Basic Electrical Circuits
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Lecture – 136

In this lesson, I will work out the sinusoidal steady state response of a first order circuit the first order system in general using the three methods, we have discussed and show you how one using the complex exponential is the easiest most convenient.

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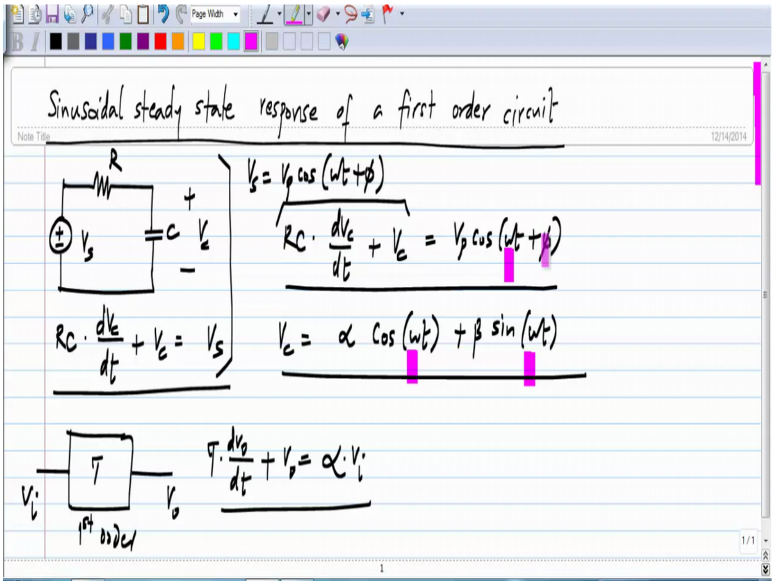

Let us take this first order circuit as an example, but the analysis applies to any first order system. So, I have V s R and C and I take V c as relevant variables again I could take anything else I wanted the deferential equation governing this R C times d V c by d t plus V c equals V s. Now, in general we could have a first order system, which some time constants tau and let say the put as V i and output V o, then the deferential equation governing this could be tau times d V o by d t plus V o equals sum multiple of V I, because it is linear.

So, I will show the analysis using this, but exactly the same can be applied to any first order circuit by normalizing it is differential equation like this the coefficient of this is one coefficient of first derivation is the time constant and we have something over here, so this can be repeated for any first order system. So, now, let say V s is V p cos omega t plus phi to be most general. Now, our differential equation says that R C times d V c by d t plus V c equals V p cos omega t plus phi.

Now, we already discuss this we have V c and it is derivative we can have some trigonometry function here either sign or cos we have sign the derivative will be cos and we have cos the derivative will sign. So, in general V c itself has to be linear combinations of sign and cos, so that this equation can be satisfied. So, V c is let say some alpha cos omega t beta sin omega t.

Now of course, this omega is exactly the same as that one, because when you differentiate it this omega will not change. So, you have to take this same frequency for this sinusoid as well. So, now, let us use this in the left hand side and try to balance the terms.

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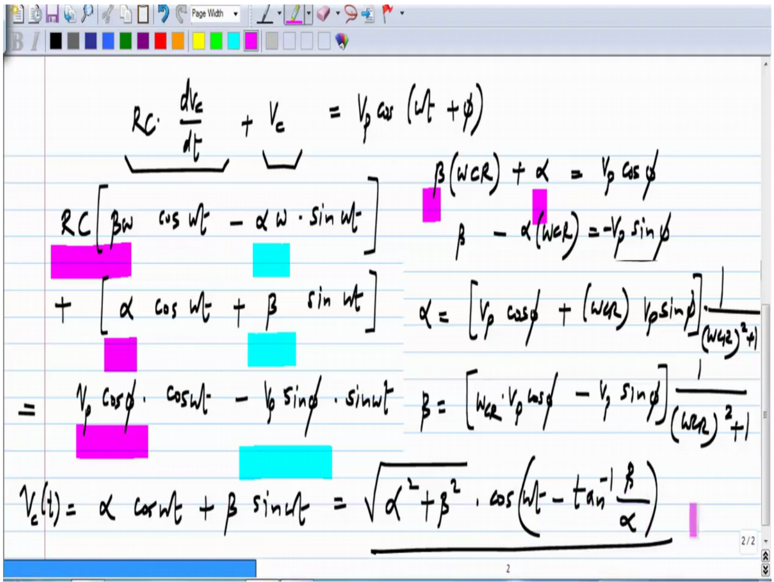

