# Basic Electrical Circuits

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## Lecture - 141

We have already seen the kind of the natural response we will get from a second order circuit when we have real and distinct a roots for the characteristic equation, we will get two experientials. Now, if we have real, but coincidence roots we have already seen it, but I will just show it any ways.

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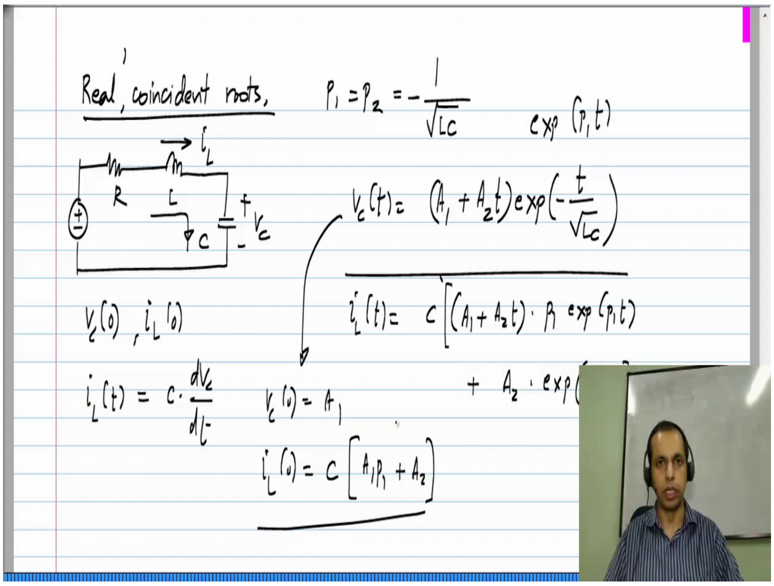


In this case p 1 will be equal to p 2 and each of them will be equal to 1 over square root L C. and in this case the response the natural response will be of course, this is for our usual R L C circuit the natural response for V c is going to be A 1 plus A 2 t exponential minus t by square root L C this is basically exponential p 1 t. Now, how do you determine the constants A 1 and A 2 as usual it is by applying into initial conditions.

We will have initials conditions V c of 0 and i L of 0 we know that i L of t which is the current in the inductor is the same as the current in a capacitor. So, it is c times d V c by d t. So, i L of t will be c times the time derivative of this, which is A 1 plus A 2 t times p 1 exponential p 1 t this is a chain rule first time differential in the this part of it plus A 2 times exponential p 1 t. So, from the first one we will get V c of 0 to be A 1 and from the second one we will get i L of 0 to be c times A 1 p 1 plus A 2 we have two equations in two unknowns and we can find both A 1 and A 2.

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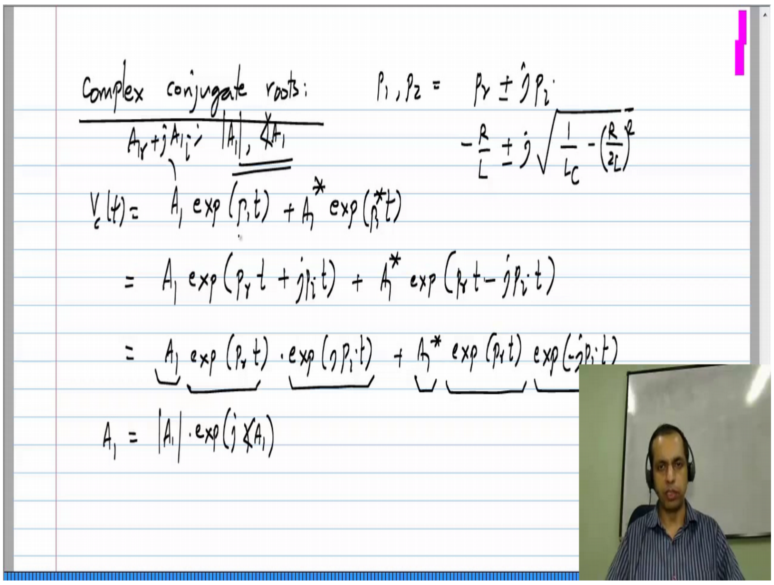


