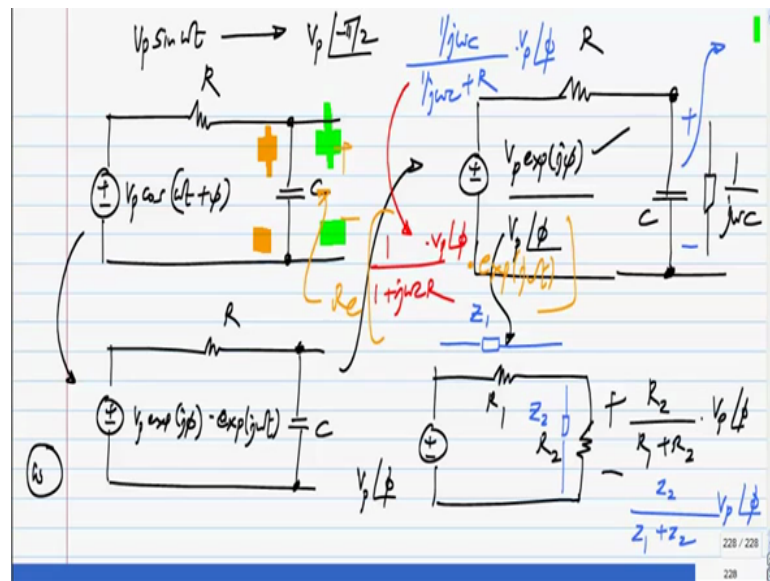


But, do not maybe I use the different symbol, so Z is some R plus $j x$ by definition this is the real part of Z and this is the imaginary part of Z . So, for that particular circuit we have, let say R_1 and L_1 this R plus $j x$ is happens to be R_1 plus $j \omega L_1$. So, the resistance of this network happens to be R_1 and the reactance of this network happens to be ωL_1 . And similarly, Y is G plus $j b$ the conductance of this network is R_1 by $R_1^2 + \omega^2 L_1^2$, what R_1 square and the susceptance b is minus ωL_1 by $R_1^2 + \omega^2 L_1^2$.

So, these are just definition, now every equidance has to constant inside if you have only an inductor then its resistance is 0 and the reactant is ωL we have only a capacitor its conductance is 0 and the susceptance is ωC and so on any questions about this if you have only a pure capacitance the admittances is $j \omega C$. So, if you write it as g plus $j b$ $G = 0$ b is ωC if you have a purely imaginary admittance, then the conductance is 0 the conductance part is 0.

If I connect some resistance R_x across it what will be the admittance R_x plus $j \omega C$ you cannot add those things R_x is resistance and ωC is the dimension of conductance $1/R_x$ plus $j \omega C$ you have to add the admittance of the resistor which is one by R_x and admittance of the capacitor, which is $j \omega C$ in this case G is this and b is that any other questions.

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So, now, this is a great simplification the fact that the voltage and current are related by some ratio some constant number that is the great simplification and it enables the analysis a circuit in a sinusoidal steady state very easily this. So, you we could do in various ways were writing out the differential equations and assume the solution of the form $a \cos \omega t + b \sin \omega t$ or expanding the cosine in terms of complex exponential or the most convenient one, which was to replace this with $V_p \exp(j\omega t + \phi)$.

Solving the differential equation with this in this case it was particularly easy because the exponential $\exp(j\omega t)$ here solutions also of the form $\exp(j\omega t)$, but now, we not even need to do this. So, what we do is everywhere we calculate only the phasors once we have the phasors we know what the real voltage is in terms of complex exponential it is the that phasor time exponential $\exp(j\omega t)$.

So, you should be given the value of frequency it is a function of ω . Now, in the actual domain that is in the actual time domain corresponding to this circuit it will be the phasor time exponential $\exp(j\omega t)$ the real part of the that whole product. So, what is the phasor corresponding to the voltage source what is this $V_p \exp(j\phi)$, so this is how phasors are usually written. So, there is a further shorthand to this normally you write it may be as $V_p \angle \phi$.

So, that suppose to mean $V_p \exp(j\phi)$ it is a complex number alternatively you could also write it as $V_p \cos \phi + j V_p \sin \phi$. Now, for us $\cos \omega t$ is the

reference if ϕ is 0; that means, that $V_p \angle \phi$ the phasor will be $V_p \angle 0$ of the phasor will be a real number. And if you want $V_p \sin \omega t$, what is the phasor corresponding to this $V_p \angle \phi$ by 2ϕ is $\pi/2$ minus $\pi/2$ $V_p \angle \phi$ minus $\pi/2$ $V_p \angle \phi$ exponential minus $j\phi/2$ any ways you can write the phasor of it.

