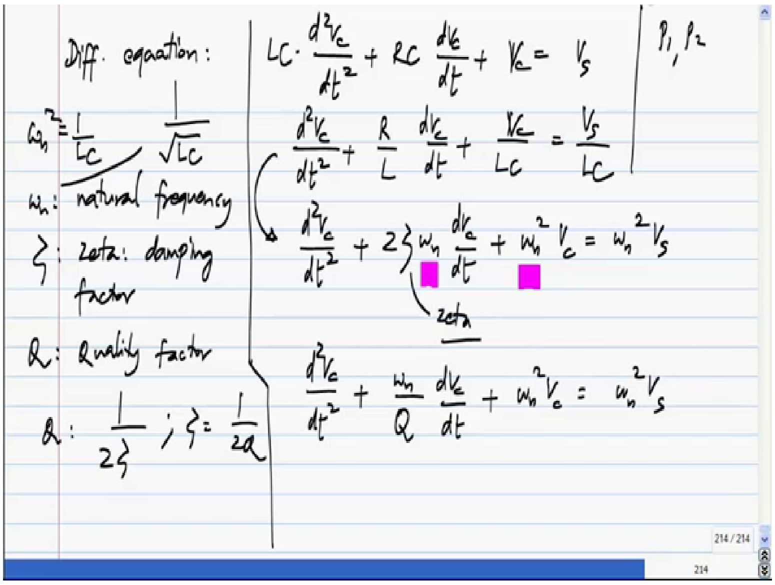
Basic Electrical Circuits

Dr Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture - 142

We have taken a specific second order circuit and set of the differential equation and analyzed it. Now, we will look at the second order differential equation with normalized coefficients. So, that any other kind of circuit can also be easily analyzed.

(Refer Slide Time: 00:16)



So, we have the differential equation as this I will write the homogeneous equation like this, this can also of course, be written as I do this just to normalized things in the way it is normally done. I think this part you probably already are familiar with from the spring mass system. All I did was to divide everything by L C that is all and I could of course, write it for the case with the input V s by L C. Now, this can be written as d square V c by d t square plus 2 zeta omega n d V c by d t plus omega n square V c equals omega n square V s.

I mean I just put out all these things this omega n square is 1 by L C it has dimensions of frequency square 1 by L C has dimensions of frequency squared. So, this omega n and omega n is called the natural frequency and this R by L you know that it has dimensions of frequency and that happens to be defined as this two times this zeta, which is the damping factor damping factor times omega n first derivative of V c and so on.

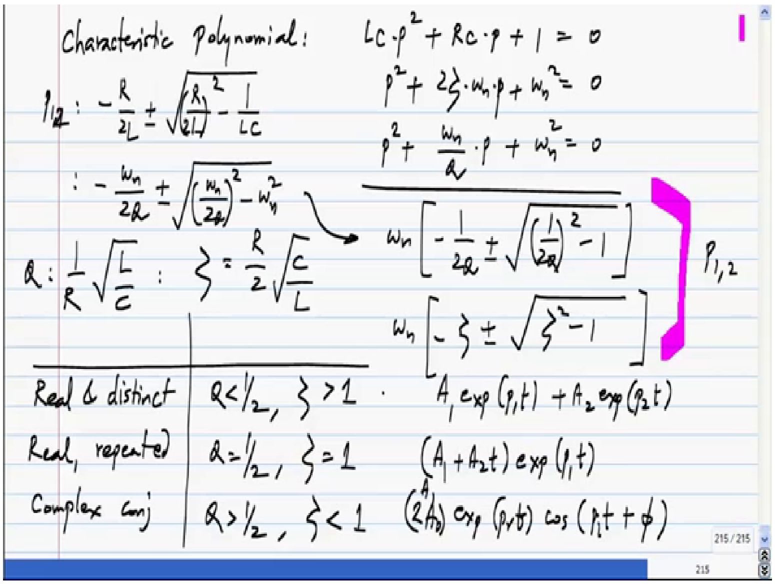
So, this is one possible way of defining the constants and sometimes you do it slightly differently d square V c by d t square plus omega n by Q d V c by d t plus omega n square V c equals omega n square V s. So, what this kind of normalization let us do is first of all do you have written for an L C circuit, but if you normalize the constants like this, if you define the constants like this it applies to any second order system that is why we do it that way that is all.

