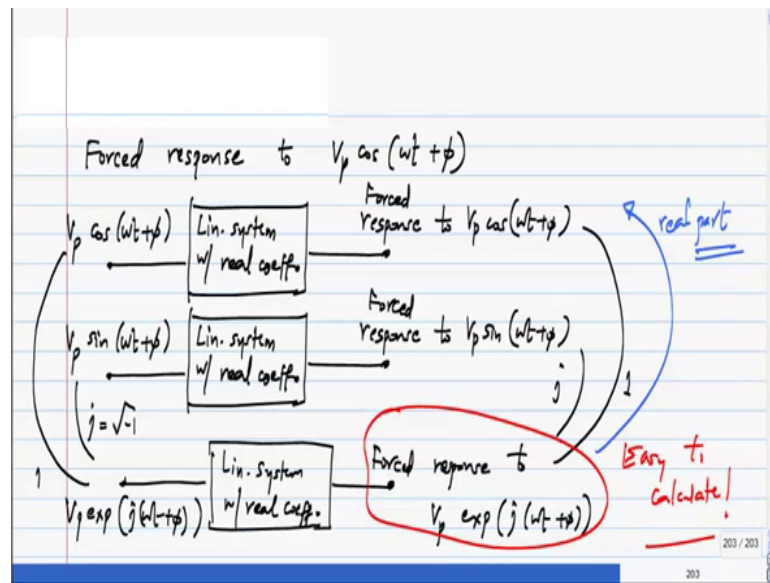


Basic Electrical Circuits
Dr Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture - 135

In the business of finding the forced response of a linear system in particular first order R C circuit to a sinusoidal signal, which can in general be represented as $\cos \omega t + \phi$ that is the sinusoidal of frequency ω and some amplitude V_p .

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So, this is the general form we will use, so for instance of the input is $\sin \omega t + \phi$ we will still think it has $V_p \cos \omega t + \phi$ by 2; that is shows the un informality later we will see that there are lots of short cuts we use we won't go through all these steps that we did putting exponential there and then, doing all those things. So, in general we came up with was that if this is fed although we took the example of an R C circuit a linear system with real coefficients linear system with real coefficients you will get some output.

I just call it response to $V_p \cos \omega t + \phi$ and if I had \sin here I would get something else to, which is response to $V_p \sin \omega t + \phi$. We also know that the forced response by the way I am talking here only of the forced response of this study state response force response to $V_p \sin \omega t + \phi$. Then, in this case what we can do is to super pose these two inputs multiplying this by 1 and this by j which is square root of minus 1 and when we do that my input becomes $V_p \exp(j \omega t + \phi)$

and it of course, goes into this same linear system with real progressions.

Clearly now, a linear system of a super position as far as the forced response is concerned. So, the response would be this times j plus that times 1 the reason for doing that is that the forced response to an exponential input is very easy to calculate we already done that, what is the forced response to an exponential input it is the same exponential input it is is the same exponential scaled by some number and that number depends on the circuit components in the circuit.

So, the point is that this is easy to calculate, so we do that, but what we really interested in is the response of $V_p \cos \omega t + 5$ that is the signal we are feeding in. Now, because the linear system had only real coefficients the only face where you get this square root of minus 1 or the j the imaginary part in by whole picture is because you multiplied it here, so to go back to this you take the real part. So, lots of analysis is based on this.

In fact, after a while its quite common especially in communication systems and so on to talk only in this complex exponential, exponential geometry you won't even here cos and sin, but you should understand, what it means the, what we are really calculating here is the response to $V_p \cos \omega t + 5$. And that can be obtained by calculating the response to $V_p e^{j\omega t} + 5$ and taking the real part reason this works is, because the linear system has real coefficients.

We had complex coefficients things could get mixed up there are actually systems like that which we won't go into at all. But, so this j which is square root of minus 1 of my respond kind of keeps the two things separately you can combine them into a exponential, but there are still separated as real and imaginary parts.

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Forced resp.

$$\frac{V_p \exp(j\omega t)}{1 + j\omega CR}$$

$$x + jy = A \exp(j\theta) = V_p \exp(j\omega t + \phi)$$

$$\frac{V_p \exp(j\omega t + \phi)}{1 + j\omega CR} = \frac{V_p \exp(j\omega t + \phi)}{\sqrt{1 + (\omega CR)^2} \cdot \exp(j \tan^{-1}(\omega CR))} = \frac{V_p \exp(j(\omega t + \phi - \tan^{-1}(\omega CR)))}{\sqrt{1 + (\omega CR)^2}}$$

$$\frac{V_p}{\sqrt{1 + (\omega CR)^2}} \cos(\omega t + \phi - \tan^{-1}(\omega CR))$$

$\text{Re}[\]$

For a particular circuit, what did we get if I apply V_s here I will get some response. So, if V_s was $V_p \exp(j\omega t)$ the forced response was, what was it. So, clearly $V_p \exp(j\omega t + \phi)$, which is the same as $V_p \cos(\omega t + \phi)$, so this whole thing in multiplying exponential $j\omega t$, so this part is like that. So, what is the response now, $V_p \exp(j\omega t)$, $1 + j\omega CR \exp(j\omega t)$ or I could put these two things together any way, which is also $V_p \exp(j\omega t + \phi)$, which is the original exponential divided by sum complex number.

Finally, $V_p \cos(\omega t + \phi)$, what is the response the real part of this and, what is that many ways to do it first of all see we are already pretty comfortable with calculations involving complex numbers there are two forms of complex numbers this is known as rectangular form, where you give the x and y coordinates of complex number or the polar form, which here should amplitude and the argument or the angle.

Now, both representations say useful clearly this is node convenient for adding complex numbers this is more convenient for multiplying complex numbers. So, if I use this here, what do I have one by one plus $j\omega CR$, what is the amplitude of this that is the amplitude and the argument is minus tan inverse ωCR there are. So, many images of doing it or you rationalize, so that you get the real denominator and then, a complex numerator and so on.

So, finally, the response to this turns out to be V_p by a square root of $1 + \omega^2 C^2 R^2$ square and $\cos(\omega t + \phi - \tan^{-1}(\omega CR))$. So, you apply a sinusoid to linear system, what you get out is sinusoid of exactly the same frequency the amplitude will be changed and the phase will be changed, how much the amplitude will be

changed and how much the phase will be changed depend on the circuit.

So, this is the amount of phase change and this is factor by, which the amplitude is changed, so that clearly depends on circuit as well as frequency. So, we can also once you have capacitors in the picture or inductors you can have a frequency selective circuits that is circuits which behave differently at different frequencies. Clearly you can see from this expression itself that if ω is very large the amplitude of the output will be very small and if ω is very small amplitude is the same as the input and very small is compared to $1/RC$.

So, if ωRC is much smaller than one the output amplitude is V_p is much more than one its inversely proportional to ω . So, we will look at these things later when we discuss sinusoidal steady state response in more detail this is known as the sinusoidal steady state response, because I mean it is the steady state response to a sinusoidal input.