So, this is known as the rectangular form this is the polar form and this is the some more compact representation of the polar form. So, the complex number had some amplitude and angle of magnitude and phase. So, the magnitude of that is the peak value of the sinusoidal the angle of that is the phase of the sinusoidal with respect to $\cos \omega t$ you can only measure the phase difference you have to had some reference signal the reference signal for us is $\cos \omega t$.

So, signal when you have phasor of this; that means, that in the actual circuit you had a signal which is, where something like this where this is ϕ . So, the point is it is just some complex number and then, they we have R and we have C basically this is like having an impedance this is the general term general symbol for the impedance and box with two wire signing of it and the impedance has a value $1/j\omega C$.

Now, because of the ratio of voltage and current is just some number every technique that you had for analyzing only the resistive circuits you can use here for instance if I had R_1 and R_2 and $V_p \angle \phi$, what is the voltage across this its R_2 by $R_1 + R_2$ times that voltage. And if you had this impedance Z_1 and Z_2 instead what would will be exactly the same Z replacing the R. So, what is it in this case, so it is Z_2 by $Z_1 + Z_2$ times $V_p \angle \phi$ and this can also be simplified as or re written as $1/(1 + j\omega C R)$ times $V_p \angle \phi$.

When you solve call the differential equation if you recall you had that $V_p \angle \phi$ exponential G getting multiplied by $1/(1 + s C R)$ that is exactly, what we have here with s substituting s is substituted by $j\omega$. So, this thus gives you the right answer, now why is this useful this is useful, because let say you apply a sinusoidal voltage to a circuit initially there will be a some response after that the transient response will die out assuming that the system is stable and after that you will get only the steady state response.

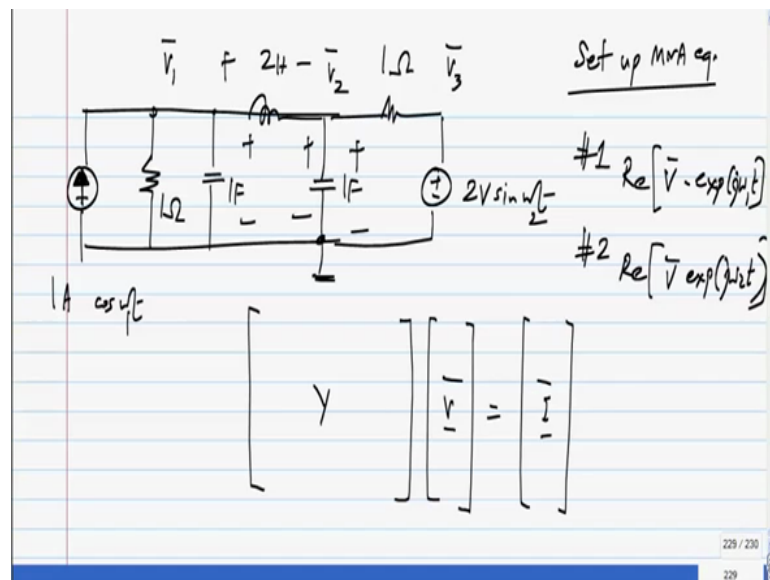
So, now, this is the straight forward way more straight forward way of calculating the steady state response compare to solving the differential equation. In fact we do not need to write the differential equation sort out we saw that first order was very easy second

order is slightly more complicated for higher order it is a bit more complicated, where as in this case I mean in this particular example I have one capacitor, what I could have 100 that is not the problem any question real part of what not real part of this, but what should we do to get the actual voltage across this steady state voltage.

So, I have to multiply this by exponential $j\omega t$ and take the real part of that whole thing that should be the voltage here up to you wait a long enough time you have to wait long enough time to for the natural response to die out. So, like I said there is no life as electrical engineer s without complex number, so we will use phasors all over the place, but you should lose connection to the actual voltages and currents in the circuits. And also you see that these numbers the numbers that are multiplying the voltage etcetera are frequency dependent.

