Basic Electrical Circuits

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Lecture - 144

We have by now; analyze the second order system quite thoroughly.

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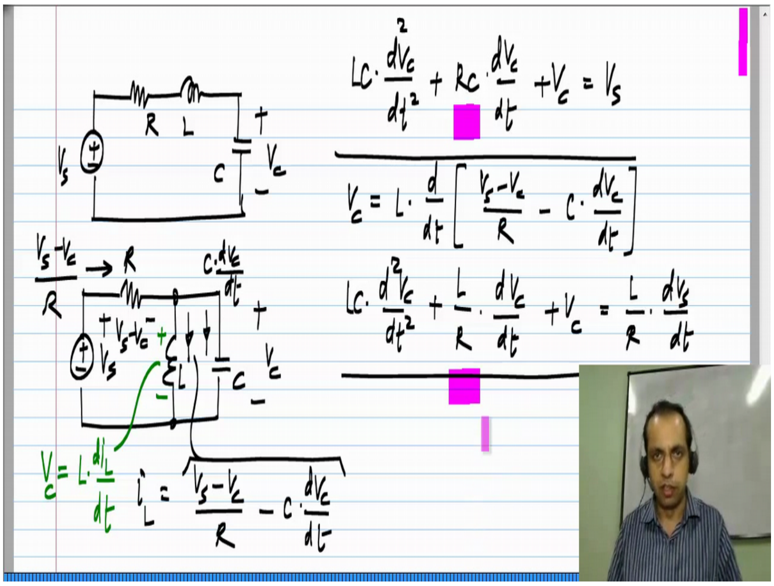

Now, the prototype circuit we took was this what I will do in this lesson as to show another kind of circuit. And also show that the formulas we derived for instance for the damping and quality factor for this circuit are not applicable universally. So, you have to look at what the circuit is write down the differential equation for it and from that from the left hand side of it derive the natural frequency and quality factor.

Let see, now in terms V c the differential equation for this was L C, second derivative of V c plus R C a first derivative of V c plus V c equals V s. Now, let me take another circuit, which also has an inductor and a capacitor, but just likely differently arranged and I will still write the differential equation in terms of the capacitors of voltage V c, but as usual you could pick any other electrical variable like the current through the inductor or current through the resistor or voltage across the resistor and so on.

Now, the current here is C d V c by d t the voltage across this is V s minus V c. So, clearly the current through this is V s minus V c divided by R and the current through this a simply the difference between the current in the resistor and current through the capacitor in the given direction. So, this current i L is V s minus V c divided by R minus

C d V c by d t and we know that the voltage across the inductor, which is the same as the voltage across the capacitor in this case.

So, V c which is also the voltage across the inductor equals L times the time derivative of the inductor current. So, now, we will substitute that substitute this whole thing for i L. So, V c is L times the time derivative of V s minus V c by R minus C d V c by d t. Now, you differentiate the terms inside this here you will get a second derivative and so on. And I will do all of that and finally, arrange all the terms, so that the variables are on the left hand side and sources on the right hand side, in fact, I suggest that you tried out and compare it to, what I have.

So, if you do that you will get L c times the second derivative of V c plus L by R times the first derivative of V c plus V c equals L by R times d V s by d t. Now, if you compare this equation to that one first of all the right hand side is different, but there is not of much consequence that depends on, where you apply the input and, which variable you consider the output. Even, if you consider just the natural response, which you get by setting V s to 0 if you said V s to 0 here you get something if is that V s to 0 here you get something else you see that the first and third terms are the same, but the term in the middle is different. So, this part here this is different from that one, so what is the consequence of that let us see.

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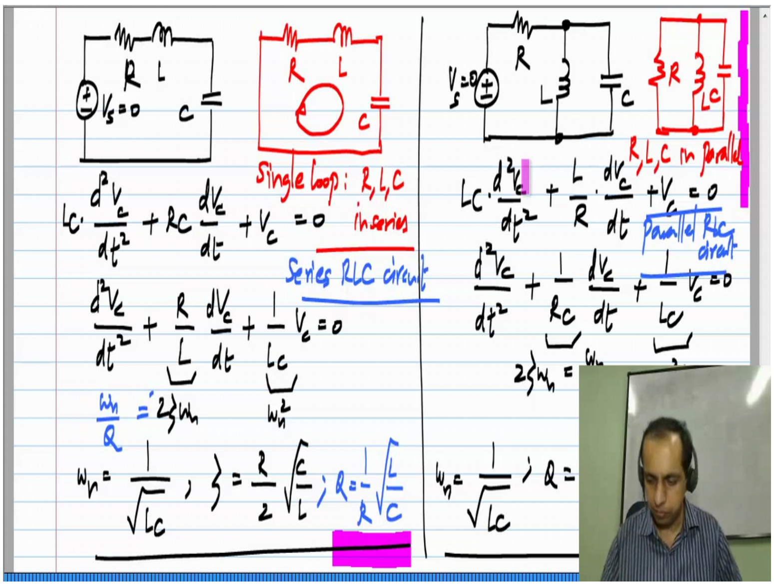

