Basic Electrical Circuits
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Lecture - 140

In the first order case we evaluated the response by solving for the differential equation sort of from scratch, in the second order case we used what is known as the characteristic equation. Now, these two are link and in this lesson we will see the link between two. First let me consider a first order system although in this lesson the focus is on the second order system.

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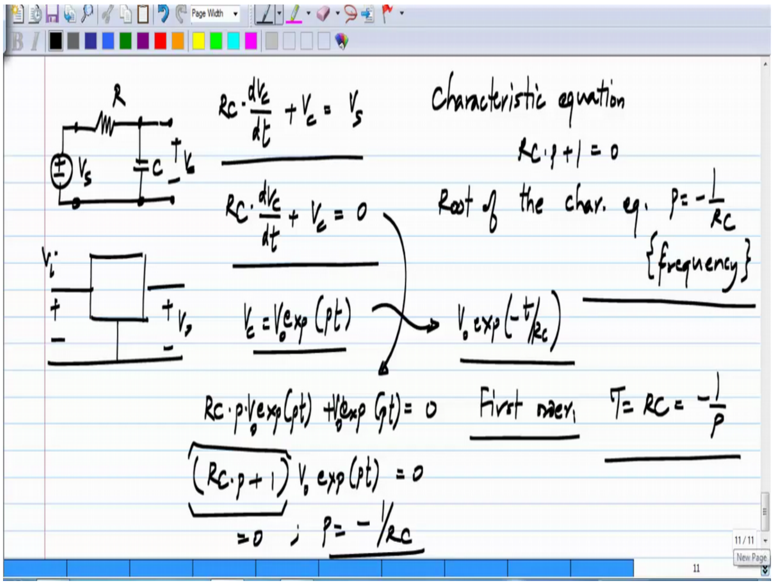

So, again we can take this circuit as an example of a general first order system. Now, in this case the differential equation for V c is given by this and the natural response is evaluated by solving for the source free case. Now, if you assume that V c is some exponential p times t substituting it in here we get R C times p times exponential p t plus exponential p t equals 0 and we can have a coefficient V naught here and that does not change the anything V naught appears both places.

Now, this will be identically equal to 0 only if this multiplying factor of the exponential is 0. So, this part has to be equal to 0 or in other words p is minus 1 by R C, so we get the result which were already very familiar with that the natural response is sum V naught exponential minus t by R C. So, this is the characteristic equation and the root of the characteristic equation p is minus 1 by R C and it has dimensions of frequency, this is the as you expect, because the argument of the exponential is p times t which should be dimensional less, so p has dimension of 1 over time or frequency.

Then, we see that the time constant in a first order case, the time constant tau which is R C is minus 1 by p minus 1 over the root of characteristic equation. So, we can use this in general for higher order equations also as we did for the second order equation. We already found the solution to the second order equation here I am trying to relate it to the first order case. So, that we gets some more insides in to our solutions.

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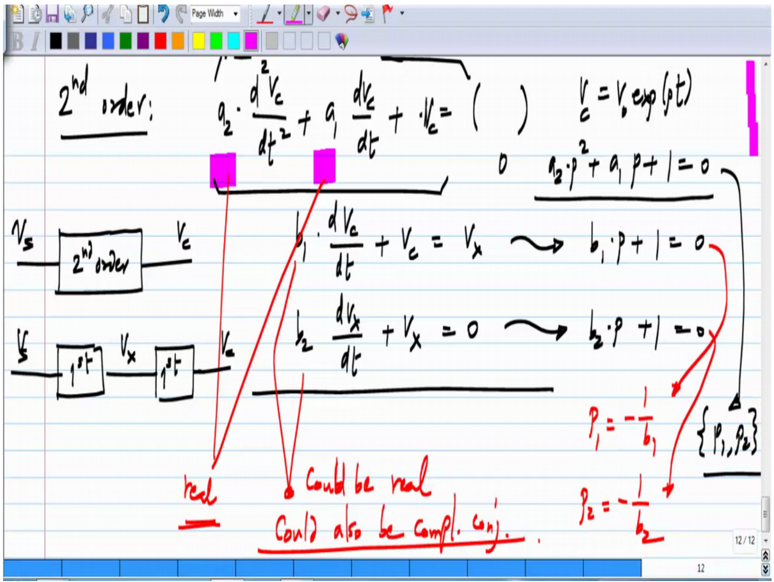

Now, in a second order case we have a second order differential equation, let us say this is the differential equation in the relevant variable. Now, for the natural response the right hand side is substituted with 0. So, I won't worry about it and I have normalize the left hand side, so that the coefficient of V c is 1 this is just for convenience. Now, this second order equation can be written as a combination of two first order equations. So, that is we go from the input to some variable V x from there to V c now is easier to understand in picture.

