Basic Electrical Circuits
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Lecture - 143

In this lesson we will consider a numerical example for calculating the natural response of a second order system. Again I will use the prototype second order system we have being using so far the series R L C circuit, but the method is applicable in general to any second order system.

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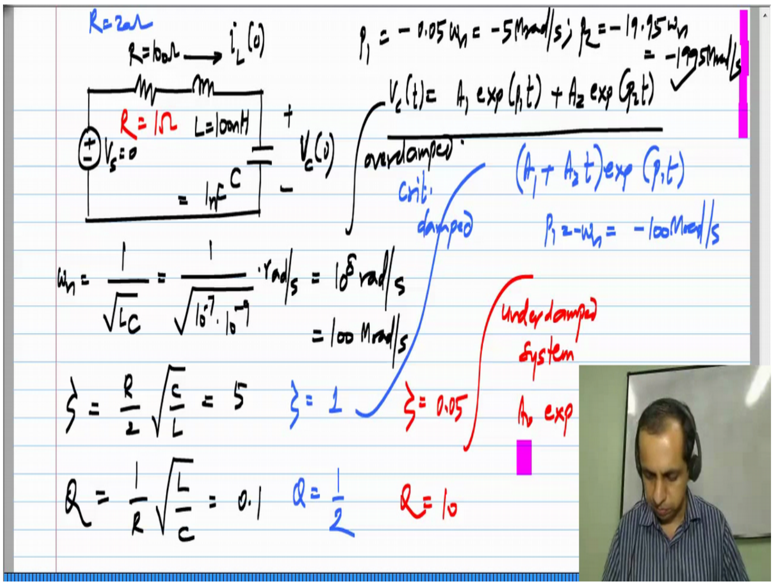

I will consider V s equal to 0 to compute the natural response and we have R L and C and initial condition on the inductor current and the capacitor voltage. Now, this is how initial conditions are normally given, but it could be something else also you could give the initial condition on the capacitor voltage and the first derivative of the capacitor voltage or initial condition on the inductor current and the first derivative of the inductor current then you have some more algebra it manipulation to do with your expressions.

So, we know that V c of t is A 1 exponential t 1 t plus A 2 exponential p 2 t if the roots are real and distinct. Now, let me take some numerical values I will take L to be 100 nanohenries and C to be 1 nanofarad. Let me say that R is 100 ohms just for the ((Refer Time: 01:37)) then we know that the natural response omega n is 1 over square root L C which is 1 over square root of L which is 10 to the minus 7 henries and C which is 10 to the minus 9 farads and this result will be in radiance per second or 10 to the 8 radiance per second in other words 100 mega radiance per second.

Now, the damping factor zeta which is R by 2 square root C by L is can be calculated alternatively you can also calculate Q which is 1 over R square root of L by C. Now, if you do it for this particular case you will find that zeta is 5 and Q is 0.1 clearly this is an over damped system that is zeta is more than one. So; that means, that the roots of the characteristic equation will be real and distinct and the natural response will be in this form.

So, instead of this let me take R to be 20 ohms in that case zeta will be 1 and the quality factor will be half. And in this case, this is critically adapt and the response of the type A 1 plus A 2 t exponential p 1 t by the way p 1 in this case will be equal to minus omega n which is minus 100 mega radiance per second. The first case that I considered with R to be 100 ohms this is over damped and in this case one of the roots p 1 is that minus 0.05 and omega n or minus 5 mega radians per second and p 2 will be at minus 19.95 omega n which corresponds to minus 1995 mega radians per second.

So, clearly in this case you see that the response will be dominated by p 1, because that is the smaller one and the exponential corresponding to that dies out more slowly in general and finally, I could take R to be let us say 1 ohm. So, in this case there are damping factor would be 0.05 and the quality factor will be 10 and this is clearly and under damped system and in this case the response will be of the type A naught exponential p r t cos p i t plus phi and we have two constant A naught and phi which are to be found out from initial conditions. I will show one particular example and you can do it for the other cases I will do it for the over damped case.

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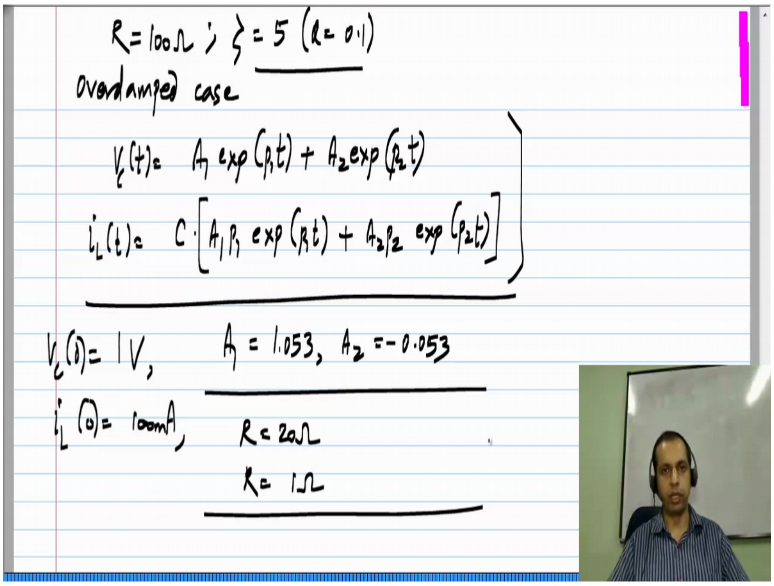

The over damped case I took which corresponded to R equals 100 ohms or zeta equals 5 and Q equals 0.1. So, we have V c of t to be A 1 exponential p 1 t plus A 2 exponential p 2 t and the current high L of t which is the same as the capacitor current is c times A 1 p 1 exponential p 1 t plus A 2 p 2 exponential p 2 t and let us take some set of initial conditions, let me say that V c of 0 is 1 volt and i L of 0 is 100 milli amp then you will get two linear equations here from which you will find that A 1 will be 1.053 and A 2 will be minus 0.053.

So, now, you have the complete natural response, so I would shown it for the under damped case, but please try it out with the same initial conditions for the other two cases with R equals 20 ohms and R equals 1 ohm as well and of course, you can also try it for some other cases and you can also plot these things and see how they look like.