So, what we will get is this is, what we have on left side and the first of these term this will give you R C times beta omega cos omega t minus alpha omega sin omega t and this term will give plus alpha cos omega t plus beta sin omega t. And finally, on the right side we have V p cos omega t plus phi. So, that has to be equal to V p cos phi time cos omega t minus V p sin phi times sin omega t.

Now, you have cos omega and sign omega on the right hand side as well and left hand side, so the cos omega t has to be balanced by sign omega t, similarly sin omega t gas to balanced by sin omega t. So, we have two equations this plus that gives you that one that one and this plus that gives you that one. So, we have beta times omega C R plus alpha equals V p cos phi and beta minus alpha times omega C R equals V p sin phi minus V p sin phi.

We have two unknowns alpha and beta we have two equations and we can solve for this, so solve for this yourself solving these two equations will get alpha to be V p cos phi plus omega C R times V p sin phi times 1 over omega C R square plus 1 and beta is omega C R times V p cos phi minus V p sin phi and divided by omega C R square plus 1. Now, so the solution is we know it is alpha cos omega t plus beta sin omega t, which can also be written as square root of alpha square beta square that will be amplitude of the sin square times cos omega t and there will be a phase shift, which is minus tan inverse beta by alpha and these are alpha and beta.

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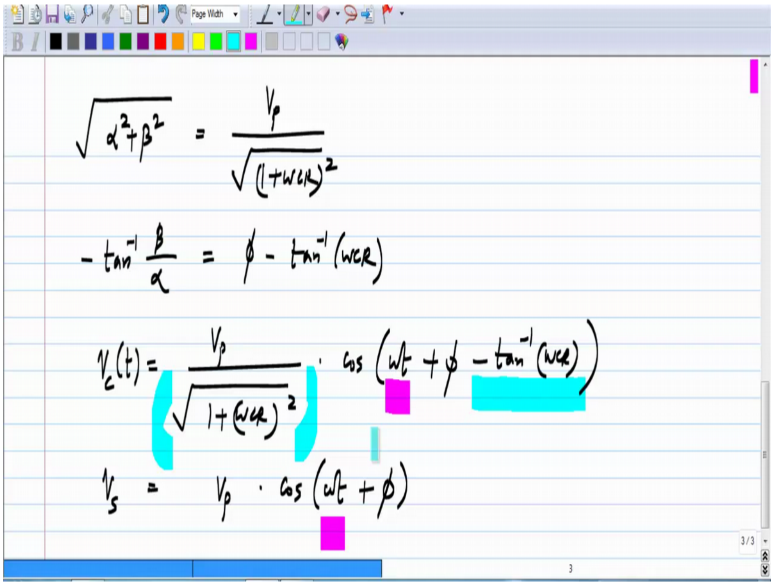