So, let us say we had a second order system with some V s is the input and V c as the output, you can think of it as two first order systems one after another with V s is the input and V c as the output and this intermediate output is called V x and for the natural response we said V s to be 0, it transfer this kind of picture makes it convenient to figure out and the nature of the solution and this case as well.

So, the characteristic equation here if you assume a solution of the type V c is sum V naught exponential p t characteristic equation would be a 2 times p square plus a 1 times p plus 1 is 0. So, every derivative term will have a p and nth order derivative will have p to the n. Now, the characteristic equation of this is b 1 times p plus 1 0 in this b 2 times p plus 1 is 0. Now, this as some roots p 1 and p 2 it has two roots, because it is a second order equation.

Now, this has a single root p 1 is minus 1 by b 1 and similarly this has another root p 2 is minus 1 by b 2. So, the two roots of the characteristic equation now become one of them is the root of this and the other one is the root of that one. Now, if you combine these two into the second order differential equation, you have to get exactly the same coefficients in a circuit with real components these coefficients will be real, but these two need not be real because this V x refers to some fictitious output which is not really accessible. So, these could be real, but would also be complex conjugates.

So, both these possibilities exists that we know already from the second order equation that this can have roots which are real and distinct real and identical or complex conjugates. Now, remember b 1 and b 2 will not be some arbitrary complex numbers there will be complex conjugates, because the eventually equation has only real coefficients. So, the two solutions p 1 and p 2 if there complex at all will be complex conjugates of each other. So, what does this mean?

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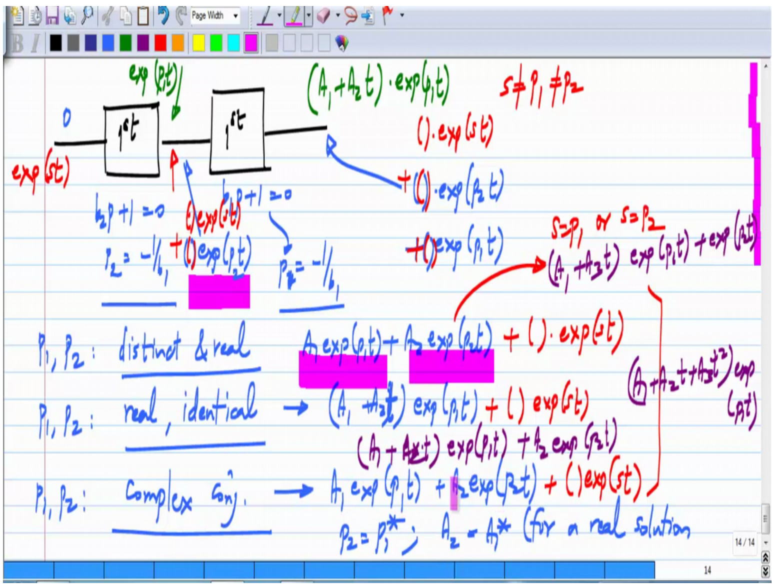

If you think of this as ((Refer Time: 08:28)) of two first order systems and let me consider first natural response, so that is the input is 0. Now, the first one here had a characteristic equation b 2 times p plus 1 0, so this p 2 is minus 1 by b 1. Now, the natural response of this would be just exponential p 2 t. Now, if you think of the second first order system it is being driven by an exponential I will omit the scaling factors every were, because I just want to point out the nature of the solution.

