Basic Electrical Circuits
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Lecture – 134

It turns out that there is an even more elegant way of finding the force response of a linear circuit to a sinusoidal input, given that we know the solution to exponential s t and we also know that these are linear circuits.

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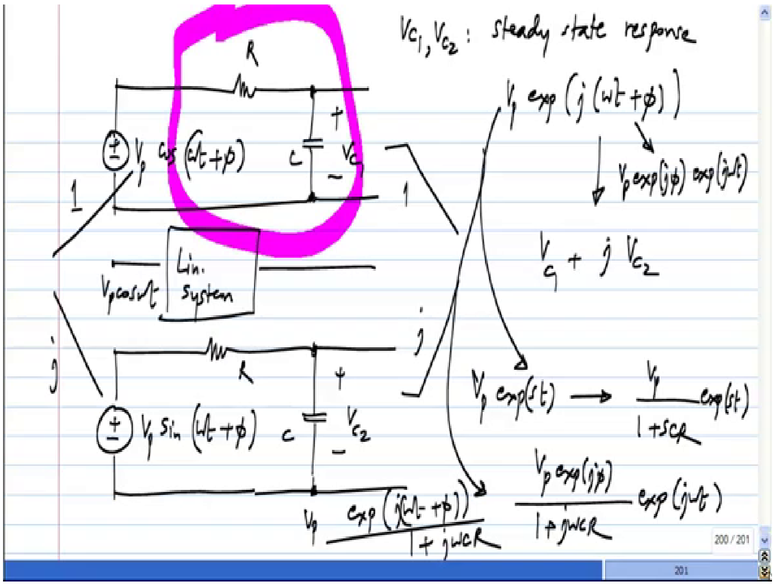

I will show an R c filter here, but this applies to any linear system of any order. I will continue with this example, but it could be, some way I could think of a general system here, where I apply V p cos omega t to some linear system. So, I will get some let say V c 1. So, here I am only considering the steady state response parts, so let say this V c 1 is the steady state and V c 2 is the steady state response. So, V c 1 is the steady state responds to V p cos omega t plus phi, V c 2 is the steady state responsively V p times sin omega t plus phi and we know that, this follows super position. So, the steady state responses follow super position.

So, what should I do now? What could I do? Let say I could weight this by some number and this by some other number and add up, that will be the response to weighting this by some number and weighting this by this number. Is not it or if I imagine this R c filter with alpha times this as the input and beta times that as the input, the steady state response will be alpha times V c 1 plus beta times V c 2. Is it part convincing? This is just straight forward super position.

So, the interesting thing is now let say this is 1 and this is j, I will take V p cos omega t plus phi plus j times V p sign omega t plus phi, this is all in your head. I mean do not worry about how we get j times V p as the voltage. So, what is the actual input that you are applying? This is equivalent to applying an input of V p exponential j omega t plus phi and the response to that is… Now, the solution to this is I already know. What is it? What is the steady state solution to V p exponential j omega t plus phi for this particular circuit?

I have two linear systems, the first one has an input V p cos omega t plus phi, second one has an input V p sin omega t plus phi, the first one has the response V c 1, second one has the response V c 2. So, if I apply to a third system, so maybe I will show those pictures.

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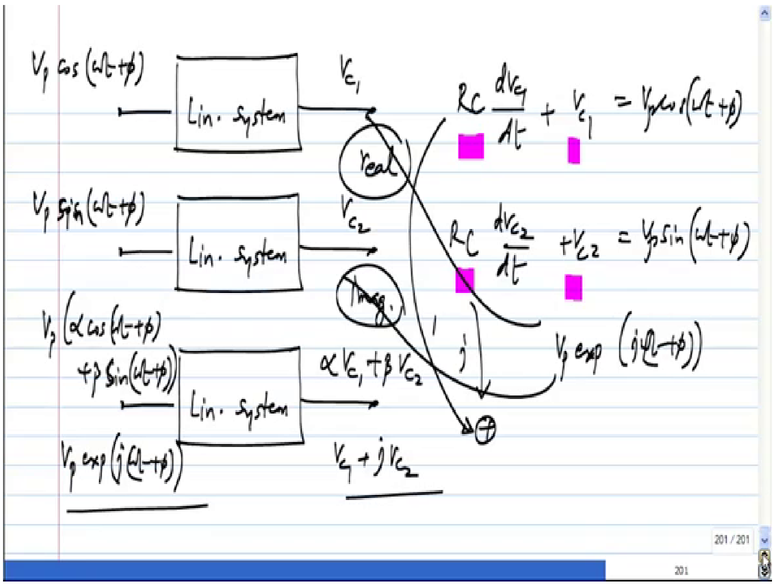