So, if we had the circuit like this R L and C, then the differential equation we have earning the natural response I will ignore the source for now, so V s equals 0. Now, we can also write this as you know as and when it is normal as like this, this term here is 2 zeta omega n and this term here is omega n square. And we know this results already the natural frequency omega n is 1 over square root L C and the damping factor zeta is R by 2 square root C by L have with alternative definition the constant this is also equal to omega n by Q and Q is 1 over R square root L by C.

Now, let us look at the other kind of circuit and let me set re as to 0 again and I will get the differential equation, which is L C second derivative of V c plus L by R first derivative of V c plus V c is 0 and normalizing it in the other manner we get. So, we get this again identifying this term with 2 zeta omega n or omega n by Q and this with omega n square we easily see that the natural frequency is exactly the same it is 1 over square root L C.

But, the quality factor, now is R square root C by L and the damping factor zeta is 1 over 2 R square root L by C. So, the natural frequency is the same, but the quality factor and damping factor have exactly the opposite kind of variation clearly in this circuit on the left side if you go on increasing R for a given L and C you will increase the damping factor or reduce the quality factor, where as in this if you go on increasing R the circuit on a right side you will reduce the damping factor and increase the quality factor.

So, what is the real difference between these two, so again said do not, let say memorize the formula for the quality factor of this and use it in any R L C circuit you will be dead wrong in this case for instance, because the Q here is exactly the reciprocal of that one. Now, these are two different kinds of circuit and these are not only two types R L C circuits, that you will see you can have R L C circuits that fit into neither this category nor that category.

But, these two are some basic types and its worthwhile knowing something about the, if you set V s set to 0 and draw the circuit with V s set to 0 will have R L and C, so you have a single loop and in that single loop you have R L and C in series. And if you do the same thing for the circuit on the right side and draw the circuit with V s set to 0, which means it is a short circuit we have R L and C in parallel there are only two nodes and R L C are in parallel this is the essential difference between these two circuits. So, this is known as the series R L C circuit and this is known as a parallel R L C circuit.

Now, why does the quality factor come out to be the opposite in these two cases quality factor some measure of quality of this circuit, what is meant by quality in this case I will not go into the derivations, but it is a measure of how much energy is stored verses how much energy is lost. Now, in this in the circuit on the left side you can see that the dead list in the extreme case when R goes to 0, then you will just have L and C in parallel with each other you can consider in to be in a series in a loop or in parallel with each other.

The point is there is no loss at all and I you had some initial energy on the inductor, let say then it will never be lost that will keep bouncing back between the inductor and the capacitor. Now, he exactly the same thing happens in the circuit on the right side if R is infinity if R is infinity, then this is an open circuit you just have L and C in parallel with each other and an initial energy on the inductor for instance will never be lost and it will be bouncing back between the inductor and the capacitor.

So, R equals 0 terms this circuit into a lost less circuit R equals infinite terms this circuit into a lost less circuit and the expression for the quality factor is consistent with that. So, if you look here R equals 0 will make Q equals infinity; that means, that this is the best quality circuit you can ever have. And similarly R equals infinity here will make Q equals infinity again denoting the best quality. And you can similarly interpret the damping factor if R equal 0 in this case damping factor 0; that means that the oscillation never sees, if you have some initial energy in the inductor let say it will got back on forth between the inductor and the capacitor. So, you measure either the inductor and the current or the capacitors voltage it will be a sinusoidal constant amplitude, which never dies out. Similarly, here exactly the same thing happens when R equals infinity the damping factor will be 0.

So, this the circuit on a left side is known as a series R L C circuit and the circuit on a right side is known as a parallel R L C circuit. Now, it is important to interpret these things carefully at least in this case when the source is present we can clearly see R L and C in series. But, such a naive interpretation by looking at the circuit may not always be possible, in this case at least when the source is present its not immoderately obvious how R L and C are in parallel, but if you set the source to 0 R L and C are parallel.

So, that is what counts you can have multiple sources in the circuit also, but when you set all the independent sources to 0 if the circuit reduces to a single loop with R L and C in series it is a series R L C circuit if it reduces to just two nodes with R L and C in parallel it is a parallel R L C circuit.