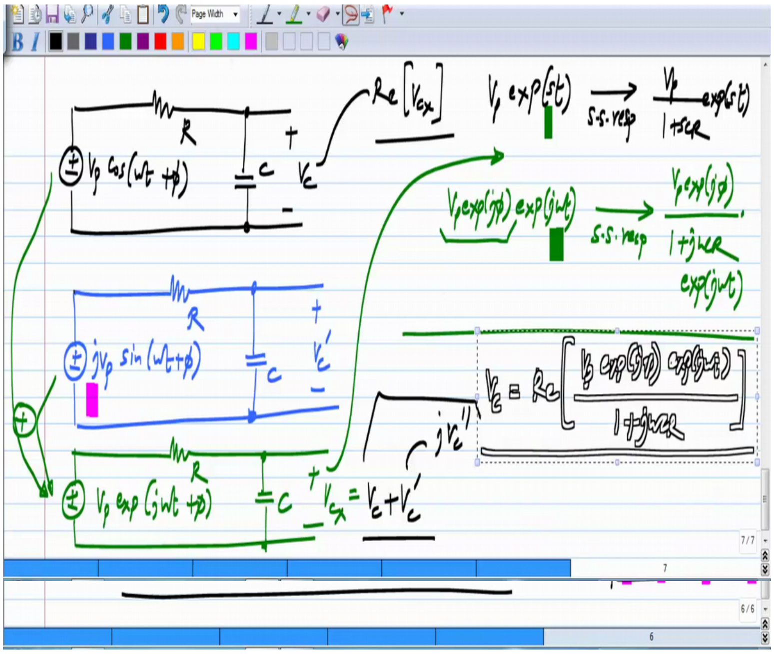
Again I will let you compute this square root of alpha square to beta square and tan inverse beta by alpha it turns out that square root of alpha square plus beta square is V p divided by square root of 1 plus omega C R square and minus tan inverse beta by alpha equals phi minus tan inverse omega C R. So, finally you will find that the steady state response is V c of t is V p by square root 1 plus omega C R square cos omega t plus phi minus tan inverse omega C R, now the input v s was V p cos omega t plus phi.

So, you can see that the output has the same frequency as the input omega and what is happened is that there is an extra phase shift the phase has been modified by that much the input phase was phi this is phi minus tan inverse omega C R the input amplitude was V p the output amplitude V p divided by some thing. So, this is in general true for any linear time variant system exited by sinusoid steady state responses will be sinusoid at the same frequency, but with the different amplitude phase.

Now, we can consider the second method using complex exponentials, which I outlined that is that if we have V p cos omega t plus phi exiting the first order system in general I will take the R C circuit as an example we want to find the steady state part of the responses. Now, we can think of this cos sin as V p by 2 exponential j omega t plus phi plus exponential minus j times omega t plus phi. Now, if we are finding out only the steady state part of response we can use super position you can think as combination of two sources exponential of plus j omega t plus phi exponential of minus j omega t plus phi.

So, if we have V p by 2 exponential j omega t plus phi it is very easy to write down steady state responses, what is V c we know that for an input of V p exponential s t the steady state responses is V p by 1 plus C R exponential s t. And it is exactly the same thing we have here the input is V p by 2 exponential j phi remember the argument here is proportional to time, so that part will isolate. So, this is equivocate to exponential j phi, which is multiple factor to exponential j omega t.

So, the steady state response corresponding to this is s here basically j omega t over there. So, we will have this part of it, which is V p by 2 exponential j phi whatever the amplitude is divided by 1 plus j omega C R times exponential j omega t. Similarly, if we have V p by 2 exponential minus j times omega t plus phi as our input, which can be written as V p by 2 exponential minus j phi exponential minus j omega t response would be V p by 2 exponential minus j phi divided by 1 minus j mega C R exp1ntial minus j omega t, so all I do is in this solution I have to put s equals minus j omega that is all. So, the steady state response is simply the sum of this and that one.



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This is simply V p by 2 exponential j phi divided by 1 plus j omega C R exponential j omega t plus V p by 2 exponential minus j phi divided by 1 minus j omega C R times exponential minus j omega t. And this part you can do the simplification yourself you will find that the answer is V p by square root of 1 plus omega C R square cos omega t plus phi minus tan inverses omega C R. Now, to be able to do this you have to flow into the usage of complex numbers in particular changing complex numbers from a polar to rectangular form and so on.

So, when you use the complex numbers you use something known as rationalization of the denominator to add up fractions. So, that you have a real number in the denominator and also you need to know rectangular to polar form conversion; that is we can express a complex number as a express j y or a exponential j theta. So, this is the real part, this is imaginary part, this is the amplitude and this is phase any of them can be used and you too use them frequently.

Now, this two methods are expected give same answer but I have already talked about the other method, which is most convenient I have V p cos omega t plus phi going into R C of an R C circuit and I want to find response V c let us imagine another circuit to, which we apply j V p sin omega t plus phi R in c and let me call this V c prime. Now, if I take a third circuit and apply V p exponential j omega t plus phi I will get some response I will call that V c x that is the steady state responses.