So, when you have capacitor and inductor you can have circuits that behaves different wave different frequency that is actually a nice speech out. So, you could design circuits that will behave some wave may be they will block certain frequencies that will allow some other frequencies and so on. So, that is how we make filters will see some examples on that later any other question about this.

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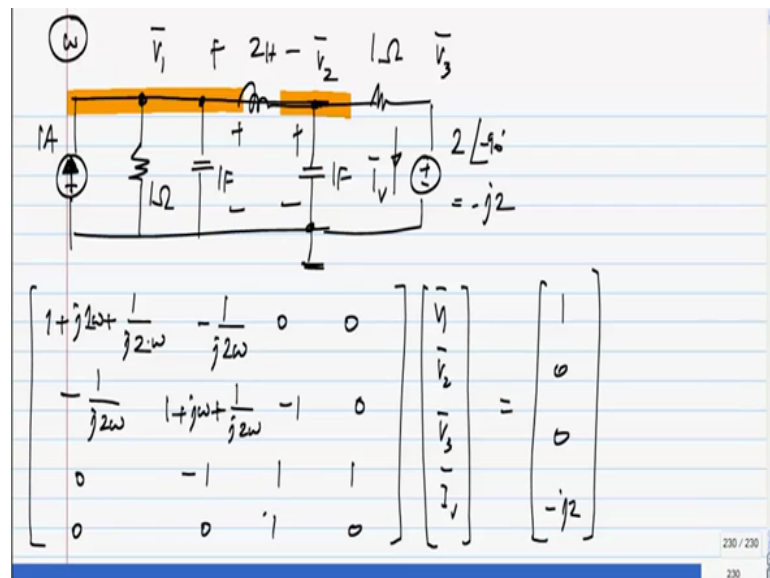
How about you go solving something like this, what said one ampere this is a current source suppose we have current source how would you go solving this superposition what all other techniques do we know. So, substitution of what I want everything here lets I have voltage across this voltage across that voltage across that everything two pole

actually two pole is the wrong thing we do not know anything inside the two pole two pole is an abstraction for, what happening outside, what do we do and what are the general solution to the circuit this is only the force response I did not tell or I am not going to say this every time.

But, in this case is the sinusoidal steady state response I am looking after, So, we have things like modifying nodal analysis, whatever we did for resistive circuit we can do it here. So, please do that please set up the, so let me say this is the reference node actually there are only two voltages V_1 and V_2 except of course, each one would be a phasor law. Because, the voltage across the inductor is V_1 minus V_2 and you also have this if you set up let say V_3 .

So, please setup the modified nodal analysis, so this is also the good opportunities to revise all those things we had long back. What I want is? Now, I call this y matrix it is a admittance matrix instead of conductance times the vector of unknown voltages which is equal to the source vector with the usual KVL the source vector does not have to be only current and unknown vector does not have to be only voltages. So, please do this; obviously, you have to use the impedance and admittance of different elements in this admittances really in nodal analysis setup.

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First of all I here exactly, what I am going to do this use a impedances and admittances impedances or admittances for every element also the phasor representation for this what is this, what is the phasor corresponding to this current source one ampere and what is

the phasor corresponding to this nodal source yeah or you can write the is minus $j 2$ and vibrate nodal analysis accept. So, formally you need four variable three voltages and this, so let us see ω and then, what is here the admittance between V_1 and V_2 .

The third $1 0$ coping and last $1 0$ and for second row minus 1 by $j 2 \omega$ and here it is somehow this is in that same plus 1 and the last 1 third 1 minus 1 last one 0 . And next one what is it 0 minus $1 1$ or minus 1 the average answer, what I get is $0 1$ or minus $1 1$ and here $0 0 1 0$ equals, what you can find all those phasors and it is multiplying it is by geometric integral real part we will get all these super positioned to do this define this oxidant variable two variable V_1 and V_2 if I convert this to.

So, everything I predicted earlier steps we can do, so I am not going to going to nodal and mesh analysis routine algorithm only thing is if the we have the admittance in conductance. Similarly, for mesh analysis instead of resistance to know the circuit analysis Thevinins theorem and some associated theorem reciprocal theorem maximum power transfer theorem all those things we can discuss with this in this sinusoidal steady state case then see what comes and some case it may be different from others and so on. And we can have essentially mostly, what happens is that wherever you had the real number before you end up with complex numbers some results will be quite interesting as well and we can have everything like two points with complex complexes parameters and so on. In fact, now it should be clear why it is called y parameter not g right y stands for admittance. So, current as function or voltages all those quantities will be admitted.