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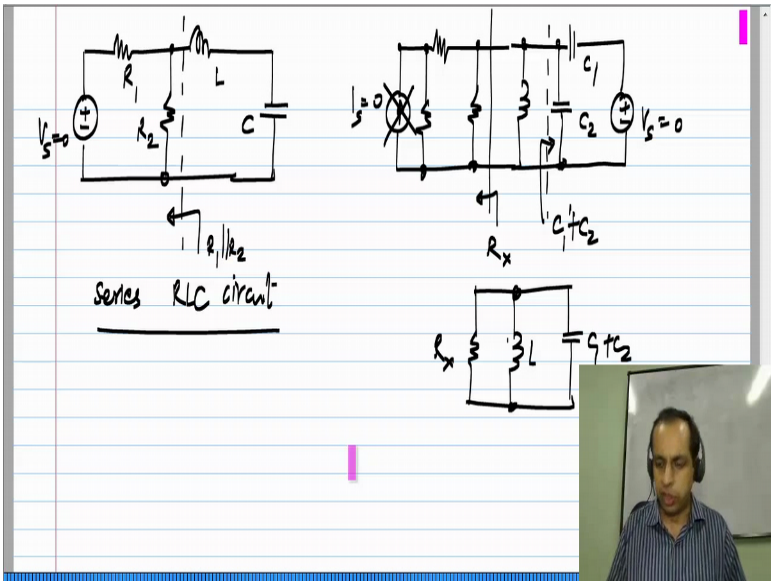

Now, because you can have multiple independent sources and you can split the component in anywhere, you can have pretty confusing looking circuit that eventually reduce to either series R L C or parallel R L C. For instance one example is this, let say what kind of circuit is it is not immediately obvious, but again if you set V s to 0, then looking them back to this you have a single resistor R 1 parallel R 2. So, you have a single resistor inductor and capacitor in series. So, this is a series R L C circuit and you can have other complicated combination as well similarly for a parallel R L C circuit.

Now, this circuit can look very complicated, but the point is, let say I call this I s and I said this to 0 this becomes an open circuit and looking back this file you have some resistor R x. And similarly, if I said this voltage source V s to 0 looking back this way I will just have the parallel combination of these two if I call the C 1 and C 2. So, eventually I have only R x l and C 1 plus C 2 in parallel and this is a parallel R L C circuit.

So, the expression for q follows R x square root of C by L in this case, so it is an important distinction and if you are really confused about, what the circuit is first of all you reduce the sources to 0 and you should be able to tell whether its series are parallel and if you cannot you finally, write the differential equation normalized properly identify the terms and find the quality factor.

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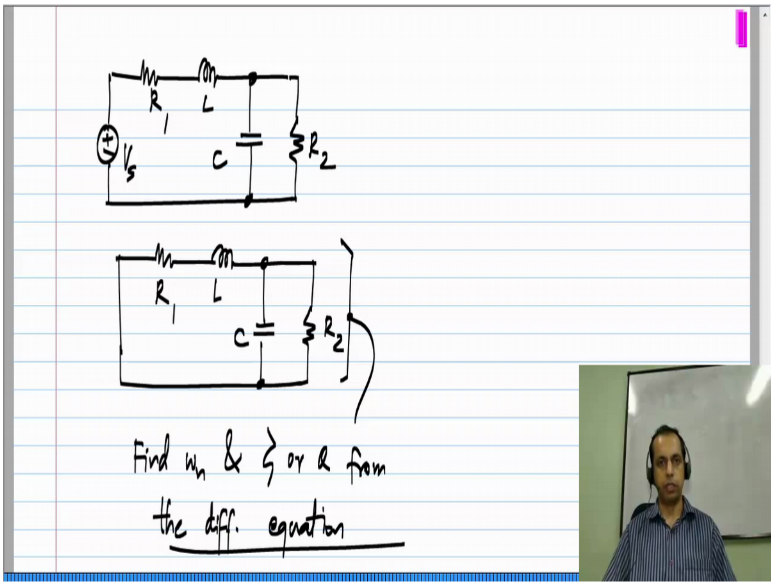

You have to do that anyway, because you can certainly have circuits with resistor capacitors and inductor basically second order circuits, which fit neither of the patterns have describe. For instance, which if I have even with V s equal 0 you neither have a single loop with R L and C and series nor do you have just two nodes with R L and C in parallel. In this case the only way is to write the differential equation you pick some variable, let say V c or i l and right the differential equation group all the variables to the left side normalized properly and find the damping factor and the natural frequency. So, that is the only way to do it and once you do that you will be able to find the natural response and force response and all are there stuff.