Basic Electrical Circuits

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Lecture – 139

We have spent a quit a bit time in first order circuits, but we also generalize some principles from there, which will be applicable to high order circuits. Now, everything that we set so far with that you can analyze any circuits with a single capacitor or a single inductor. And I say single capacitor or inductor, let us assume that you have done all the parallel series combinations possible with the source being null, with the source being null you will effectively get a single capacitor or single inductor; that is a first order circuit.

So, that, all of that you can analyze now with what we have. Is that okay? There will be one time constant that you can find by finding the Thevenin resistance across the terminals of the capacitor or the inductor. Now, what we have not done so far is to put both the inductors and capacitors together, so let us do that and see what happens.

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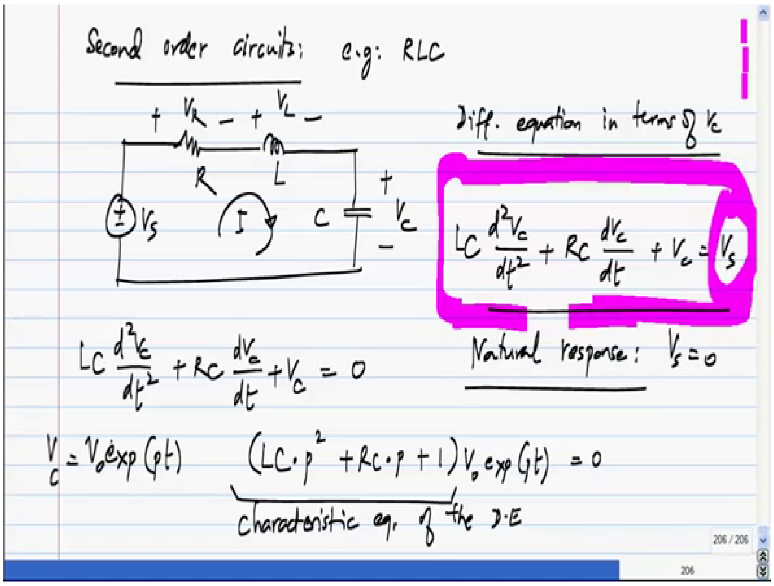

So, far we have only resistors and inductors or resistors and capacitors, now we will have everything together. Now, we have some current in the loop, the way I have chosen the circuit is to put all three elements in series of course, not necessary we can have it in different ways and we have a single loop with a loop current and we have a number of variables V R, V L and V C. So, first of all please write down the differential equation in terms of V C, please do that.

Student: I will also derive the differential equation; you can compare your answer to what I get.

You do get second order differential equation, which is…

Student: L C times second derivative of V C plus R C times time derivative of V C plus V C equals V S.

So, you get that and all you do is, KVL around this loop V R plus V L plus V C equals V S and then, express V R and V L in terms of V C. This is V R, C times d v c d t is the current and that times R is the voltage across the resistor and this L times derivative of that is the voltage across the inductor. For that we can write the differential equation for the circuit in terms of V R or V L. What could be it in terms of V R? What would we get?

Student: Again please derive the differential equation in terms of V R and compare it to what I get. You will get exactly the same terms on the left hand side, because the circuit is the same L C times second derivative of V R with respect to time plus R C, first derivative of V R with respect to time plus V R equals R C times first derivative of V S, the Source Voltage.

As usual you get the same homogeneous part of the question. So, only the right hand side that is changed and if you do it, if you try to V L you will again get the exact same thing; that is the left part is the same, the differential equation and the right hand side will be different. So, what it means of course, is that the natural response will have the same form for any quantity in the circuit and you can also write it in terms of I, which is also very easy, I is simply V R divided by R. So, you just divide this by R and you will get the differential equation in terms of y.

So, this is the differential equation governing the circuit and as usual, first we will find natural response or the solution to the homogeneous equation and what is that. What you have to do? Set V S to 0, so we will be left with…

Student: L C times second derivative of V C with respect to time plus R C, first derivative of V C with respect to time plus V C equals 0.

What is the form of the responsive expect from this? Exponential, I mean that is the standard solution to any linear differential equation; that simply, because the derivative of an exponential is an exponential. So, you sum up any number of derivatives together, you expect that all of them will have some exponential as a function of time, so the only way I will cancel is by having all the coefficients in the appropriate values. So, if I do assume that V C is something; that means p times t here and just for keeping dimensions correct I will use V node, but that will cancel out anywhere. So, what do I have? What is the first term?

Student: If I substitute this V C in to the differential equation.

L C times p square plus R C times p plus 1 times V naught exponential p t equals 0. The only way this is going to happen is, if that quadratic equation equals 0. I think we have done things of this type, this is known as the characteristic equation of the differential equation. When we did it for the first order circuit, we did not do it directly like this. We manipulated the variables to get a homogeneous equation and from there we got the answers, but exactly the same procedure will work.