So, now, the solution here is the response the total response to an exponential input, the input to the second block is exponential p 2 times t and this has a characteristic equation b 1 p plus 1 equal 0 whose root is p 1 is minus 1 by p 1. So, the force response of this consist of it is natural response as well as the input exponential. Now, let us assume first that p 1 and p 2 are distinct and real, the distinct will necessarily real then this will consist of first of all the input exponential scaled by some value. So, it will have something times exponential p 2 t plus it is own natural response which is something times exponential p 1 t.

So, the natural response of the first stage is the driving input to the second stage and that also appears as it is. Now, let us imagine a second case, where p 1 and p 2 are real and identical this happens in the quadratic equation when a 2 square of 4 times a 1. So, then what happens is this p 1 and p 2 are both identical. So, the output here which is the natural response of the first order system is exponential p 1 t, because p 2 is the same as p 1.

We know that when a system is driven by it is own natural response, the output will be out of form A 1 plus A 2 t times exponential p 1 t it will get multiplied by this t which grows with time. Because, a system has a certain natural frequency if it is executed by it is on natural response then the multiplying factor trends to low up over time. So, this gives you the idea that the solution in this case is of the form A 1 plus A 2 t exponential p 1 t there is only one p 1 p 2 equals p 1 in the other case it is A 1 exponential p 1 t plus A 2 exponential p 2 t.

Finally, when p 1 and p 2 are complex conjugates then also it is the same as the distinct real case except that p 1 and p 2 are related in some way. Now, remember here the output will be exponential p 2 t which is a complex number really, but this is some fictitious case, this is not an accessible point in the circuit we have just mathematically split up the second order system into two first order systems, but here he will get exponential p 2 t plus exponential p 1 t.

So, we will have A 1 exponential p 1 t plus A 2 exponential p 2 t and p 2 will be p 1 conjugate which also means that A 2 has to be A 1 conjugate for a real solution. So, that is how we get different cases for the natural response of a second order system. Now, let us consider the force response and let me again consider some exponential s t as the input. So, now, you can imagine many different cases first I will take is different from p 1 which is also different from p 2, so there all distinct.

So, then at this point what will we get, we get the total response to an exponential. So, instead of exponential p 2 t we would have got something times exponential s t plus something times exponential p 2 t and here we will get the input exponentials scaled by some number. So, we will get something times exponential s t plus something times exponential p 2 t plus the natural response of the second stage which is something times exponential p 1 t. So, the total response would be A 1 exponential p 1 t plus A 2 exponential p 2 t plus something times exponential s t and this something can be evaluated by substituting exponential s t in the solution to the differential equation and solving for the resulting equation.

Now, similarly if I have a exponential s t as the input when p 1 and p 2 are complex conjugates of each other and I will assume that s is not equal to either p 1 or p 2 we will simply get plus exponential s t. And similarly, if the p 1 and p 2 are real and identical, but s is not equal to p 1 we will simply get plus exponential s t, but these are not the only possibilities for instance, in this case I could have s equals p 1 or s equals p 2. So, if s equals p 1 we will get A 1 plus A 3 t times exponential p 1 t plus exponential p 2 t.

Similarly, if s equals p 2 we will have this coefficient of t multiplying exponential p 2 t. And similarly, in this case in the second case p 1 and p 2 are all of the identical if s happens to be equal to p 1 then we will get A 1 plus A 2 t plus A 3 t square exponential p 1 t. So, we get t square turn as well and finally, in this case it is possible that s is equal to either p 1 or p 2 and again we have the same cases here, we could get A 1 plus A 2 t exponential p 1 t plus A 2 exponential p 2 t and so on. So, there are many different possibilities.

Now, I am just going to only qualitatively outline all these other exotic possibilities finally, when it comes to force response of more complicated systems, we will not be evaluating them in a general way we can do that by using exponential s t and later after you learn about Laplace transforms you will see that any signal can be represented as an integral of exponential s t over s over difference frequencies that will let you solve the equations more generally.

In our case we will only consider the dc input case and the sinusoidal input case, the sinusoidal input case it terms out can be solve much more easily without even writing down the differential equation that we will come to later, but they should give you an idea of the nature of other solutions when you have a second order system, mainly you can have two separate exponentials with real frequencies p 1 and p 2 or you can have A 1 plus A 2 t times exponential p 1 t and finally, you can have this case where you have complex conjugates which will eventually add up to some sinusoidal.