So, let say the response is V c 1 to V p cos omega t plus phi and the response is V c 2 to V p sin omega t plus phi and so if I have V p times alpha cos omega t plus phi plus beta times sin omega t plus phi. The steady state response would be alpha times V c 1 plus beta time V c 2 that is. So, clearly this falls for any alpha and beta, so I will use alpha equal to 1 and beta equals to j. So, the response to V p exponential j omega t plus phi would be V c 1 plus j times V c 2.

So, what is the response of R circuit to V p ((Refer Time: 05:10)) exponential j omega t plus phi? What is the response to V p exponential s t? What was it, the study state response? V p by 1 plus SCR exponential s t. So, this is nothing but, V p exponential j phi exponential j omega t. So, the solution to this is V p exponential j phi by 1 plus j omega C R exponential j omega t and I can put the phi back to there, I can write it as also V p exponential j omega t plus phi by 1 plus j omega C R.

The answer I already know that is the advantage here, I did the super position, the input turns out to be V p exponential j omega t plus phi and answer to that I already know, then what.

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If I write the differential equation separately, this one is R c d v c 1 by d t plus V c 1 equals V p cos omega t plus phi and R c d v c 2 by d t plus V c 2 is V p sin omega t plus phi, the key here is these coefficients are all real, if those coefficient for imaginary which is a perfectly valid linear differential equation we could not do this, then you would get cross them, I mean this can give you that and so on.

The only place we get this square root of minus 1 is where we do the super position, we multiply this by 1 and we multiply this by j and then we add it up. So, the only place where this j appears is when it multiples this one, because of that when you compute the response to V p exponential j omega t plus phi, the real part is the response here and the imagine part is the response there. The key thing here is that, this is the liner system whose differential equation has only real coefficients.

Now of course, any circuit will follow that because all our component values are real, but it is possible to construct differential equations with I mean I can just put a j here, we can find the solution to that. But, then you cannot say that the real part is coming from there, because you could also get a complex part just because of j that inside the system. So, this is the real part and this is the imaginary part, is this fine. So, this is just a way of superposing cos and sin to get complex exponential and the solutions to complex exponential in general are simpler, algebraically compare to the solutions to sinusoidal.

There is the same solution of course, in a spirit, but the formulas look a little more messy, because I had this alpha cos omega t plus beta sin omega t and then I have to find constants and so on. Now, this is an easier way of doing it of course, while doing this you will get the same answer. So, I strongly urge you to try all three methods, this is first of all a very, very simple circuit. So, you put a solution as alpha cos plus beta sin, find the constants, you decompose the cos as exponential plus j omega exponential minus j omega, find the solution. And finally, you do this you will get the solution. What is the solution then? What is V c 1((Refer Time: 08:44)? What is the force response? What is the real part of this?

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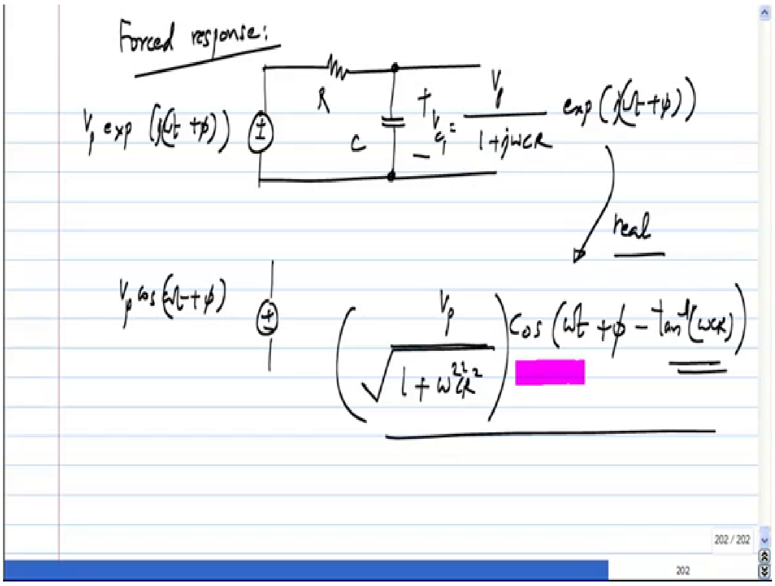