So, in general case it is called y parameters similarly the one side you have resistance that were called Z because Z is for impedance any questions about any of this that is not the linear circuits we cannot use any of this no I mean you can analyze it in any way you want, but your question is different if I put a divert here that is a non-linear circuit. So, I have to analyze it as a non-linear circuit any questions about this frequency of, which that is the very interesting question.

So, let say this was $\cos \omega t$ and this was $\omega 2 t$ I still want to find the steady state voltages across all the elements, what should I do I have to use super position. So, I have to superpose cases, where I have voltage of all excitation of one particular frequency only and other particular frequency. And then, calculate them separately and then I can add the final voltages that is the final voltages, which are in the time domain.

So, I take first of all from the first set I calculate the phasor time exponential $j\omega t$ this will be give me some voltage. Let say this voltage across this capacitor and in the second case I get real part of V exponential $j\omega t$ that will give me some other voltage those two voltage I can add up when I am doing sinusoidal steady state analysis I have to take one frequency at a time any other questions I thought somebody ask something here.

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$\omega = 10^3$
 $1\text{mA} \cos(10^3 t) : v_c = \sin(10^3 t) \text{V} \quad i = C \frac{dv}{dt}$ Solution for $t > 0$
 $\text{Circuit: } 1\text{mA} \text{ source, } 1\mu\text{F} \text{ capacitor, } v_c = 0 \text{ at } t = 0$ $\frac{1}{j\omega C} = -j10^3 \Omega$
 $1\text{mA} \sin(10^3 t) : v_c = [1 - \cos(10^3 t)] \text{V}$
 $1\text{mA} \cos(10^3 t) \quad \sin(10^3 t)$
 $1\text{mA} \angle 0 \times (-j10^3 \Omega) = -j1 \text{V}$
 $1\text{mA} \sin(10^3 t) \quad [1 - \cos(10^3 t)] \text{V}$
 $-j1 \text{mA} \times (-j10^3 \Omega) = -1 \text{V}$

So, let us try another small problem I think you may have solved this in one of the earlier tutorial I do not remember. So, forgot this all phasor analysis phasor sinusoidal steady state analysis all of these just from the fundamental relationships $I = C \frac{dv}{dt}$ you calculate this rather. So, you may be, so these three rows please do it for this excitation and these two please do it for 1 milli amp sine $10^3 t$ the initial condition on the capacitor is 0.

So, what I want is to solution for t greater than 0, so please do it for 1 of you do it for cos rest of you do it for sine extremely easy and please tell me the solution I think this was in the first tutorial or something else for the solution to this in this case V_c happens to be what 1 minus $\cos 10^3 t$ volts.

In this case $\sin 10^3 t$ volts fine what is the phasor corresponding to this, what is the phasor corresponding to that 1 milli amp $\cos 10^3 t$, what is the phasor 1 milli amp ω is of course, 10^3 , what is the impedance of this at that frequency 1

over $j\omega C$, which is $-j10$ to the 3 ohm again do not forget the units and this current time voltage is the current times impedance is the voltage.

So, what is the first let me list out the inputs and outputs when it was 1 milli amp $\cos 10$ to the 3 t the output was $\sin 10$ to the 3 t and the input in terms of phasor was 1 milli amp angle 0 if you want to express it and this times $-j10$ to the 3 ohms this should have been how much $-j$ volts at that frequency and when you have this other 1 milli amp $\sin 10$ to the 3 t the actual voltage is 1 minus $\cos 10$ to the 3 t volts.

So, what is the phasor corresponding to this input $-j1$ milli amp and that times the impedance it is $-j10$ to the 3 ohm $-j1$ kilo should be how much $-j$ volts that is what $-j1$ bits. So, now, what you calculate from time the domain and using the phasor analysis are they consistent yes. So, $\sin 10$ to the 3 t is $-j$ volts that is what about this here I got $-j1$ volt and there I got something else.

What is the, what is time domain, what is the correspondent to this phasor here the phasor is $-j1$ volt what is the voltage $\cos \omega t$ ω here is 10 to the 3 volts, but I have got something extra here why is that. So, this sinusoidal steady state analysis suppose to be useful right what happen here please thing over it will discuss it tomorrow it is not very difficult.