In this case this Q is known as the quality factor and; obviously, Q is 1 by 2 zeta or zeta is 1 by 2 Q is it fine. Now, from the characteristic polynomial we can find the values of p p 1 and 2 and exponential p 1 t and exponential p 2 t will be in general the natural responses. Now, we can classify the types of natural responses, we have already found those types, there are three types one where you have a combination two exponentials, one you have a single exponential multiplied by A 1 plus A 2 t and the third one where you have an exponentially modulated sinusoid that is you have a sinusoid cos omega t plus phi multiplied by exponential something.

So, these constants the natural frequency omega n and the damping factor zeta or the natural frequency omega n and the quality factor Q are some properties of the system, just like for a first order system we say something is the time constant what is the time constant, it is what appears in the exponential. So, similarly here this omega is n some constant, it is a second order differential equations. So, we have two constants we can either leave the constants as they are in terms of component values or in a more general way we can define this natural frequency omega n and the damping factor.

We could also instead of this just use the values of p we could simply use p 1 and p 2. So, this is a little more general way of associating with any other second order system that is all. So, there is nothing special here all I did was to redefine variables in this page if you look at it all I have done is that.

(Refer Slide Time: 04:52)



Now, what were the conditions the characteristic polynomial was L C times p square plus R C times p plus 1 equals 0 which alternately could also be written as 2 times zeta times omega n times p plus omega n square is 0 I divide by L C that is all and we get we know that p 1 and 2 what are they, what was the values minus R by 12 plus or minus square root of R by 2 L whole squared minus 1 by L C in terms of this new constant what is that omega n by 2 Q that is one way of doing that plus or minus what do you have omega n by 2 Q square minus omega n square.

So, now, you can kind of take this omega n outside plus minus square root of or if you express it in terms of damping factor, what is it I mean you can see the convenience already this omega n which is the frequency that comes out and then whether the roots are real or repeated or complex conjugate or determined only by this damping factor zeta or the quality factor Q.

So, we already saw that this real and distinct roots when does this happen, when R by 2 L whole squared is more than 1 by L C like by the way what is the expression for this quality factor Q in terms of L C and so on. In this case it is 1 by R square root L by C quality factor and the damping factor is R by 2 square root of C by L. So, you can see that if the value of R becomes larger and larger for the same L and C you will have higher damping factor or lower quality factor.

Now, real and distinct roots this corresponds to what in terms of quality factor what is that when do you have real and distinct roots in terms of quality factor Q less than half or equivalently the damping factor greater than 1 that is why it is called the damping factor and then you have real and repeated roots if the quality factor is equal to half or damping factor exactly equals 1 and finally, the complex conjugate roots when Q is more than half and the damping factor is less than 1.

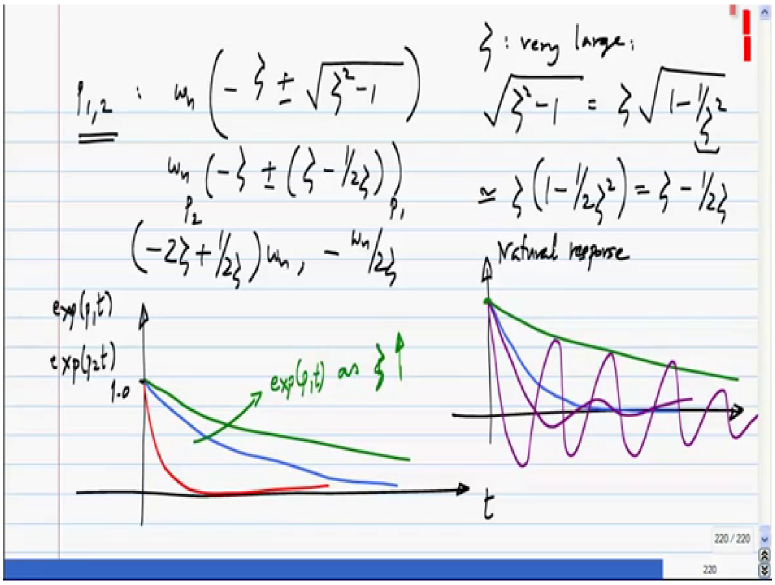
So, this damping factor and quality factor are normally used constants. That is why I defined both of them. So, in this case you will get a response of the type A 1 exponential p 1 t plus A 2 exponential p 2 t, where p 1 and p 2 can also be expressed in terms of omega n and Q or zeta like this. So, these are the values of p 1 and 2 and in this case you will have A 1 plus A 2 t exponential p 1 t I will just call that P 1 both roots are p 1 and in this we will have we can call it 2 A naught or is a single constant A naught it does not matter it is the point is that there is some constant there exponential p r t cos p i t plus phi.