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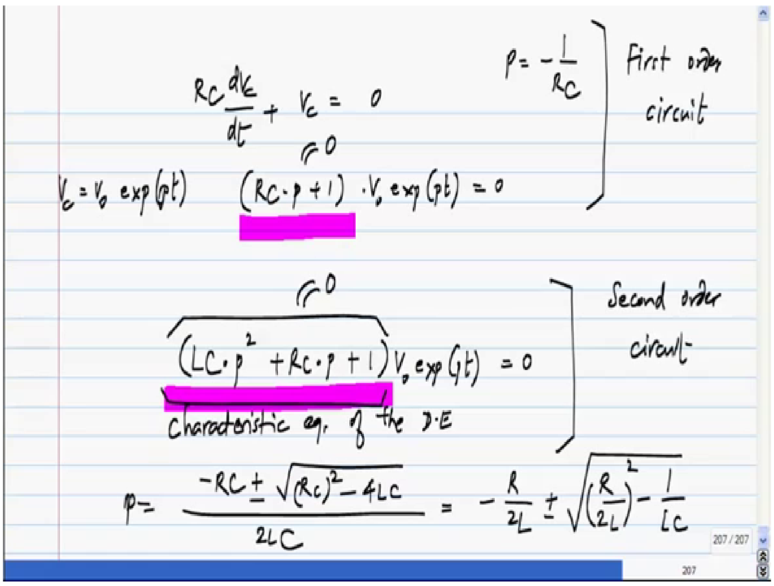

Because, what was the differential equation in that case.

Student: R C times first derivative of V C plus V C equals 0.

This was the homogeneous equation and if I assume V C is V naught exponential p times t, I would get R C times p plus 1 V naught exponential p t equals 0. So, values of p which satisfy this whole thing equal 0 will satisfy the differential equation; obviously, p is minus 1 by R C. What are the dimensions of p? Frequency, exponential p t is the argument of the exponential should be dimensionless. So, p times t is dimensionless, p is frequency.

So, if you solve the characteristic equation you get possible frequencies of the exponential. So, similarly here you have a quadratic equation you will get two solutions and, what are the solutions, so this part has to be equal to 0. So, at two ails of p whatever.

Student: We get the standard solution to the quadratic equation, which is minus R C plus or minus square root of R C square minus four times L C divided by two times L C which can also be written as minus R by 12 plus minus square root of R by 2 l whole square minus 1 by L C.

So, you can write it in either form, so there are two values of p that' is satisfy this, what is the mean for the natural response there are two modes two natural modes for the circuit. So, p 1 and p 2 and both are possible exponential p 1 t and exponential p 2 t and actual natural response will be some linear combination of the 2. Because, for the linear differential equation of two possible solution any linear combination as also a solution, so in general. So, let us say a one exponential p 1 t plus a 2 exponential p 2 t those are the solutions to this homogeneous differential equations and how do you find a 1 and a 2 initial condition.

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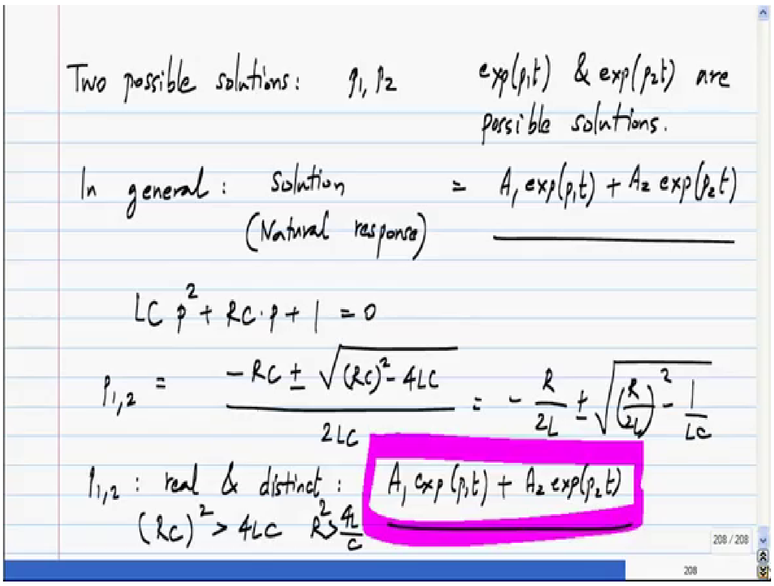

There are some conditions that you have to further check, so now we have a coordinate equations, so what are the possibilities for the roots. So, both p 1 and p 2 are real in which, case you simply get this two distinctive exponentials, so that is the solution in that case and you find a 1 and a 2 from initial conditions. And then, it is possible that p one and p 2 are identical and real and finally, p 1 and 2 are complex conjugate each other.

Now, you can also have p 1 and 2 to be complex conjugate each other. So, in p 1 2 real and distinct this is the natural response and when does this happen when will they real understand R C square greater than 4 L C or in other words R square greater than 4 L by C. So, in general they will be two explanations.

Student: Sir, in the natural response, now you we to considered to other cases as well when p 1 and p 2 are real and identical and when their complex conjugates of each other, we will first in another lesson look at a second order system as a scatted of two first order systems and come back to these issues, because that will help us understand the solutions even better.