We see that this case here exponential j omega t plus phi is clearly a super position of these two cases. So, this V c x is nothing but, because if I add this two V p times cos omega t plus phi plus j times sin omega t plus phi at V p times exponential omega t plus phi, so V c x is simply V c plus V c time. Now, everywhere we have only real coefficient real R real C the only place, where j a square root of minus 1 appears is over them.

So, in this V c x we have V c, which is purely real and V c prime, which is purely imaginary V c prime will be j times some V c the whole prime and the answer we want to find as V c and that can be is easily found us real part of V c x. Now, the reason to do it as usual because the responses to an exponential is very easily determined for a linear differential equation. So, you do it with the particular kind of coefficient with j omega instead of sinusoid of omega and then, you find the real part and that is a solution this even easier than the previous case, where the super post two complex exponential.

We do not even need 2 1 is enough the response of this can be written down again quite easily, because we know that V p exponential s t has a steady state responses remember I am only talking about steady state responses here V p by 1 plus s C R times exponential s t. Now, this here, which is V p exponential j phi exponential j omega t as a steady state responses they cannot exponential j phi to be short of part the multiplying factor I will have V p exponential j phi divided by 1 plus j omega C R.

So, s s j omega that is all s is j omega over that, so this times exponential j omega t. So, what is V c after all V c is real part of V p exponential j phi exponential j omega t by 1 plus j omega C R like I said you have to be fluent with converting from rectangular to polar format 1 I will, so that here how it is useful, so this is the solution.

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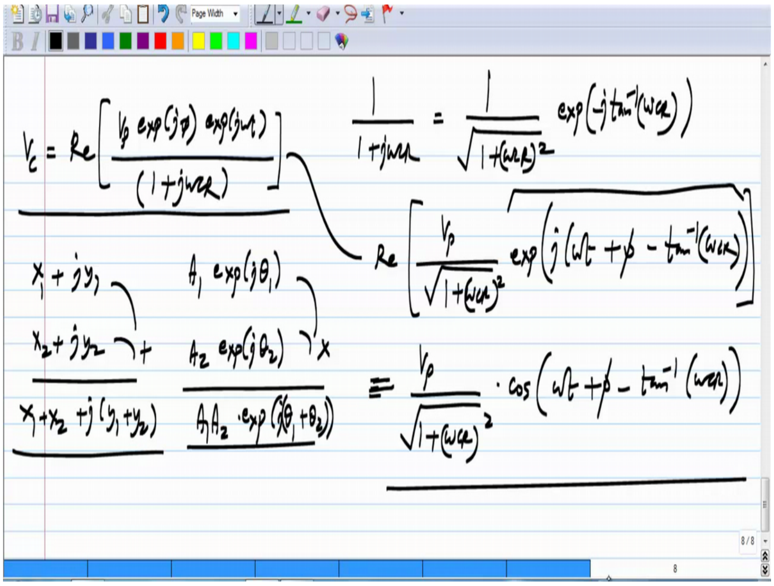

So, first of all this rectangular form right, let us say you have complex number it is most conveniences when you want to add them because the real parts adds and there must remaining parts also add with each other to give you the final real part, so addition is very easy with the rectangular form. On the other hand the polar form makes it very easy to carry out multiplication. So, the product of this two product of this 2 a 1 a 2 times exponential j theta 1 plus theta 2, so this is why you need to know both and go back and forth between them.

So, I have this part in the rectangular form I know that 1 by 1 plus j mega C R has an amplitude, which is 1 over square root 1 plus omega C R square and phase, which is because this is in the denominator minus j times tan inverses omega C R. So, if I use that in here all the amplitudes gets multiplied together after different complex numbers and all the phases get added together. So, this transfer to be real part of V p by square root of 1 plus omega C R square exponential i have j omega t j phi and minus j that inverse omega C R.

So, it will be j omega t plus phi minus tan inverse omega C R and taking a real part of something like this, this is very easy we have a single complex exponential inside, which means that the real part of it is co sin of the same argument without this j. So, this is nothing but, V p by 1 plus omega C R square times cos omega t plus phi minus tan inverses omega C R. So, again exactly the same answer as before as we should expect it is the same circuit with the same input, but this third method is most convenient. So, that why is used all the time.