Now, we have complex conjugate roots we will have p 1 and p 2 to be some real part plus minus j times the same imaginary part and the real part will be minus R by L, the imaginary part you know the expression for this will be plus minus j times square root of 1 over L C minus R by 2 L square. So, again the response will be of the form A 1 exponential p 1 t plus A 2 exponential p 2 t, now in this case P 2 is of course, p 1 conjugate. So, A 2 will also be A 1 conjugate, so that the whole expression is real.

Now, if I substitute the form of p 1 and p 2 I will have p r times t plus j p i times t plus A 1 conjugate exponential p r times t minus j p i times t which after factoring out experiential p r t outside we will have exponential j p i t plus A 1 conjugate exponential p r t exponential minus j p i t, this is a real number exponential p r t and is common to the two terms here.

Now, the other part you can see that we have A 1 times exponential j p i t it is some complex number, the other part A 1 conjugate times experiential minus j p i t is also a complex number and it is complex conjugate of the first one. Now, the most convenient way to simplify this further is to represent A 1 in polar. So, let us say A 1 is the magnitude of A 1 exponential j times the angle of A 1.

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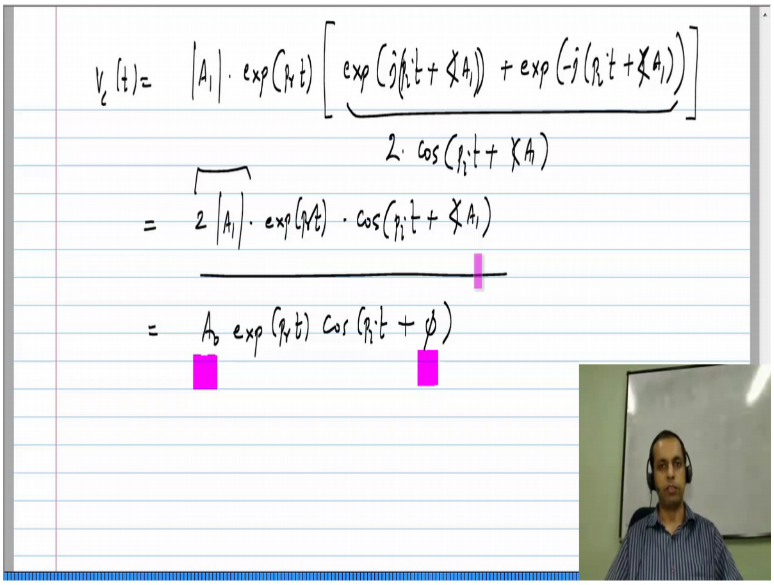

Whatever that is substituting in that what we will get is V c of t is the magnitude of A 1 exponential p r t this will be common factor to the whole thing and then we will have exponential j p i t plus angle of A 1 plus exponential minus j p i t plus angle of A 1 and you realize that this is nothing but, 2 times cos p i t plus angle of A 1. So, this whole thing can be written as 2 times absolute value A 1 exponential p r t and cos p i t plus angle of A 1.

So, again you have two constants you can think of them originally we had A 1 and A 2 now we have A 1 and A 1 conjugate, but A 1 itself has the real and imaginary part A 1 r and j i 1 or the magnitude and the angle. So, we can represent that in any way you want, but finally, will have two equation from two initial conditions and you can find these two numbers.

Typically it is easiest we can this two does not have much significant here this whole thing is some constant A naught experiential p r t cos p i t plus some angle phi. So, we have two constant A naught and phi which we can find from the initial conditions. So, these are the responses when we have real coincidence roots and when we have complex conjugate roots for the characteristic equation of the second order system.