If this is the source R and C, the solution is V p by 1 plus j omega c R exponential j omega t plus phi, this is the force response again we are only talking about the force response here and if you had V p cos omega t plus phi as the excitation, the response would be the real part of this. So, what is the real part of this?

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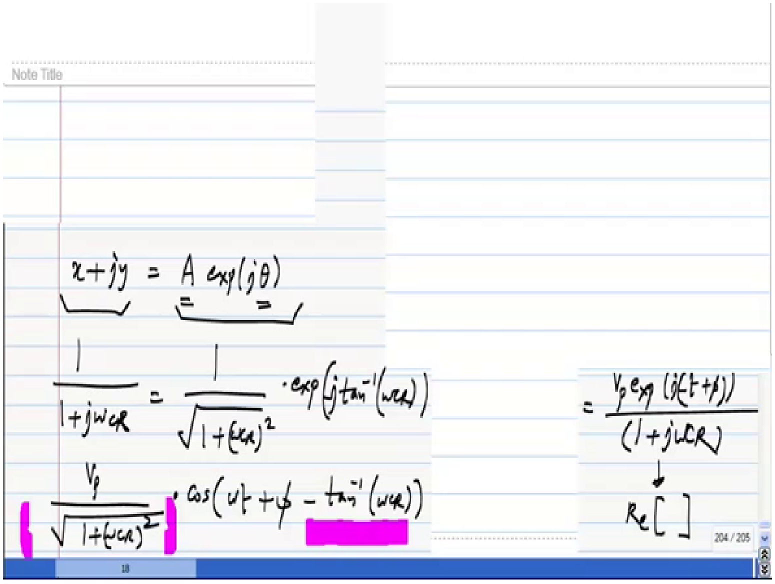

The real part of this and what is that many ways to do it, first of all I assume we are already pretty comfortable with calculations involving complex numbers, there are two forms of complex numbers, this is not as a rectangular form, where review the x and y coordinates of the complex number or the polar form it is here to amplitude and the argument or the angle.

Now, both representations say useful clearly this is more convenient for adding complex numbers, this is more convenient of multiplying complex numbers. So, if I use this here what do I have 1 by 1 plus j omega c R. What is the amplitude of this?

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Amplitude.

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And that is the amplitude and the argument is minus tan inverse omega c R. There are so many ways of doing it. ((Refer Time: 10:52)) So, you get the real denominator and then a complex numerator and so on. So, finally, the response to this turns out be V p by square root of 1 plus omega c R square and cos of omega t plus phi minus time inverse omega c R.

So, you apply a sinusoidal to a linear system, what you get out with this, a sinusoidal with exactly the same treatment c, the amplitude will be deep changed and the face will be change. How much the amplitude will be change and then how much the face will be change depend on the circuit? So, this is the amount of face says and this is the factor by which the amplitude is change. So, that clearly depends on the circuit as well as frequency.

So, you can also once you have capacitors into the picture or inductors you can have frequency selective circuits that is circuits which behave differently and different frequencies. Clearly you can see from this expression itself that the omega is very large the amplitude of the output will be very small and if omega is very small, amplitude is the same as the input and very small is compare to 1 by R c. So, if omega c had is much smaller then 1 the output amplitude is V p is much smaller then 1 it inversely proportional to omega ((Refer Time: 12:20)).

So, the response the real part of this as a cosine cos amplitude is V p divided by square root of 1 plus omega square c square R square and whose argument is omega t plus phi minus tan inverse omega c R. Most important thing you notice here is that if you apply a sinusoidal to a linear system, the force response will be a sinusoidal of the exactly the same frequency, you cannot generate any new frequency from the linear system, you applied cos of omega t you are only going to get cos omega t, it is amplitude is change it is face is change that is all. But, this is the very, very important accept of linear systems that will be useful for everything both for analysis and measurement and testing.