Basic Electrical Circuits
Dr Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture - 137

Now, you can consider the total response of a first order system to sinusoidal excitation. This is not very different from when we had DC or exponentials, but I just go through it in detail once.

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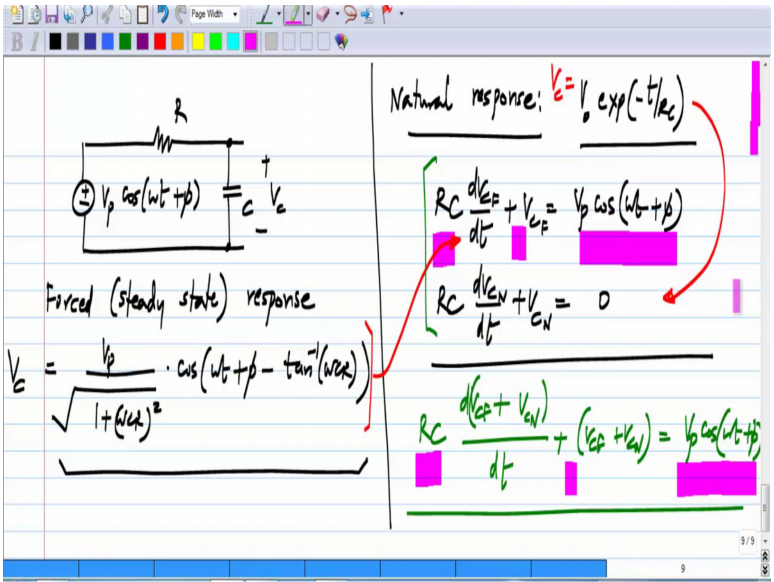

So, when you have again taking the same circuit, but with the reminder of that, this is applicable to any first order system, when you have V p plus negative plus 5. We know that force response to be C the steady state response, we have calculated this in many, many ways. It is V p by square root of 1 plus omega C R square cos omega t plus 5 minus tan inverse omega C R. You get a sinusoidal the same frequency, but with the amplitude and phase modified.

Now, the natural response is of the form V naught exponential minus t by R C. So, this is the response of the R C circuit when no input is given to it, this will always be present. Now, this you know from basics of linear differential equation, that the total response can be the steady state response plus a scaled version of the natural response. Now, what is the idea here? We have the differential equation governing the system R C d v c by d t plus V c is V p cos omega t plus 5. What is this mean for this function to be the solution to V c? This value of V c will satisfy this particular deferential equation.

Now, let us consider R C d v c by d t plus V c equal 0. Now, just to differentiate between these two, let me put a subscript F here, denoting the force response and N here, denoting the natural response. Now, this function here with any value of V naught will satisfy that particular equation. Now, let us say we add these two equations and because the differential is a linear operator, we can add the arguments and so on.

We have R C, the time derivative of V C F plus V C N by d t plus V C F plus V C N equals V p cos omega t plus 5. So; obviously, this V c of here the force response plus this natural response V C N will also satisfy the original differential equation. You see that this differential equation here is exactly the same as this, the coefficients are the same and right hand side is the same. So, when you compute a force response, that will surely satisfy the differential equation, but that plus any scaled version of the natural response will also satisfy the differential equation, because the natural response after all satisfies the differential equation for 0 input.

So, in general the solution is the force response of the particular solution that you compute plus a scaled version of the natural response. What is the scaling factor you have that, depends on the initial conditions.

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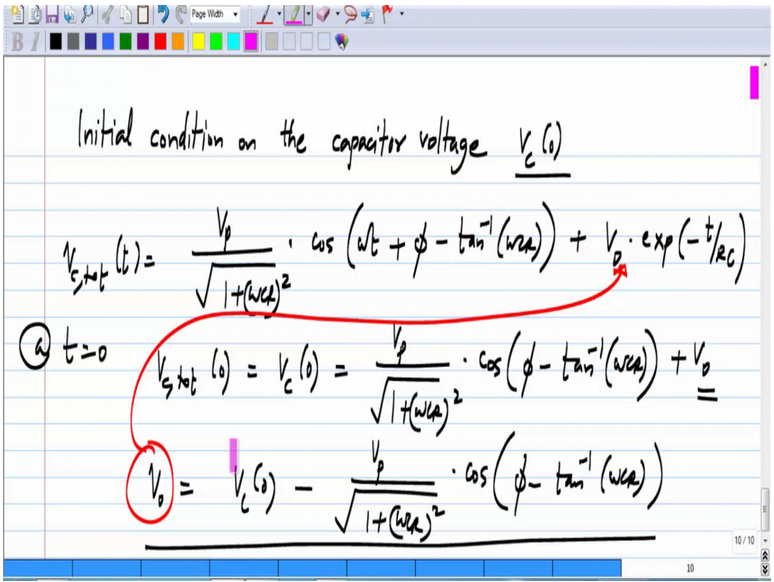

So, in this particular case, let us say that the initial condition on the capacitor voltage is V c of

0 is V c of 0. Now, we know that the total response is the particular solution, there is a unique particular solution of course, cos omega t plus 5 minus tan inverse omega C R plus this scaled version of the natural response V naught is unknown it. Now, we substitute t equal 0 and we find V c total of 0, which has to be equal to the initial condition that is given V c of 0 and that is equal to substituting t equal 0. In this case I get V p by square root of 1 plus omega C R square cos 5 minus tan inverse omega C R plus V naught.

So, this unknown coefficient V naught of the natural response is V c of 0 minus V p by 1 plus omega C R square cos 5 minus tan inverse omega C R. This you substitute in to the solution, this has to be substituted in there to get the total response. Now, an interesting thing is and sometimes it turns out to be useful, that for a given input that is V p omega and 5 are given, the input is V p cos omega t plus 5.

You can choose this V c of 0 the initial condition on the capacitor; such that the natural response is 0. The initial of condition on the capacitor happen to be exactly equal to this number V p by square out 1 plus omega C R square times cos 5 minus tan inverse omega C R, then V naught will be 0 and there will not be any transients at all. So, if you start from the right initial conditions and what are the right initial conditions as specific to the input you have, you can have node transient response and you will immediately get the steady state response from the circuit. And sometimes in circuit design this is useful, this is like you do not have any transients, you start off the circuit from the right state, so that you will have only the steady state response.