

**Basic Electrical Circuits**  
**Dr Nagendra Krishnapura**  
**Department of Electrical Engineering**  
**Indian Institute of Technology Madras**

**Lecture - 145**  
**Forced Response of a Second Order System**

(Refer Slide Time: 00:05)

Forced response of a 2<sup>nd</sup> order system

open circuit C  
short circuit L

$V_s$ : constant  $2V$   $V_c = \text{final value} = V_s$

$\zeta = 1/10$   
 $Q = 5$

Steady state response + natural response

$$V_c(t) = V_s + A_1 \exp(p_1 t) \cos(\beta_1 t + \phi)$$

$C \cdot \frac{dV_c}{dt} \Big|_{t=0} = S_0 A$

$$V_c(0) = 2V = V_s + A_1 \cos(\phi)$$

So, now, I can go on to the forced response of a second order system, because we have already discussed the forced response of first order in great detail, this should not be very difficult. Again, I will take particular circuit as examples, but it understood that this applies to anything else. Let say  $V_s$  is constant what do you expect the force response to be of  $V_c$  constant; and in this case  $V_s$  itself how do you find it, in case of  $R_c$  circuit, we had some algorithms finding the final value. When you have both  $I(s)$  and  $c(s)$  what do you do, open circuit the capacitors and short circuit the inductors. So, you cannot have a dc voltage across an inductor, because if you do that a current will be growing. So, if you want have a dc steady state the inductor voltage should be zero and the capacitor current should be zero, so that is to find the final values, you open circuit the capacitor and short circuit inductors, this you would have any way for  $RL$  circuits as well, but for  $RLC$  circuit it is the same way.

So, if I have a constant input, so the steady state response  $V_c$  what it is, what is the final value of the response, I mean not the steady state response of the function of time. The final value is... And let say I was interested in the current also, the inductor current of

this, what is the final value - 0. So, now, we know that total response is steady state response plus natural response. So, here we will get  $V_s$  plus one of the three types of the natural response. So, let say  $L$  is 100 Nano Henry and  $C$  is 1 Nano Farad and  $R$  is let say two ohms. First of all is this critically amp circuit or under damp circuit and over damp circuit please find out, what is the formula for quality background damping factor from that we should be able quickly tell, it is under damped, what is the damping factor damping factors is what is the quality factors, damping circuit 1 by 10 quality factors 5, it is under damped. So, you expect the response of the type  $P_r$  and  $\phi$  themselves be expressed in terms of  $\omega_n$  and  $\zeta$  and  $Q$ . So, I would not to do that here.

Now, how do you found these constants, so let say I tell you that initially carries 2 volts, and this current initially is 5 milli amps. So, how would you find the constant  $A_1$  and  $\phi$ ,  $t$  equals to 0. So, I will get  $V_c$  of zero, which is 2 volts is  $V_s$  plus  $A_1 \cos \phi$  and then we have to differentiate it. Now how is it related to what I have given here this is the current. So, in this case, the current  $C$  times the derivative of this voltage. So,  $C$  times  $dV_c/dt$  at  $t$  equal to 0 is 5 milli amps. And if you differentiate this, you get you can differentiate this whole thing and then said  $t$  equal to zero. And you have two equations; from that you can find the two constants.

Similarly, if this was instead 20 ohms what kind of system would it be,  $Q$  will be exactly half and it will be critically damp. And if I fifty ohms, it will be over damped. So, as an exercise, please take these three values and find the natural response with these initial conditions that is 2 volts and 5 milli amps so that should I thing give you a drill in all the steps that are involved and finding the forced response with a dc input that is a constant input.