So, you have to find this constant I mean I could as well call this some A 1 or something ((Refer Time: 10:02)) two it does not have any significance, you have to find the multiplying factor for this whole thing that is all. So, now, what is the value of p r minus zeta omega n many ways to express it minus R by 2 L this p r if this part is negative the square root is purely imaginary and then this is the real part and this p r is negative that is the point.

So, what happens what is the kind of response you have, the exponential will decay. So, it will be a sinusoid, but whose amplitude will go on decaying. And then if you examine this p 1. What is the value of p 1? If zeta equals 1 minus zeta omega n it will be some negative number also I mean it is the same minus R by 2 L. So, then also the responsible eventually will decay and finally, for this real and distinct case you can evaluate that both roots will be negative.

So, when you have this negative sign both terms are negative; obviously, the result is negative, when you have this positive sign you have some negative number and another positive number which is smaller than that, so you will have both negative roots. So, this circuit is also always stable. So, in this case also the natural response raised on with time let us now examine the roll of the damping factor in the natural response of the second order system.

(Refer Slide Time: 11:32)



The roots will be omega n minus zeta zeta square minus 1. Now, if zeta is very large then what happens is that this square root of zeta square minus 1 can be written as zeta times square root of 1 minus 1 by zeta square, where this 1 by zeta square is a small number. So, this is approximately zeta times 1 minus, you know this approximation square root of 1 plus x is 1 plus x by 2 which is zeta minus 1 by 2 zeta. So, this roots can be approximated as omega n minus zeta plus minus the two values will be basically when it is minus minus 2 zeta 1 by 2 zeta I will set z is very large. So, that part is small what is the other root minus omega n by 2 zeta.

So, these are the two roots and let say on the same axis I plot let us say I call this V 2 and this is V 1 exponential p 1 t exponential p 2 t what will I look like relatively let say I start both of them with unity exponential p 1 t and exponential p 2 t what will they look like, it will up to start from 1 and decay to 0, but how will that do that which one will go to 0 or pick up p 2. So, exponential p 2 t would do that and exponential p 1 t would do that.

For a given omega n the larger of the value of zeta this exponential p 1 t gets keeps getting slower and slower as zeta increases. So, we will have two exponentials in the natural response one of them will die out very quickly, other one will take a very long time to die out this is fine. So, what happens is, so let say you start with I mean you look at the natural response and for large values of zeta it will be dominated by this exponential p 1 t. So, it will be very, very slow it will almost like a single exponential that is very slow as the damping factor reduces and when you reach critical damping you get some nice response like that.

Now, what happens when zeta becomes very small? So, what will happen is that it will go to go out quickly like that, but then it will ring back and forth. And then, if zeta becomes very, very small what happens is that it becomes just as bad as the other case when zeta is very large. So, it will go rapidly, but then it will start doing this, because what is the real part of p 1 and p 2 when they are complex conjugates by omega and zeta.

So, when zeta becomes very small; that means, that zeta omega in this very small number and that is the exponential that is modulating the sinusoid. So, that exponential is dying out very, very slowly. So, if you are looking for a clean settling critically damp response the one to look for, in many cases you look for the system to have a critically damp response of course, it depends on what system it is, but that is the reason.

So, for a given omega n very large value of zeta means that it will not ring like this, but it will be ((Refer Time: 16:14)) it will settle very slowly and for a very small value of zeta it will thought of changing very rapidly, but then this sinusoidal part will stay for long time. On the other hand if you have critically damp stuff it just goes down and then settles cleanly. So, it gives you short of the cleanest settling, so if you look for damping factors which are around the critically damped value in many cases.